Enter your solutions in the space provided or on the back page!
Papers that do not comply with this rule will not be marked!

STUDENT #: ____________________

NAME: ________________________

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/1331 FALL 2018
Dr. A. Czajkowski

Assignment 5: ENTROPY

1. An ice tray contains 500 g of liquid water at 0°C. Calculate the change in entropy of the water as it freezes slowly and completely at 0°C. (It is a one-line problem so please fit this into the space provided here)

$$\Delta S = \frac{mL}{T} = \frac{0.5 \cdot (3.3 \cdot 10^5)}{273}\frac{J}{K} = 604.40\frac{J}{K}$$

2 A 1.00-kg iron horseshoe is taken from a forge at 900°C and dropped into 4.00 kg of water at 10.0°C. Assuming that no energy is lost by heat to the surroundings, determine the total entropy change of the horseshoe-plus-water system.

Q3 SOLUTION:

$c_{\text{iron}} = 448 \text{ J/kg} \times {}^\circ\text{C}$; $c_{\text{water}} = 4\,186 \text{ J/kg} \times {}^\circ\text{C}$ since $Q_{\text{cold}} = -Q_{\text{hot}}$:

we have $4.00\text{ kg}(4\,186 \text{ J/kg}\times{}^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00\text{ kg})(448 \text{ J/kg}\times{}^\circ\text{C})(T_f - 900^\circ\text{C})$

which yields $T_f = 33.2^\circ\text{C} = 306.2\text{ K}$

$$\Delta S = \int_{283\text{ K}}^{306.2\text{ K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1173\text{ K}}^{306.2\text{ K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1173}\right)$$

$$\Delta S = (4\,186 \text{ J/kg}\cdot\text{K})(4.00\text{ kg})(0.078\,8) + (448 \text{ J/kg}\cdot\text{K})(1.00\text{ kg})(-1.34)$$

$$\Delta S = \boxed{718 \text{ J/K}}$$

3 What change in entropy occurs when a 27.9-g ice cube at –12°C is transformed into steam at 115°C?
We assume a constant specific heat for each phase. As the ice is warmed from –12°C to 0°C, its entropy increases by:

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\text{ K}}^{273\text{ K}} \frac{m c_{\text{ice}} dT}{T} = m c_{\text{ice}} \int_{261\text{ K}}^{273\text{ K}} T^{-1} dT = m c_{\text{ice}} \ln T\Big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S = 0.027\,0\text{ kg}(2\,090 \text{ J/kg}\cdot{}^\circ\text{C})(\ln 273\text{ K} - \ln 261\text{ K}) = 0.027\,0\text{ kg}(2\,090 \text{ J/kg}\cdot{}^\circ\text{C})\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.54 \text{ J/K}$$

As the ice melts its entropy change is $\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{0.027\,0\text{ kg}(3.33\times 10^5 \text{ J/kg})}{273\text{ K}} = 32.9 \text{ J/K}$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{m c_{\text{liquid}} dT}{T} = m c_{\text{liquid}} \ln\left(\frac{T_f}{T_i}\right) = 0.027\,0\text{ kg}(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{373}{273}\right) = 35.3 \text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + m c_{\text{steam}} \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.027\,0\text{ kg}(2.26\times 10^6 \text{ J/kg})}{373\text{ K}} + 0.027\,0\text{ kg}(2\,010 \text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{388}{373}\right) = 164 \text{ J/K} + 2.14 \text{ J/K}$$

The total entropy change is $(2.54 + 32.9 + 35.3 + 164 + 2.14) \text{ J/K} = \boxed{236 \text{ J/K}}$.

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Assignment 5: ENTROPY

4 A heat engine operates between two reservoirs at T_2 = 600 K and T_1 = 3 50 K. It takes in 1 000 J of energy from the higher-temperature reservoir and performs 250 J of work. Find (a) the entropy change of the Universe ΔS_U for this process and (b) the work W that could have been done by an ideal Carnot engine operating between these two reservoirs. (c) Show that the difference between the amounts of work done in parts (a) and (b) is $T_1 \Delta S_U$.

a) $\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{-1000}{600} + \frac{750}{350}\frac{J}{K} = 0.476\frac{J}{K}$

b) e $= \frac{|W|}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$ so for the Carnot engine $e = 1 - \frac{|T_c|}{|T_h|} = \frac{5}{12}$ and thus

$W = \frac{5}{12}1000J = 416.667J \Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{-1000}{600} + \frac{583.33}{350}\frac{J}{K} = 0\frac{J}{K}$

Work difference is 166.67J, while $T_c \Delta S = T_1 \Delta S = 350(0.47618) = 166.7\,J$

5 a) Find the speed of the fastest N_2 molecule that can be found in 10 moles of air at 20C.
We use the Maxwell-Boltzmann for 7.8mole of N_2 and dv=1m/s

$N_v = NP_v = 0.78 \cdot 10 \cdot P_v = 7.8 \cdot 4\pi \cdot \left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}} dv$

Change v until N_v=1

Ans: v_{max}=3109(1)m/s

b) Express this speed in terms of v_{mp} at this temperature

$v_{rms}(N_2, 293K)$=417m/s, v_{max}=/ v_{rms} =7.46

6 Question 6 is supposed to be solved/answered individually – Help Centre has been instructed not to help students with it! Please treat this question as a puzzle – I invoke an honor system!

a) The photon of energy of 5eV is absorbed by the 50000 liters tank of water at temperature of 20C. What is resulting change in the entropy?

The tank will not change its temperature (293K) as result of absorbing the Q=5eV=8x10^{-19}J of heat.
The entropy change is thus given by:

$$\Delta S = \frac{Q}{T} = \frac{8}{293} \cdot 10^{-19}\frac{J}{K} = 2.73 \cdot 10^{-21}\frac{J}{K}$$

b) As result of certain process the number of possible microstates realizing certain microstate changed from the initial value of 10^{27} to the final value of 10^{36}. What was the change of the entropy?

$$\Delta S = k\ln W_f - k\ln W_i = k\ln\frac{W_f}{W_i} = \frac{8.31}{6.02} \cdot 10^{-23} \ln 10^9 = 2.86 \cdot 10^{-22}\frac{J}{K}$$