

ASSIGNMENT 3:

First Law of Thermodynamics,
Heat and Work in Gas Processes
Kinetic Theory of Gases

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/31 Fall 2018
Dr. A. Czajkowski

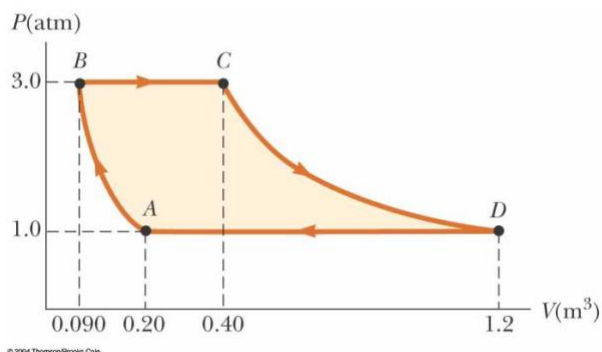
Released: Sept 28,

Due: Oct 5

6PM

STUDENT #: _____
NAME: _____

1 A sample of a gas goes through the process shown. From A to B, the process is adiabatic; from B to C, it is isobaric with 100 kJ of energy entering the system by heat. From C to D, the process is isothermal; from D to A, it is isobaric with 150 kJ of energy leaving the system by heat. Determine the difference in internal energy $E_{int,B} - E_{int,A}$.



$$W_{BC} = -P_B (V_C - V_B) = -3.00 \text{ atm} (0.400 - 0.090) \text{ m}^3$$

$$= -94.2 \text{ kJ}$$

$$\Delta E_{int} = Q + W$$

$$E_{int,C} - E_{int,B} = (100 - 94.2) \text{ kJ}$$

$$E_{int,C} - E_{int,B} = 5.79 \text{ kJ}$$

Since T is constant $E_{int,D} - E_{int,C} = 0$

$$W_{DA} = -P_D (V_A - V_D) = -1.00 \text{ atm} (0.200 - 1.20) \text{ m}^3$$

$$= +101 \text{ kJ}$$

$E_{int,A} - E_{int,D} = -150 \text{ kJ} + (+101 \text{ kJ}) = -48.7 \text{ kJ}$ since change of internal energy for the whole cycle is 0, we can write:

$$E_{int,B} - E_{int,A} = -[(E_{int,C} - E_{int,B}) + (E_{int,D} - E_{int,C}) + (E_{int,A} - E_{int,D})] = -(5.79 + 0 - 48.7) \text{ kJ} = 42.91 \text{ kJ}$$

2 A) One mole of an ideal gas is heated slowly so that it goes from the PV state (P_0, V_0) , to $(3P_0, 3V_0)$, in such a way that the pressure is directly proportional to the volume. How much work is done on the gas in the process?

$$|W| = 2P_0V_0 + \frac{1}{2}(2P_0)(2V_0) = 4P_0V_0$$

Ans: $W = -4P_0V_0$

B) As a 1.00-mol sample of a monatomic ideal gas expands adiabatically, the work done on it is -2500 J . The initial temperature and pressure of the gas are 500 K and 3.60 atm . Calculate (iii) the final temperature, and (iv) the final pressure.

(iii) $W = nC_V(T_f - T_i)$ so that $-2500 \text{ J} = 1 \text{ mol} \cdot \frac{3}{2} \cdot 8.314 \text{ J/mol} \cdot \text{K} (T_f - 500 \text{ K})$ and $T_f = \boxed{300 \text{ K}}$

(iv) $P_i V_i^\gamma = P_f V_f^\gamma$ and thus $P_i \left(\frac{nRT_i}{P_i}\right)^\gamma = P_f \left(\frac{nRT_f}{P_f}\right)^\gamma$ $T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$

$$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f} \text{ and } P_f = P_i \left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)} = 3.60 \text{ atm} \left(\frac{300}{500}\right)^{5/2} = \boxed{1.00 \text{ atm}}$$

3 Use the ideal gas equation to fill the alternative expressions for work and heat below:

There are many correct entries—here are few examples (obtained using $pV=nRT$ in substitutions)

Process	Work		Heat	
	V=const	0	0	$nC_V \Delta T$
P=const	$-p(V_f - V_i)$	$-nR(V_f - V_i)$	$nC_p \Delta T$	$\frac{C_p p(V_f - V_i)}{R}$
T=const	$-nRT \ln \frac{V_f}{V_i}$	$-nRT \ln \frac{p_i}{p_f}$	$nRT \ln \frac{V_f}{V_i}$	$nRT \ln \frac{p_i}{p_f}$
Q=0	$\frac{1}{\gamma-1}(p_f V_f - p_i V_i)$	$\frac{nR}{\gamma-1}(T_f - T_i)$	0	0

4 Using the approach demonstrated during the lecture show that for $pV^\gamma = \text{const.}$ for adiabatic gas process. (Present your derivation on the opposite site of this page). DETAILS OF THIS CALCULATION WERE GIVEN IN LECTURE.

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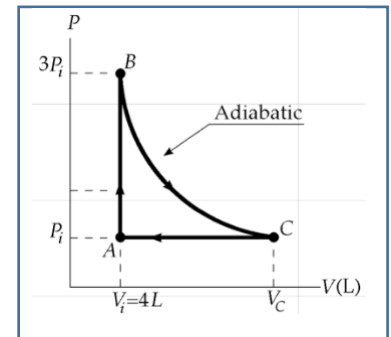
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5 A 4 liter sample of a diatomic gas with $\gamma=1.4$ confined to a cylinder, is carried through a closed cycle. The gas is initially at $p=1.00\text{atm}$ and $T=200\text{K}$. First, its pressure is tripled under constant volume. Then it expands adiabatically to its original pressure. Finally the gas is compressed isobarically to its original volume.

- draw pV diagram of this cycle
- determine the volume of the end of the adiabatic expansion
- find the temperature of the gas at the start of the adiabatic expansion
- find the temperature at the end of the cycle
- what was the net work done on the gas for this cycle



(a) See the diagram at the right.

(b) $P_B V_B^{\gamma} = P_C V_C^{\gamma}$ so that $3P_i V_i^{\gamma} = P_i V_C^{\gamma}$

so that $V_C = (3^{1/\gamma}) V_i = (3^{5/7}) V_i = 2.19 V_i$ $V_C = 2.19(4.00 \text{ L}) = \boxed{8.77 \text{ L}}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$ which gives $T_B = 3T_i = 3(200\text{K}) = 600\text{K}$

(d) After one whole cycle, $T_A = T_i = 200\text{K}$

(e) Using the expressions for work (THE TABLE!), we get:

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 0 + \frac{1}{\gamma - 1} (p_C V_C - p_B V_B) - p_i (V_A - V_C)$$

$$W_{ABCA} = \frac{1}{\frac{5}{2} - 1} (p_i (3^{5/7} V_i) - (3p_i) V_i) - p_i (V_i - (3^{5/7} V_i)) = 818.7\text{J} + 483.5\text{J} = -335.5\text{J}$$

(e) USING THE EXPRESSIONS FOR HEAT:

$$Q_{AB} = nC_V dV = n \left(\frac{5}{2} R \right) (3T_i - T_i) = (5.00) nRT_i, \quad Q_{BC} = 0 \text{ as this process is adiabatic}$$

$$P_C V_C = nRT_C = P_i (2.19 V_i) = (2.19) nRT_i \text{ so } T_C = 2.19 T_i,$$

$$Q_{CA} = nC_P dT = n \left(\frac{7}{2} R \right) (T_i - 2.19 T_i) = (-4.17) nRT_i$$

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17) nRT_i = (0.829) nRT_i$$

$$(DE_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$

For the whole cycle

$$W_{ABCA} = -Q_{ABCA} = -(0.829) nRT_i = -(0.829) P_i V_i$$

$$W_{ABCA} = -(0.829) (1.013 \times 10^5 \text{ Pa}) (4.00 \times 10^{-3} \text{ m}^3) = \boxed{-336 \text{ J}}$$

6 Given is distribution of speeds of cars at 417 Highway as measured by OPP.

- Is this a discrete or continuous distribution?
- Find the V_{mp} , V_{rms} , V_{avg} .
- Find the probability that a randomly picked car will have speed larger than 125km/h.
- Find the probability that a randomly picked car will have speed larger than 85km/h and less than 115km/h.

- discrete
- $V_{mp}=120\text{km/h}$; $V_{rms}=127\text{km/h}$; $V_{avg}=125\text{km/h}$
- $P(v>125) = 0.49$
- $P(85<v<115) = 0.30$

