Université d'Ottawa
Faculté de génie

École de science informatique
et de génie électrique

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L'Université canadienne
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University of Ottawa
Faculty of Engineering

School of Electrical Engineering
and Computer Science

ELG3106 Applied Electromagnetism **Professor:** H. Schriemer **Date:** Nov. 2nd, 2013
ELG3506 Électromagnétisme Appliqué **Professeure :** K. Hinzer **Date:** 2 nov. 2013

Instructions

- Alloted time : 80 minutes
- Closed book exam. Useful formulas are listed at the end of the exam.
- Programmable calculators are not authorized.
- Answer all questions. Explain clearly how you got to the final answer. Solutions without adequate will not be considered. Do not forget to indicate units.
- The mark for each question is indicated.
- Give back ALL material to the professor.

Instructions

- Temps alloué : 80 minutes
- Examen à livres fermés. Des formules utiles sont fournies à la fin de l'examen.
- Calculatrices pré-programmables non autorisées.
- Répondre à toutes les questions. Expliquer clairement comment vous êtes arrivés à la solution finale. Les solutions sans justification adéquate ne seront pas prises en compte. Ne pas oublier de donner les unités.
- Les valeurs des questions sont telles qu'indiquées.
- Remettre TOUT le matériel de la professeure.

Q1/	**/ 8**
Q2/	**/ 10**
Q3/	**/ 2**

Final Mark / Note finale:

/20

Question 1

A uniform plane wave whose electric field is of the form

Une onde plane uniforme dont le champ électrique est donné par

$$\mathbf{E}(z,t) = \hat{\mathbf{a}}_x 7.5 e^{-0.004z} \cos\left(0.3\pi \times 10^8 t - 0.5\pi z\right) \quad \text{V/m}$$

is propagating in a homogeneous isotropic nonmagnetic medium.
se propage dans un milieu homogène, isotrope et non magnétique.

(a) Write the electric field in phasor notation.

Écrivez le champ électrique sous forme de phaseur.

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{a}}_x 7.5\, e^{-0.004z} e^{-j0.5\pi z} = \hat{\mathbf{a}}_x E_0\, e^{-\gamma z}$$

Therefore

$E_0 = 7.5$ V/m

$\gamma = \alpha + \beta \quad \Rightarrow \quad \alpha = 0.004$ Np/m $\quad \beta = 0.5\pi$ rad/m

(b) What are the wave frequency, the wavelength, and the phase velocity?

Quelles sont la fréquence, la longueur d'onde et la vitesse de phase de l'onde ?

$$\omega = 2\pi f = 0.3\pi \times 10^8 \text{ rad/s} \quad \Rightarrow f = 0.15 \times 10^8 = 15 \text{ MHz}$$

$$\beta = \frac{2\pi}{\lambda} = 0.5\pi \text{ rad/m} \quad \Rightarrow \quad \lambda = \frac{2\pi}{0.5\pi} = 4 \text{ m}$$

$$u_p = \frac{\omega}{\beta} = \frac{0.3\pi \times 10^8}{0.5\pi} = 0.6 \times 10^8 \text{ m/s}$$

(c) What is the intrinsic impedance of the medium?

Quelle est l'impédance intrinsèque du milieu ?

$$\beta = \omega\sqrt{\mu\varepsilon} \quad \Rightarrow \quad \varepsilon = \frac{\beta^2}{\mu\omega^2} = \frac{(0.5\pi)^2}{(4\pi\times10^{-7})(0.3\pi\times10^{8})^2} = 2.21\times10^{-10} \text{ F/m}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{4\pi\times10^{-7}}{2.21\times10^{-10}}} = 75.4\,\Omega$$

(d) What is the conductivity of the medium? Is the medium a low-loss medium, a good conductor, or somewhere in between? Justify your answer.

Quelle est la conductivité du milieu ? Est-ce un milieu à faibles pertes, un bon conducteur ou entre les deux ? Justifiez votre réponse.

It is not a good conductor because $\alpha \neq \beta$

It is not lossless because $\alpha \neq 0$

Assume it is low-loss and check:

$$\alpha = 0.5\sigma\sqrt{\mu/\varepsilon} \quad \Rightarrow \quad \sigma = 2(0.004)\sqrt{\frac{2.21\times10^{-10}}{4\pi\times10^{-7}}} = 1\times10^{-4}$$

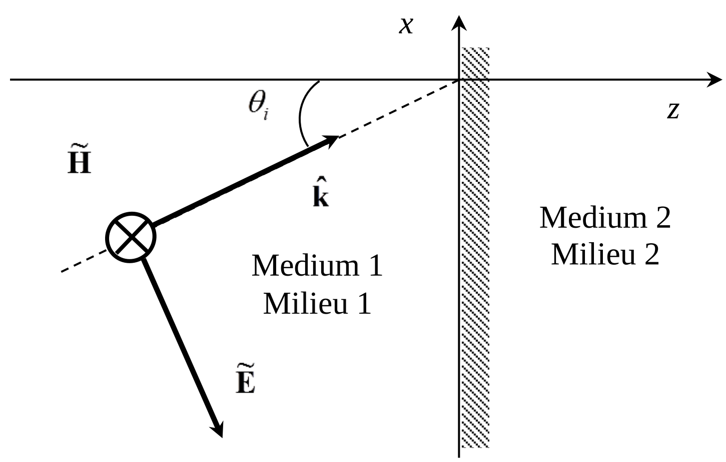
$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\varepsilon\omega} = \frac{1.06\times10^{-4}}{(2.21\times10^{-10})(0.3\pi\times10^{8})} = 5.1\times10^{-3} \ll 1$$

Yes, it is a low-loss medium.

Question 2

Consider a uniform plane wave obliquely incident from medium 1 onto medium 2, as shown below; the incident angle is θ_i = 30°. The constitutive parameters of medium 1 are ε_r = 4 and μ_r = 16, while those of medium 2 are ε_r = 9 and μ_r = 27. The magnitude of the incident magnetic field is H_{io} = 0.1 A/m, and its direction is $-\hat{\mathbf{a}}_y$ as shown in the figure.

Soit une onde plane uniforme obliquement incidente d'un milieu 1 à un milieu 2 (voir figure ci-dessous). L'angle d'incidence est θ_i = 30°. Les paramètres du milieu 1 sont ε_r = 4 et μ_r = 16, tandis que ceux du milieu 2 sont ε_r = 9 et μ_r = 27. L'amplitude du champ magnétique incident est H_{io} = 0.1 A/m et sa direction est $-\hat{\mathbf{a}}_y$ comme montré sur la figure.



(a) Is this wave transverse electric or transverse magnetic? Justify your answer.
Cette onde est-elle transverse électrique ou magnétique ? Justifiez votre réponse.

This is a TM wave, since **H** is normal to the plane of incidence

(b) Find the transmitted angle.

Déterminez l'angle transmis.

$$\sqrt{\mu_1\varepsilon_1}\sin\theta_i = \sqrt{\mu_2\varepsilon_2}\sin\theta_t$$

$$\Rightarrow \sin\theta_t = \sin\left(30^\circ\right)\sqrt{\frac{\mu_{r1}\varepsilon_{r1}}{\mu_{r2}\varepsilon_{r2}}} = (0.5)\sqrt{\frac{(16)(4)}{(27)(9)}} = (0.5)(0.5132) = 0.2566$$

Therefore $\theta_t = 14.9^\circ$

(c) Find the reflection and transmission coefficients.

Déterminez les coefficients de réflexion et de transmission.

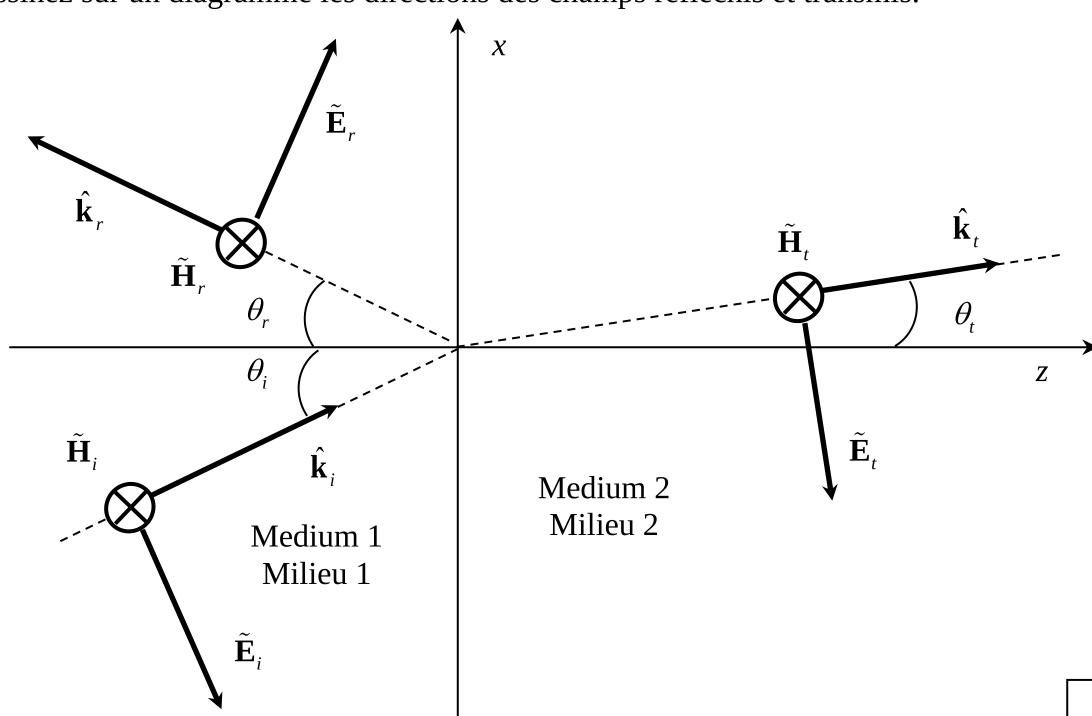
$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{\sqrt{27/9}\cos(14.9^\circ) - \sqrt{16/4}\cos(30^\circ)}{\sqrt{27/9}\cos(14.9^\circ) + \sqrt{16/4}\cos(30^\circ)}$$

$$= \frac{(1.7321)(0.9665)-(2)(0.866)}{(1.7321)(0.9665)+(2)(0.866)} = \frac{1.6741-1.732}{1.6741+1.732} = \frac{-0.0579}{3.4061} = -0.017$$

$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{2\sqrt{27/9}\cos(30^\circ)}{\sqrt{27/9}\cos\theta_t + \sqrt{16/4}\cos\theta_i} = \frac{3}{3.4061} = 0.88$$

(d) Sketch the reflected and transmitted field directions on a diagram.

Dessinez sur un diagramme les directions des champs réfléchis et transmis.



Question 3

A plane wave in air with

Une onde plane dans l'air ayant

$$\mathbf{E}^i = \hat{\mathbf{y}} 20 e^{-j(3x+4z)} \quad \text{V/m}$$

$$\Gamma_\perp = -0.41$$

$$\theta_i = 36.87°$$

is incident upon the planar surface of a dielectric material, with $\varepsilon_r = 4$, occupying the half space $z \geq 0$.

est incidente sur une surface plane d'un diélectrique, ayant $\varepsilon_r = 4$, occupant the demi- espace $z \geq 0$.

(a) Find the reflected electric field in phasor form.

Trouvez l'expression du champ électrique réfléchi en forme phaseur.

Since/puisque $\mathbf{E}^r_\perp = \hat{\mathbf{y}} \mathbf{E}^r_{\perp 0} e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$ V/m

and using the relation/et employant la relation $E^r_0 = \Gamma_\perp E^i_0$

$$\mathbf{E}^r = -\hat{\mathbf{y}} 8.2 e^{-j(3x-4z)}$$

Here we use the fact that $\theta_i = \theta_r$ and that the z-direction has been reversed.

(b) Find the reflected magnetic field in phasor form

Trouver l'expression du champ magnétique réfléchi décrite en forme phaseur.

$$\mathbf{H}^r = -(\hat{\mathbf{x}}\cos\theta_i + \hat{\mathbf{z}}\sin\theta_i)\frac{8.2}{\eta_0} \ \mathrm{e}^{-j(3x-4z)}$$

$$\mathbf{H}^r = -(\hat{\mathbf{x}} 17.4 + \hat{\mathbf{z}} 13.06) \ \mathrm{e}^{-j(3x-4z)} \quad \text{mA/m}$$

Extra space / Espace supplémentaire

Equation sheet / Page de formules

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma|\, e^{j\theta_\Gamma} \qquad S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \nabla \cdot \mathbf{D} = \rho_v \qquad \nabla \cdot \mathbf{B} = 0$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad k = \omega\sqrt{\mu\varepsilon}$$

$$\Gamma_\perp = \Gamma_{TE} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad \tau_\perp = \tau_{TE} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\Gamma_{||} = \Gamma_{TM} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \tau_{||} = \tau_{TM} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$

Dans l'air/ in air $\eta = 120\,\pi = 377\ \Omega$

$$\varepsilon_0 = 8.85\times10^{-12} \simeq \frac{1}{36\pi}\times10^{-9}\ \text{F/m} \qquad \varepsilon = \varepsilon_r\varepsilon_0$$

$$\mu_0 = 4\pi\times10^{-7}\ \text{H/m}$$

$$\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega}$$

$$\nabla^2\mathbf{E} + k_c{}^2\mathbf{E} = 0 \qquad \nabla^2\mathbf{H} + k_c{}^2\mathbf{H} = 0 \qquad k_c = \omega\sqrt{\mu\varepsilon_c} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\mathbf{S} = \mathbf{E}\times\mathbf{H}$$

$$\tilde{\mathbf{S}}_{av} = \tfrac{1}{2}\,\text{Re}[\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*]\ (\text{W/m}^2) \qquad \tilde{\mathbf{S}}_{av}^i(z) = \hat{\mathbf{z}}\,\frac{|E_{i0}|^2}{2\eta_1^*}$$

$$\tilde{\mathbf{S}}_{1av}(z) = \hat{\mathbf{z}}\,\frac{|E_{i0}|^2}{2\eta_1^*}\left(1-|\Gamma|^2\right) \qquad \tilde{\mathbf{S}}_{2av}(z) = \hat{\mathbf{z}}\,\frac{|\tau|^2\,|E_{i0}|^2}{2\eta_2^*}$$

$$\vec{H} = \frac{1}{\eta}\vec{a}_k\times\vec{E} \qquad \lambda = \frac{2\pi}{k} \qquad \omega = 2\pi f \qquad \Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$

$$S = \frac{|\tilde{\mathbf{E}}_i|_{\max}}{|\tilde{\mathbf{E}}_i|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad \tau = 1+\Gamma$$

$$l_{\max} = \frac{\theta_r\lambda_1}{4\pi} + n\frac{\lambda_1}{2}$$ **(n=0,1,2….) où** $\theta_r = 0$ **si** $\eta_1 > \eta_2$ **et** $\theta_r = \pi$ **si** $\eta_1 < \eta_2$

	General Case *Cas général*	*Lossless* *Milieu sans pertes*	*Low loss* *Milieu à faibles pertes*	*Good conductor* *Bon conducteur*
α (Np/m)	$\omega\left\{\frac{1}{2}\mu\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right\}^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f\mu\sigma}$
β (rad/m)	$\omega\left\{\frac{1}{2}\mu\varepsilon'\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f\mu\sigma}$
η_c (Ω)	$\sqrt{\frac{\mu}{\varepsilon'}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$
u_p (m/s)	$\frac{\omega}{\beta}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\sqrt{\frac{4\pi f}{\mu\sigma}}$