

**ADM 2304 – Final Examination**  
**APPLIED STATISTICAL METHODS IN BUSINESS**  
**Winter 2014**

PRINT YOUR NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

SECTION (circle one): M    N    P    Q    R    S

**Exam:**                    **9 pages (including cover page)**

**Appendices:**            **9 pages of Minitab output and 4 pages of statistical tables**

***Complete all tests with hypotheses, test statistic, critical value, decision, and conclusion. Use the 0.05 significance level unless otherwise indicated. Explain your answers if asked.***

Calculators and one sheet of notes (8.5 x 14 in.) are allowed.

Please hand back all exam materials (exam booklet, appendices and tables), but keep your personal sheet of notes for future reference.

Question	Value	Mark
1	6	
2	8	
3	4	
4	10	
5	15	
6	22	
<b>Total</b>	<b>65</b>	

**Statement of Academic Integrity**

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: \_\_\_\_\_

**Question 1. [ 6 marks ]**

A shortage of long term care capacity has long hindered our health care system from providing appropriate care to Canadian citizens. One issue is whether the age of entry into long term care is related to the living arrangements of residents before entry. The following data were compiled:

	<60	60-69	70-79	80-89	90-99	>100	Totals
Alone	14	48	140	524	278	12	1016
Non-Private Residence	16	28	98	285	192	8	627
Spouse/Life Partner	22	99	374	783	232	4	1514
With Family	37	32	127	311	209	10	726
<b>Totals</b>							

The data outlines four potential pre-admission living arrangements – alone, non-private residence, living with spouse and living with family. The columns represents the age of residents at time of admission. Using the partially completed output in the Appendix A, perform the appropriate hypothesis test to determine if there is indeed a relationship between the age of entry and the living arrangement prior to admission. Use a 5% significance level. (You should show the calculation of the missing chi-square contribution.)

**Question 2. [ 8 marks]**

- (a) Using the same data, test whether the proportions of residents entering long term care before the age of 70 are different for those living alone versus those living with family. Use a 5% significance level and the critical value approach.

[4]

- (b) Calculate a 95% confidence interval for the difference between the proportion of residents entering long term care before the age of 70 among those living alone and the proportion of residents entering before the age of 70 among those living with family. Does this confirm your conclusion in part (a)?

[3]

- (c) What key assumption justifies the use of the Z distribution in this question?

[1]

**Question 3. [ 4 marks ]**

Another issue of whether appropriate care can be provided is whether there has been a shift in the age distribution of incoming long term care residents. Some changes (such as increased severe obesity and increased dementia) might lead to earlier entry into long term care while increased life expectancy might mean that people require long term care later. In 2008, the **population** of residents entering long term care had the following distribution:

	<60	60-69	70-79	80-89	90-99	>100
2008	2.96%	5.45%	18.21%	48.71%	23.57%	1.10%

In 2012, a random sample of residents entering long term care were distributed as follows:

	<60	60-69	70-79	80-89	90-99	>100	Total
2012	33	59	234	649	383	17	1375

Perform the appropriate hypothesis test to determine whether there has been an overall shift in the distribution of the age of residents at time of admission into long term care over the 4 year period. Use a 1% significance level. Your test must consider all the age groups.

**Question 4. [ 10 marks ]**

Appendix B shows the average salaries for male and female assistant professors in the Faculty of Arts at 22 different universities. Use the 0.05 level of significance in all tests.

- (a) Explain whether the two samples are independent or paired.

[1]

- (b) With specific reference to the appropriate boxplot(s), explain whether a parametric or non-parametric test is more appropriate to test for a difference in average salaries (mean or median).

[2]

- (c) Without regard to your answer in (b), perform the appropriate non-parametric test.

[3]

- (d) Without regard to your answer in (b), perform the appropriate t-test.

[3]

- (e) Explain how the paired t-test results are equivalent to the two-way analysis of variance results.

[1]

**Question 5. [ 15 marks ]**

A consumer organization wanted to examine the effect of age and gender on the amount (in hundreds of dollars) used car dealerships were willing to pay for a used car. Twelve “owners” (six male and six female) were recruited from each of three age groups (young, middle and senior). Each “owner” was randomly assigned to one of thirty-six used car dealers and brought the same car to solicit a cash offer. The data and analyses can be found in Appendix C.

- (a) Examine the residual plot and explain whether the model assumptions are justified.

[2]

- (b) Describe the effects of age and gender on the average offer, using the interaction plot. Explain whether age and gender interact with each other in how they affect the offer.

[3]

- (c) Test whether the effect of age depends on gender. Use the 5% level of significance.

[3]

- (d) Now test the effect of age on the average offer. Use the 5% level of significance.

[3]

- (e) Using a Bonferroni family confidence level of 95%, can you conclude that middle age “owners” receive higher cash offers than either of the other two age groups?

[4]

**Question 6. [ 22 marks ]**

To model the Canadian/U.S. dollar exchange rate ('ExchangeRt'), stock indices for various sectors of the economy are used as predictors. Some of the variables used in the four models shown in Appendix D are:

- ExchangeRt: The Canadian dollar equivalent to the U.S. dollar.
- Energy: Index of energy stocks, including oil and gas.
- Financial: Index of Banks and other financial institutions
- Industrial: Industrial Index
- Materials: Index of Lumber, Potash and other materials
- M&M: Index for Metals and Mining sector
- Utilities: Index for the power production sector
- IT: Information technology sector index

The following questions pertain to **Model 1** [ 4 marks ]

- (a) What problem(s) can you identify in the residual plots?  
[1]
- (b) What is the simple linear regression equation which relates 'ExchangeRt' to the predictor variable 'Energy'?  
[1]
- (c) Explain why the negative coefficient of the 'Energy' index makes economic sense.  
[1]
- (d) What is the correlation coefficient between 'ExchangeRt' and 'Energy'?  
[1]

Comparison of models [3 marks ]

- (e) Between Model 2 and Model 3, which is the better model? Explain briefly.  
[2]
- (f) The high Variance Inflation Factors suggest that there may be a problem with multicollinearity. Describe how this might affect the regression coefficients.  
[1]

Answer the following questions using **Model 4** [ 15 marks ]

- (g) Explain if the linear model assumptions are justified.  
[2]
- (h) How might you improve the fit of Model 4? (Only one suggestion will be marked.)  
[1]
- (i) Test whether this model is useful. Use the 5% level of significance.  
[3]

- (j) Test at the 1% level of significance whether the 'Financials' index is important to the model. State your conclusion carefully.  
[3]
- (k) Explain the meaning of the estimated coefficient of the 'Financials' index? Be as precise as possible.  
[2]
- (l) Using Model 4, calculate a 99% interval prediction for 'ExchangeRt' when the 'Financials' index is 190 and the 'Materials' index is 400.  
[2]
- (m) If 'ExchangeRt' is regressed on the residuals from Model 4, the R-square is 0.0416. How is this related to the R-square of Model 4? Explain the relationship.  
[2]

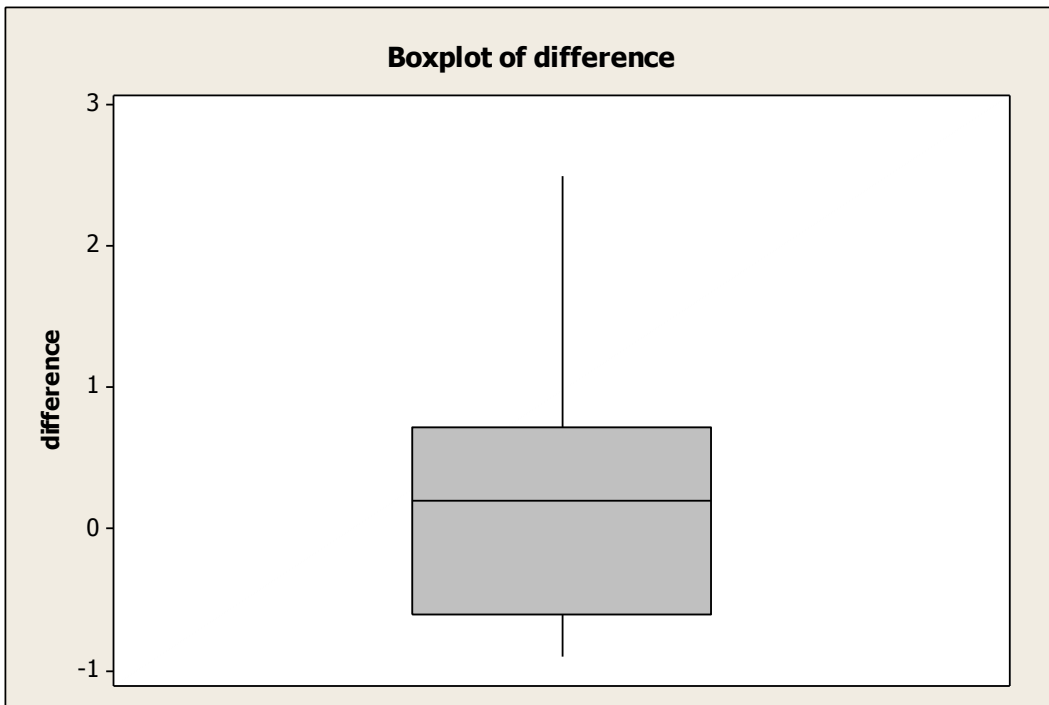
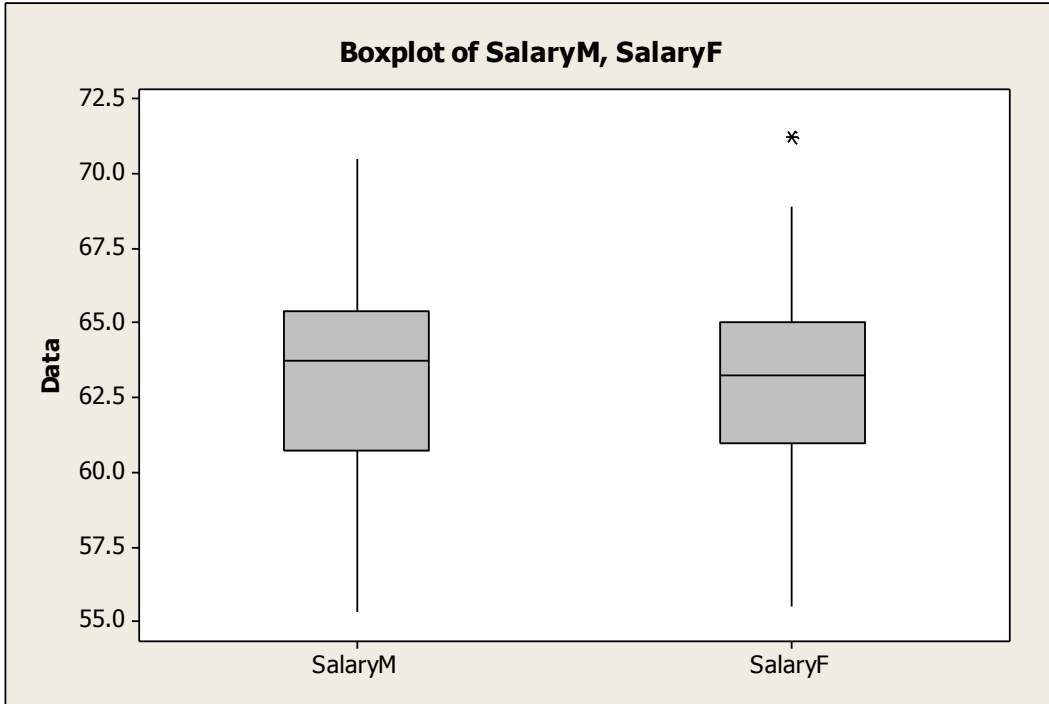
### Appendix A

Expected counts are printed below observed counts  
 Chi-Square contributions are printed below expected counts

	<60	60-69	70-79	80-89	90-99	>100	Total
1	14	48	140	524	278	12	1016
	23.29	54.16	193.36	497.93	238.37	8.90	
	3.704	0.701	14.726	1.365	6.590	1.083	
2	16	28	98	285	192	8	627
	14.37	33.42	119.33	307.28	147.10	5.49	
	0.185	0.880	3.812	1.616	13.704	1.147	
3	22	99	374	783	232	4	1514
	34.70	80.71	288.14	741.99	355.20	13.26	
	4.649	4.145	25.585	2.267	42.733	6.464	
4	37	32	127	311	209	10	726
	_____	38.70	138.17	355.80	170.33	6.36	
	_____	1.161	0.903	5.641	8.780	2.088	
Total	_____	207	739	1903	911	34	_____

### Appendix B

University	SalaryM	SalaryF	difference
1	64.5	63.9	0.6
2	60.5	61.2	-0.7
3	65.1	65	0.1
4	65.7	64.2	1.5
5	61.5	62.4	-0.9
6	64.4	64.1	0.3
7	62.1	62.7	-0.6
8	60.7	59.9	0.8
9	63.7	61.2	2.5
10	65.3	65.5	-0.2
11	60.7	60.2	0.5
12	64.2	64.8	-0.6
13	69.6	68.7	0.9
14	60.5	60	0.5
15	63.8	63.8	0
16	61.7	62.4	-0.7
17	62.8	61.7	1.1
18	68.5	68.9	-0.4
19	70.5	71.2	-0.7
20	55.3	55.5	-0.2
21	58.6	58	0.6
22	65.8	65.1	0.7



**Two-Sample T-Test and CI: SalaryM, SalaryF**

Two-sample T for SalaryM vs SalaryF

	N	Mean	StDev	SE Mean
SalaryM	22	63.43	3.55	0.76
SalaryF	22	63.20	3.60	0.77

Difference = mu (SalaryM) - mu (SalaryF)  
 Estimate for difference: 0.231818  
 95% CI for difference: ( \_\_\_\_\_, \_\_\_\_\_ )  
 T-Test of difference = 0 (vs not =): T-Value = \_\_\_\_\_ P-Value = \_\_\_\_\_ DF = \_\_\_\_\_  
 Both use Pooled StDev = 3.5738

**Paired T-Test and CI: SalaryM, SalaryF**

Paired T for SalaryM - SalaryF

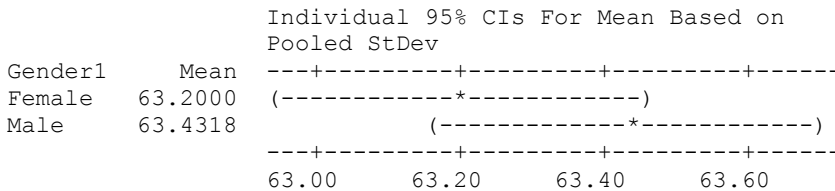
	N	Mean	StDev	SE Mean
SalaryM	22	63.4318	3.5462	0.7561
SalaryF	22	63.2000	3.6012	0.7678
Difference	22	0.231818	0.845935	0.180354

95% CI for mean difference: ( \_\_\_\_\_, \_\_\_\_\_ )  
 T-Test of mean difference = 0 (vs not = 0): T-Value = \_\_\_\_\_ P-Value = \_\_\_\_\_

**Two-way ANOVA: Salary versus University, Gender1**

Source	DF	SS	MS	F	P
University	21	528.914	25.1864	70.39	0.000
Gender1	1	0.591	0.5911	1.65	_____
Error	21	7.514	0.3578		
Total	43	537.019			

S = 0.5982 R-Sq = 98.60% R-Sq(adj) = 97.14%



**Wilcoxon Signed Rank Test: difference**

Test of median = 0.000000 versus median not = 0.000000

	N	Test for	Wilcoxon	P	Estimated
difference	N	Test	Statistic		Median
difference	22	21	143.5	0.339	0.1500

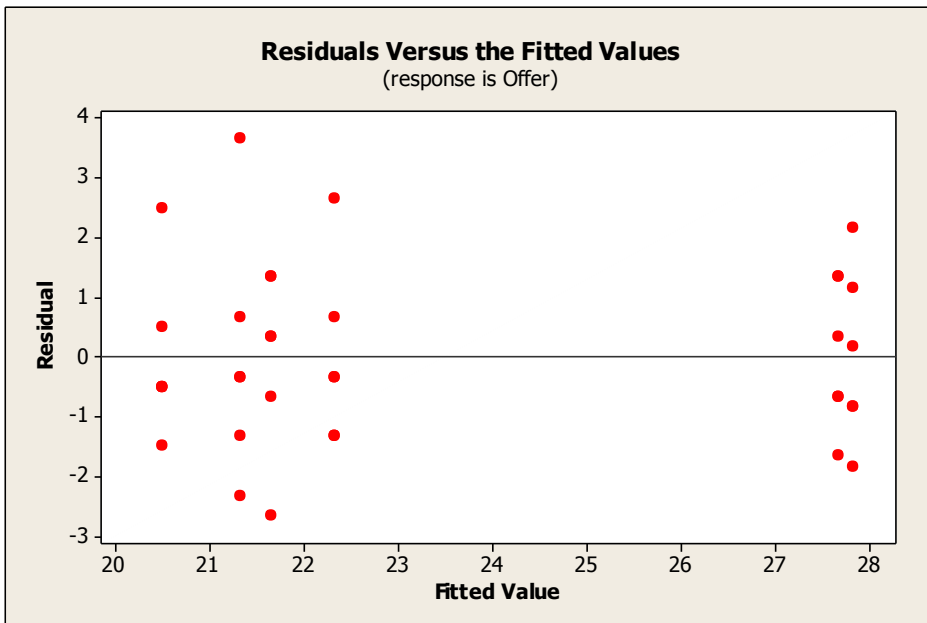
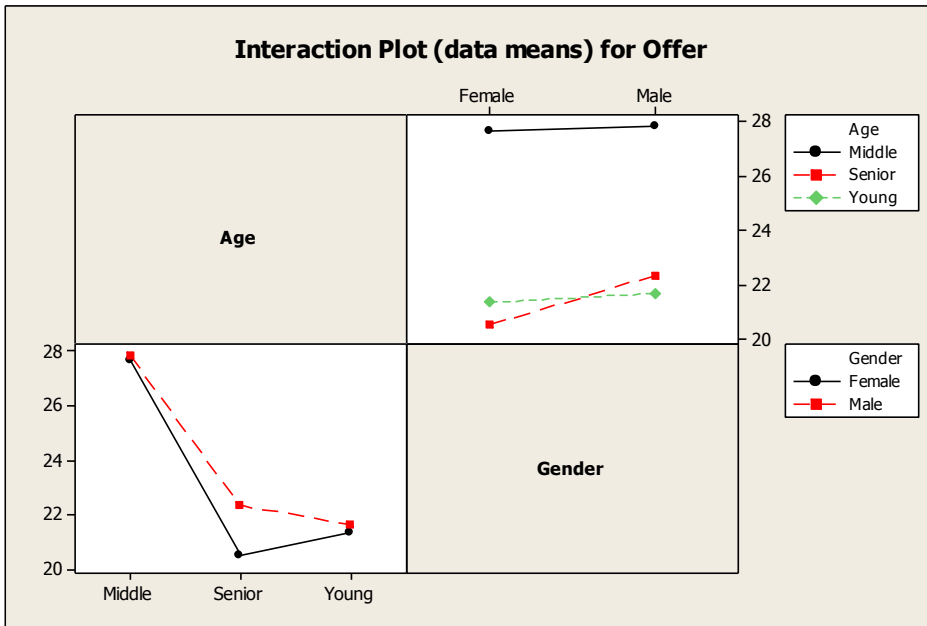
**Mann-Whitney Test and CI: SalaryM, SalaryF**

	N	Median
SalaryM	22	63.750
SalaryF	22	63.250

Point estimate for ETA1-ETA2 is 0.300  
 95.0 Percent CI for ETA1-ETA2 is (-1.700,2.400)  
 W = 510.0  
 Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.7336  
 The test is significant at 0.7335 (adjusted for ties)

### Appendix C

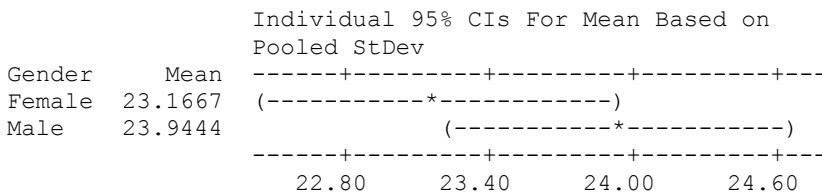
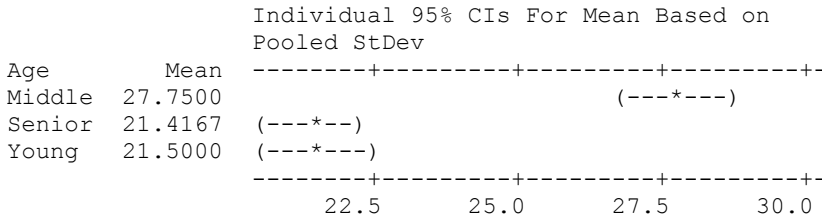
Age	Male			Female		
Young	21	23	19	21	22	20
	22	22	23	21	19	25
Middle	30	29	26	26	29	27
	28	27	27	28	27	29
Senior	25	22	23	23	19	20
	21	22	21	21	20	20



**Two-way ANOVA: Offer versus Age, Gender**

Source	DF	SS	MS	F	P
Age	2	316.722	158.361	66.29	_____
Gender	1	5.444	5.444	2.28	_____
Interaction	2	5.056	2.528	1.06	_____
Error	30	71.667	2.389		
Total	35	398.889			

S = 1.546 R-Sq = 82.03% R-Sq(adj) = 79.04%



**Appendix D**

**Model 1**

**Regression Analysis: ExchangeRt versus Energy**

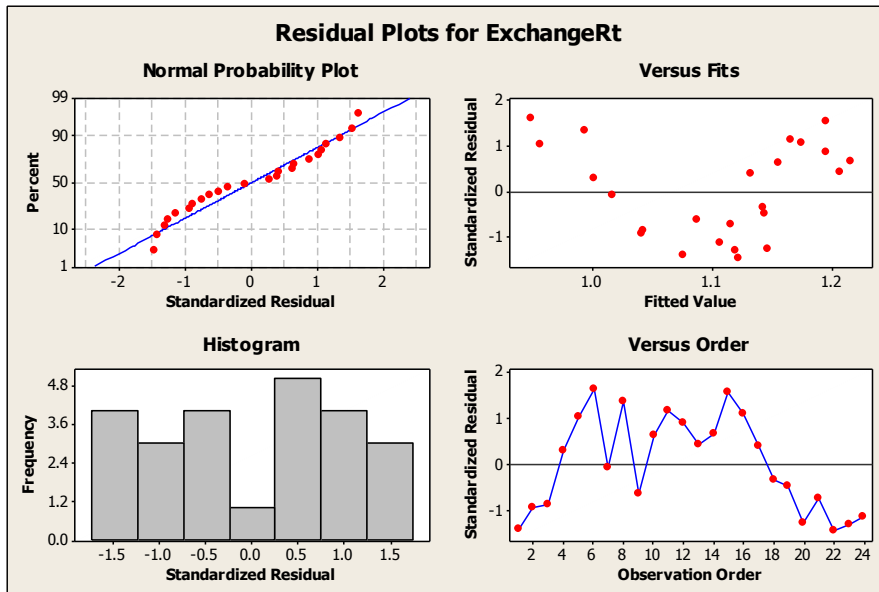
The regression equation is  
 ExchangeRt = 1.43 - 0.00109 Energy

Predictor	Coef	SE Coef	T	P
Constant	1.42989	0.04184	34.17	0.000
Energy	-0.0010857	0.0001358	_____	_____

S = 0.0472508 R-Sq = \_\_\_\_\_ R-Sq(adj) = 73.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	0.14275	0.14275	_____	_____
Residual Error	22	0.04912	0.00223		
Total	23	0.19187			



### Model 2

#### Regression Analysis: ExchangeRt versus Financials, Industrials, ...

The regression equation is

$$\text{ExchangeRt} = 1.68 - 0.00200 \text{ Financials} + 0.00174 \text{ Industrials} - 0.000337 \text{ Materials} - 0.000077 \text{ M\&M} - 0.00132 \text{ Utilities}$$

Predictor	Coef	SE Coef	T	P	VIF
Constant	1.67814	0.04162	40.32	0.000	
Financials	-0.0019995	0.0004481	-4.46	0.000	10.135
Industrials	0.001739	0.001199	1.45	0.164	25.996
Materials	-0.0003365	0.0001866	-1.80	0.088	8.325
M&M	-0.00007700	0.00004241	-1.82	0.086	7.617
Utilities	-0.0013209	0.0006196	-2.13	0.047	17.081

S = 0.0184611    R-Sq = 96.8%    R-Sq(adj) = 95.9%

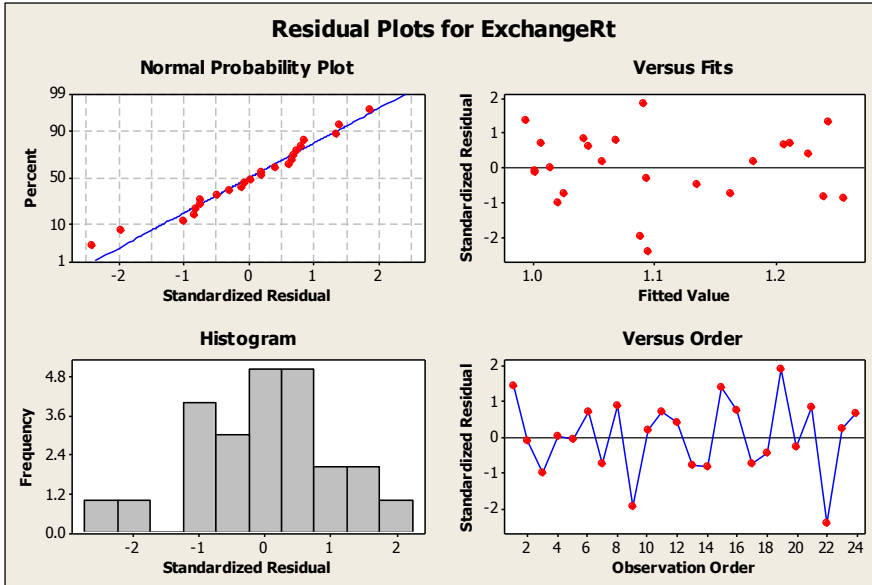
#### Analysis of Variance

Source	DF	SS	MS	F	P
Regression	5	0.185735	0.037147	109.00	0.000
Residual Error	18	0.006135	0.000341		
Total	23	0.191869			

#### Unusual Observations

Obs	Financials	ExchangeRt	Fit	SE Fit	Residual	St Resid
22	166	1.05485	1.09439	0.00857	-0.03954	-2.42R

R denotes an observation with a large standardized residual.



**Model 3**

**Regression Analysis: ExchangeRt versus Financials, Industrials, ...**

The regression equation is

$$\text{ExchangeRt} = 1.68 - 0.00202 \text{ Financials} + 0.00131 \text{ Industrials} - 0.000398 \text{ Materials} - 0.000083 \text{ M\&M} - 0.00132 \text{ Utilities} + 0.00223 \text{ IT}$$

Predictor	Coef	SE Coef	T	P	VIF
Constant	1.68310	0.04215	39.93	0.000	
Financials	-0.0020195	0.0004506	-4.48	0.000	10.158
Industrials	0.001310	0.001291	1.01	0.325	29.911
Materials	-0.0003983	0.0001991	-2.00	0.062	9.395
M&M	-0.00008297	0.00004309	-1.93	0.071	7.794
Utilities	-0.0013223	0.0006223	-2.12	0.049	17.081
IT	0.002230	0.002426	0.92	0.371	12.485

S = 0.0185411    R-Sq = 97.0%    R-Sq(adj) = 95.9%

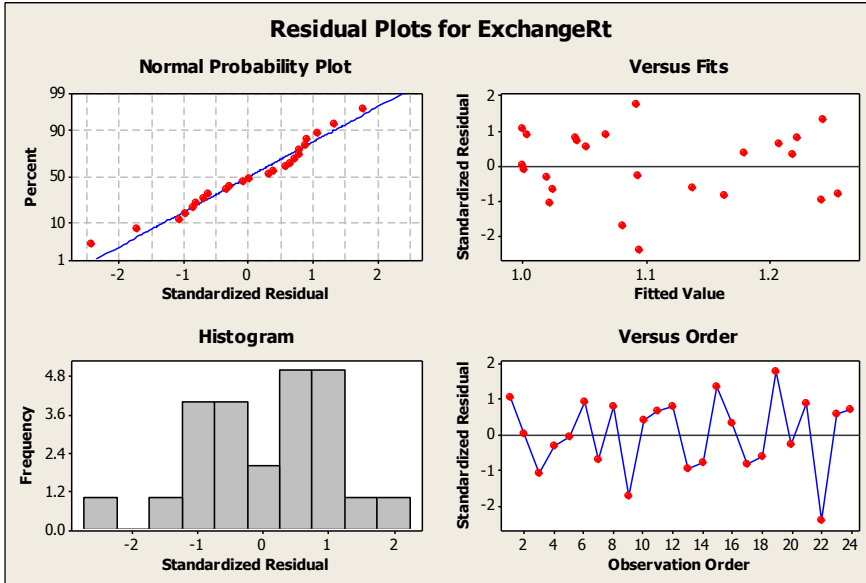
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	6	0.186025	0.031004	90.19	0.000
Residual Error	17	0.005844	0.000344		
Total	23	0.191869			

Unusual Observations

Obs	Financials	ExchangeRt	Fit	SE Fit	Residual	St Resid
22	166	1.05485	1.09486	0.00862	-0.04001	-2.44R

R denotes an observation with a large standardized residual.



**Model 4**

**Regression Analysis: ExchangeRt versus Financials, Materials**

The regression equation is

$$\text{ExchangeRt} = 1.65 - 0.00226 \text{ Financials} - 0.000559 \text{ Materials}$$

Predictor	Coef	SE Coef	T	P	VIF
Constant	1.64602	0.02504	65.74	0.000	
Financials	-0.0022571	0.0002247			2.285
Materials	-0.0005589	0.0001032	-5.41	0.000	2.285

S = 0.0194924 R-Sq = 95.8% R-Sq(adj) = 95.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.183890	0.091945		
Residual Error	21	0.007979	0.000380		
Total	23	0.191869			

Source	DF	Seq SS
Financials	1	0.172755
Materials	1	0.011135

Unusual Observations

Obs	Financials	ExchangeRt	Fit	SE Fit	Residual	St Resid
22	166	1.05485	1.10422	0.00411	-0.04937	-2.59R

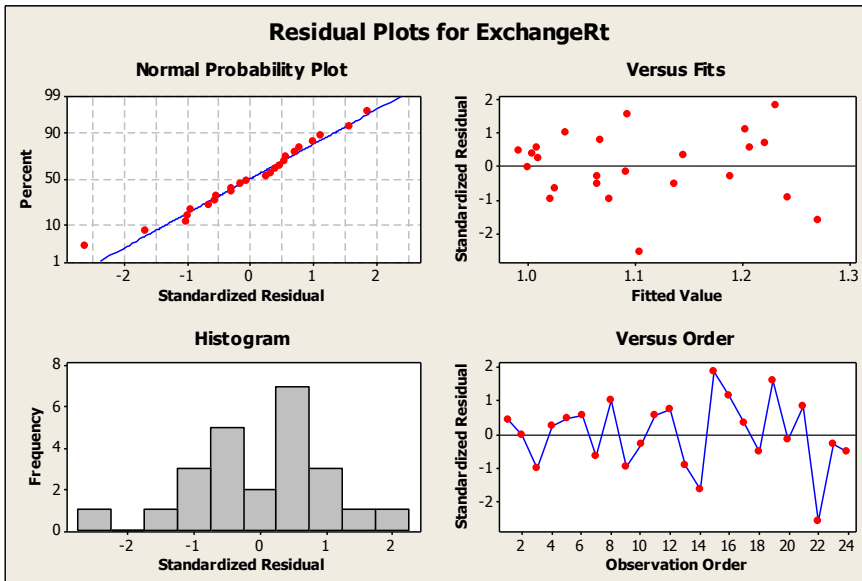
Predicted Values for New Observations

New Obs	Fit	SE Fit	99% CI	99% PI
1	0.99361	0.00768	( , )	( , )

Values of Predictors for New Observations

New Obs	Financials	Materials
1	190	400

**Residual Plots for ExchangeRt**



**Best Subsets Regression: ExchangeRt versus Energy, Financials, ...**

Response is ExchangeRt

Vars	R-Sq	R-Sq(adj)	Mallows Cp	S	I	F n	i d M U	n u a t	a s t i	E n t e l	n c r r i	e i i i t	r a a a M i	g l l l & e I	y s s s M s T
1	90.0	89.6	32.8	0.029476		X									
1	82.0	81.2	75.6	0.039653			X								
2	95.8	95.4	4.1	0.019492		X	X								
2	93.1	92.4	18.8	0.025195				X	X						
3	96.0	95.4	5.2	0.019584		X		X	X						
3	96.0	95.4	5.3	0.019618		X		X	X						
4	96.4	95.7	4.9	0.018991		X		X	X	X					
4	96.2	95.4	6.0	0.019524		X	X		X	X					
5	96.8	95.9	5.0	0.018461		X	X	X	X	X					
5	96.8	95.9	5.1	0.018556		X		X	X	X	X				
6	97.0	95.9	6.2	0.018541		X	X	X	X	X	X				
6	96.8	95.7	6.8	0.018905		X	X	X	X	X	X				
7	97.0	95.7	8.0	0.019021		X	X	X	X	X	X	X			

Standard Normal Distribution										
P( Z < z ) (z negative)										
Second decimal place in z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	<b>z</b>
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

**Standard Normal Distribution**

**P( Z < z ) (z positive)**

Second decimal place in z

<b>z</b>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

**Student's t distribution**

df	$t_{\alpha}$										
	$\alpha = P(t > t_{\alpha}) = \text{one-tail probability}$										
	0.100	0.050	0.025	0.010	0.009	0.008	0.005	0.001	0.0005	0.0004	0.0001
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73

Treat df = 1000 as infinity

**Chi-square distribution**

$$\chi^2_{\alpha;v}$$

$$\alpha = P(\chi^2 \geq \chi^2_{\alpha;v})$$

v	0.995	0.990	0.980	0.975	0.950	0.900	0.100	0.050	0.025	0.020	0.010	0.005
1	0.00	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	5.41	6.63	7.88
2	0.01	0.02	0.04	0.05	0.10	0.21	4.61	5.99	7.38	7.82	9.21	10.60
3	0.07	0.11	0.18	0.22	0.35	0.58	6.25	7.81	9.35	9.84	11.34	12.84
4	0.21	0.30	0.43	0.48	0.71	1.06	7.78	9.49	11.14	11.67	13.28	14.86
5	0.41	0.55	0.75	0.83	1.15	1.61	9.24	11.07	12.83	13.39	15.09	16.75
6	0.68	0.87	1.13	1.24	1.64	2.20	10.64	12.59	14.45	15.03	16.81	18.55
7	0.99	1.24	1.56	1.69	2.17	2.83	12.02	14.07	16.01	16.62	18.48	20.28
8	1.34	1.65	2.03	2.18	2.73	3.49	13.36	15.51	17.53	18.17	20.09	21.95
9	1.73	2.09	2.53	2.70	3.33	4.17	14.68	16.92	19.02	19.68	21.67	23.59
10	2.16	2.56	3.06	3.25	3.94	4.87	15.99	18.31	20.48	21.16	23.21	25.19
11	2.60	3.05	3.61	3.82	4.57	5.58	17.28	19.68	21.92	22.62	24.72	26.76
12	3.07	3.57	4.18	4.40	5.23	6.30	18.55	21.03	23.34	24.05	26.22	28.30
13	3.57	4.11	4.77	5.01	5.89	7.04	19.81	22.36	24.74	25.47	27.69	29.82
14	4.07	4.66	5.37	5.63	6.57	7.79	21.06	23.68	26.12	26.87	29.14	31.32
15	4.60	5.23	5.98	6.26	7.26	8.55	22.31	25.00	27.49	28.26	30.58	32.80
16	5.14	5.81	6.61	6.91	7.96	9.31	23.54	26.30	28.85	29.63	32.00	34.27
17	5.70	6.41	7.26	7.56	8.67	10.09	24.77	27.59	30.19	31.00	33.41	35.72
18	6.26	7.01	7.91	8.23	9.39	10.86	25.99	28.87	31.53	32.35	34.81	37.16
19	6.84	7.63	8.57	8.91	10.12	11.65	27.20	30.14	32.85	33.69	36.19	38.58
20	7.43	8.26	9.24	9.59	10.85	12.44	28.41	31.41	34.17	35.02	37.57	40.00

**Fisher's F distribution**

$$F_{\alpha;v_1;v_2}$$

$$\alpha = P(F \geq f_{\alpha;v_1;v_2}) = 0.050$$

v <sub>2</sub>	1	2	3	4	5	6	7	8	9	10	11	12
10	5.0	4.1	3.7	3.5	3.3	3.2	3.1	3.1	3.0	3.0	2.9	2.9
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.90	1.86
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80