

- [1] 1. Find the domain of the function  $g(x) = \frac{e^{\sqrt{1-2x}}}{x^2 - 9}$ .

**solution:** We need to make sure that the expression in the square root is non-negative, and we need to make sure that the denominator is not zero. So we must have  $1 - 2x \geq 0$  which gives  $x \leq 1/2$  and we also need to have  $x^2 - 9 \neq 0$  which gives  $x \neq \pm 3$ . So the domain is  $(-\infty, -3) \cup (-3, 1/2]$ .

- [2] 2. Using the definition of the derivative, find  $\frac{d}{dx}\sqrt{9-2x}$ .

**solution:**

$$\begin{aligned}
 & \lim_{h \rightarrow 0} \frac{\sqrt{9-2(x+h)} - \sqrt{9-2x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{9-2(x+h)} - \sqrt{9-2x}}{h} \times \frac{\sqrt{9-2(x+h)} + \sqrt{9-2x}}{\sqrt{9-2(x+h)} + \sqrt{9-2x}} \\
 &= \lim_{h \rightarrow 0} \frac{(\sqrt{9-2(x+h)})^2 - (\sqrt{9-2x})^2}{h(\sqrt{9-2(x+h)} + \sqrt{9-2x})} \\
 &= \lim_{h \rightarrow 0} \frac{(9-2(x+h)) - (9-2x)}{h(\sqrt{9-2(x+h)} + \sqrt{9-2x})} \\
 &= \lim_{h \rightarrow 0} \frac{-2h}{h(\sqrt{9-2(x+h)} + \sqrt{9-2x})} \\
 &= \lim_{h \rightarrow 0} \frac{-2}{(\sqrt{9-2(x+h)} + \sqrt{9-2x})} \\
 &= \lim_{h \rightarrow 0} \frac{-2}{(\sqrt{9-2(x+0)} + \sqrt{9-2x})} \\
 &= \lim_{h \rightarrow 0} \frac{-2}{(2\sqrt{9-2x})} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{(\sqrt{9-2x})}
 \end{aligned}$$

- [4] 3. Evaluate each of the limits. Show your work!

a)  $\lim_{x \rightarrow 2} \frac{x-2}{2x^2-4x}$

**solution:**

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x-2}{2x^2-4x} &= \lim_{x \rightarrow 2} \frac{x-2}{2x(x-2)} \\ &= \lim_{x \rightarrow 2} \frac{1}{2x} \\ &= \frac{1}{2(2)} \\ &= \frac{1}{4}\end{aligned}$$

b)  $\lim_{x \rightarrow \infty} \frac{x^3 - x - 7}{2x^3 + \sqrt{x}}$

**solution:**

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^3 - x - 7}{2x^3 + \sqrt{x}} &= \lim_{x \rightarrow \infty} \frac{x^3 - x - 7}{2x^3 + \sqrt{x}} \times \frac{1/x^3}{1/x^3} \\ &= \lim_{x \rightarrow \infty} \frac{1 - 1/x^2 - 7/x^3}{2 + 1/x^{5/2}} \\ &= \lim_{x \rightarrow \infty} \frac{1 - 0 - 0}{2 + 0} \\ &= \frac{1}{2}\end{aligned}$$

- [2] 4. Suppose that  $x^3 + x^2y - 2y^5 = 5$ . Find  $y' = \frac{dy}{dx}$  in terms of  $x$  and  $y$ .

**solution:**

$$\begin{aligned}x^3 + x^2y - 2y^5 &= 5 \\ 3x^2 + 2xy + x^2y' - 10y^4y' &= 0 \\ x^2y' - 10y^4y' &= -3x^2 - 2xy \\ y' &= \frac{-3x^2 - 2xy}{x^2 - 10y^4}\end{aligned}$$

- [2] 5. Let  $f(x) = 2x^3 - x$ . Find the equation of the line tangent to  $f$  at the point  $x = 1$ .

**solution:** First we find the derivative:  $f'(x) = 6x^2 - 1$ . The slope is  $f'(1) = 5$ . The value of the function is  $f(1) = 1$ . Then we have the line.

$$\begin{aligned}(y - y_0) &= m(x - x_0) \\ (y - 1) &= 5(x - 1) \\ y &= 5x - 4\end{aligned}$$

[6] 6. Differentiate each of the following. Show your work! You do not need to simplify your answer.

a)  $u(t) = (t^2 + 5)^7(e^t + \pi)$

**solution:**

$$7(t^2 + 5)^6(2t)(e^t + \pi) + (t^2 + 5)^7(e^t)$$

b)  $g(x) = \frac{\tan(x)}{x + 2^x}$ .

**solution:**

$$\frac{\sec^2(x)(x + 2^x) - \tan(x)(1 + 2^x \ln(2))}{(x + 2^x)^2}$$

c)  $w(r) = \ln(\sqrt{1 - r^2})$ .

**solution:**

$$\frac{1}{\sqrt{1 - r^2}} \frac{1}{2} (1 - r^2)^{-1/2} (-2r)$$

[2] 7. Use logarithmic differentiation to find the derivative of  $f(x) = (x^2 + 2)^{\cos(x)-1}$ . Give your answer in terms of  $x$  (not  $f$ ), but you do not need to simplify.

**solution:** First take  $\ln$  of both sides.

$$\begin{aligned} \ln(f) &= \ln\left((x^2 + 2)^{\cos(x)-1}\right) \\ &= (\cos(x) - 1) \ln(x^2 + 2) \end{aligned}$$

Now differentiate implicitly, using product rule and chain rule. Remember to substitute  $f$  at the end.

$$\begin{aligned} \frac{1}{f} f' &= (-\sin(x)) \ln(x^2 + 2) + (\cos(x) - 1) \frac{1}{x^2 + 2} (2x) \\ f' &= (x^2 + 2)^{\cos(x)-1} \left( (-\sin(x)) \ln(x^2 + 2) + (\cos(x) - 1) \frac{1}{x^2 + 2} (2x) \right) \end{aligned}$$

[3] 8. A rectangle is changing width and height continuously. When the width is 20cm and the area is  $100\text{cm}^2$ , the width is increasing at a rate of  $3\text{cm/s}$  and the area is increasing at a rate of  $10\text{cm}^2/\text{s}$ . At this moment, what is the rate of change of the height?

**solution:** Consider a rectangle of width  $w$ , height  $h$  and area  $A$ . We know that  $A = wh$ . Differentiating we get  $\frac{dA}{dt} = \frac{dw}{dt}h + w\frac{dh}{dt}$ . When the width is 20 and the area is 100 the height

must be  $100/20 = 5$ . At this moment we know  $\frac{dw}{dt} = 3$  and  $\frac{dA}{dt} = 10$ .

$$\begin{aligned}\frac{dA}{dt} &= \frac{dw}{dt}h + w\frac{dh}{dt} \\ 10 &= (3)(5) + (20)\frac{dh}{dt} \\ \frac{dh}{dt} &= \frac{10 - 15}{20} \\ &= -\frac{1}{4}\end{aligned}$$

Note that the height is decreasing, since its derivative is negative.