Department of Mathematics and Statistics

Course	Number	Section(s)
Mathematics	209	All

Examination	Date	Pages
Midterm	October 2018	2

TIME	Course Examiner
1 h 30 min	A. Kokotov

Special Instructions:

- Only approved calculators are allowed

1. **[12]** Find the limits

   (A) $\lim_{x\to -1} \frac{x^2+5x+4}{x^2+3x+2}$

   (B) $\lim_{x\to 1} \frac{x^2-1}{\sqrt{x+1}-\sqrt{2}}$

   (C) $\lim_{x\to -\infty} \frac{3x^3+2x^2+5x-7}{5x^3-100x^2+25}$

2. **[8]**

   A) Find the coordinates of all the points of the graph of the function

$$y = x^4 - 8x^2 + 1$$

   where the tangent line is horizontal.

   B)At what annual rate of interest compounded continuously must money be invested to triple in 7 years?

3. [12] Find the derivatives of the following functions. DO NOT SIMPLIFY.

(A) $\frac{13x+1}{\sqrt{x^5}} - x^3e^{3x}$

(B) $(2x^4 + x + 1)(\frac{1}{\sqrt{x}} - e^x)$

(C) $\frac{x^2}{\ln(\ln x)} + e^{\sqrt{x}}$

(D) $(\ln x + \sqrt{x^3+1})^{2018}$

4. [6] Find the equation of the tangent line at the point $x = 1, y = 1$ to the curve

$$2x^2y^4 + 3x^3y = 5$$

5. [4] Find the value of the differential dy at $x = 5$ with change in x as 0.3, where

$$y = \ln(3x+1) + \frac{x}{x+1}$$

6. [4] A point is moving on the graph $x^3 + 4y^5 = 12$. When the point is at $(2, 1)$ its x-coordinate is decreasing. Does the y-coordinate of the point decrease or increase at that moment?

7. [4] The total cost in dollars of producing x computers is

$$C(x) = 6000 + 30x - 0.05x^2$$

(A) Find the total cost and the marginal cost at a production level of 100 computers and interpret the results.

(B) Find the average cost and the marginal average cost at a production level of 100 computers and interpret the results.

MATH. 209.

Midterm Solutions.

Fall 2018.

1.A. $\lim_{x \to -1} \dfrac{x^2+5x+4}{x^2+3x+2} = \dfrac{0}{0}$ I.F.

$$\lim_{x \to -1} \frac{(x+4)(x+1)}{(x+2)(x+1)} = \lim_{x \to -1} \frac{x+4}{x+2} = \frac{3}{1} = 3.$$

B. $\lim_{x \to 1} \dfrac{x^2-1}{\sqrt{x+1}-\sqrt{2}} = \dfrac{0}{0}$ I.F.

$$\lim_{x \to 1} \frac{x^2-1}{\sqrt{x+1}-\sqrt{2}} \left(\frac{\sqrt{x+1}+\sqrt{2}}{\sqrt{x+1}+\sqrt{2}} \right)$$

$$\lim_{x \to 1} \frac{(x^2-1)(\sqrt{x+1}+\sqrt{2})}{(x+1)-(2)}$$

$$\lim_{x \to 1} \frac{(x+1)(x-1)(\sqrt{x+1}+\sqrt{2})}{x-1}$$

$$\lim_{x \to 1} (x+1)(\sqrt{x+1}+\sqrt{2}) = 4\sqrt{2}.$$

C. $\lim\limits_{x \to -\infty} \dfrac{3x^3+2x^2+5x-7}{5x^3-100x^2+25} = \dfrac{-\infty}{-\infty} = \dfrac{\infty}{\infty}$ I.F.

$$\lim_{x \to -\infty} \frac{x^3\left(3+\frac{2}{x}+\frac{5}{x^2}-\frac{7}{x^3}\right)}{x^3\left(5-\frac{100}{x}+\frac{25}{x^3}\right)}$$

$$\lim_{x \to -\infty} \frac{\left(3+\frac{2}{x}+\frac{5}{x^2}-\frac{7}{x^3}\right)}{\left(5-\frac{100}{x}+\frac{25}{x^3}\right)} = \frac{3+0+0-0}{5-0+0}$$

$$= \frac{3}{5}.$$

2. A. $y = x^4 - 8x^2 + 1$

$y' = 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0, \pm 2.$

B. $A = Pe^{rt}$ $\quad t = 7\text{yrs.}$

$r\,?$

$3P = Pe^{r(7)}$

$3 = e^{7r}$

$\ln(3) = \ln e^{7r} = 7r \ln e = 7r$

$r = \frac{\ln 3}{7} \approx 0.15694$

$\therefore$

$r \approx 15.69\%.$

3\. A. $y = \dfrac{13x+1}{x^{5/2}} - x^3 e^{3x}$

$$y = 13x^{-3/2} + x^{-5/2} - x^3 e^{3x}$$

$$y' = -\frac{39}{2}x^{-5/2} - \frac{5}{2}x^{-7/2} - \left[x^3(3e^{3x}) + e^{3x}(3x^2)\right]$$

$$y' = \frac{-39}{2}x^{-5/2} - \frac{5}{2}x^{-7/2} - 3x^3 e^{3x} - 3x^2 e^{3x}.$$

B. $y = (2x^4 + x + 1)\left(\dfrac{1}{\sqrt{x}} - e^x\right)$

$$y = (2x^4 + x + 1)(x^{-1/2} - e^x)$$

$$y' = (2x^4 + x + 1)\left(-\frac{1}{2}x^{-3/2} - e^x\right) +$$

$$(x^{-1/2} - e^x)(8x^3 + 1).$$

C. $y = \dfrac{x^2}{\ln(\ln x)} + e^{\sqrt{x}}$

$$y' = \frac{\ln(\ln x)(2x) - x^2\left(\frac{1}{\ln x}\left(\frac{1}{x}\right)\right)}{(\ln(\ln x))^2} + \frac{1}{2}x^{-\frac{1}{2}}e^{\sqrt{x}}.$$

D. $y = \left(\ln x + \sqrt{x^3+1}\right)^{2018}$

$y' = 2018\left(\ln x + \sqrt{x^3+1}\right)^{2017}\left[\frac{1}{x} + \frac{1}{2}(x^3+1)^{-1/2}(3x^2)\right].$

4. $2x^2y^4 + 3x^3y = 5 \qquad (1,1)$

$\left[2x^2(4y^3y') + y^4(4x)\right] + \left[3x^3(y') + y(9x^2)\right] = 0$

$8x^2y^3y' + 4xy^4 + 3x^3y' + 9x^2y = 0$

$8x^2y^3y' + 3x^3y' = -4xy^4 - 9x^2y$

$y'(8x^2y^3 + 3x^3) = -4xy^4 - 9x^2y$

$y' = \dfrac{-4xy^4 - 9x^2y}{8x^2y^3 + 3x^3}.$

$m_T = y'(1,1) = \dfrac{-13}{11}.$

$\therefore \quad y - y_1 = m(x - x_1)$

$y - 1 = -\dfrac{13}{11}(x-1)$

$y = -\dfrac{13}{11}x + \dfrac{24}{11}.$

5. $f(x) = \ln(3x+1) + \dfrac{x}{x+1}$ $\qquad x = 5$, $dx = 0.3$.

$$f'(x) = \frac{3}{3x+1} + \frac{(x+1)(1) - x(1)}{(x+1)^2}$$

$$f'(x) = \frac{3}{3x+1} + \frac{1}{(x+1)^2}.$$

$$dy = f'(x)\,dx.$$

$$dy = f'(5)(0.3) = \left(\frac{3}{16} + \frac{1}{36}\right)(0.3) = 0.0646.$$

6. $x^3 + 4y^5 = 12.$ $\qquad (2,1)$

$\dfrac{dx}{dt} < 0$

$\dfrac{dy}{dt}$?

$$3x^2\frac{dx}{dt} + 20y^4\frac{dy}{dt} = 0$$

$$20y^4\frac{dy}{dt} = -3x^2\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-3x^2}{20y^4}\frac{dx}{dt} = \frac{-3(2)^2}{20(1)^4}\frac{dx}{dt} > 0$$

y-coordinate is increasing.

7. $C(x) = 6000 + 30x - 0.05x^2$

A. $C(100) = 6000 + 30(100) - 0.05(100)^2 = \$8,500.$

$C'(x) = 30 - 0.1x$

$C'(100) = 30 - 0.1(100) = \$20.$

Cost of producing 100 computers is \$8,500.

approximate " " " one more computer after 100 are produced is \$20.

B. $\bar{C}(x) = \frac{C(x)}{x} = \frac{6000 + 30x - 0.05x^2}{x}$

$\bar{C}(x) = \frac{6000}{x} + 30 - 0.05x$

$\bar{C}(100) = \frac{6000}{100} + 30 - 0.05(100) = \$85.00$

$\bar{C}'(x) = -\frac{6000}{x^2} - 0.05.$

$\bar{C}'(100) = -\$0.65.$

average cost of producing 100 computers is \$85.00.

approximate average cost of producing one more computer after 100 produced is $-\$0.65$.