UNIVERSITY OF BRITISH COLUMBIA
FACULTY OF APPLIED SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

Test #4, October 28, 2010

MECH 221

Suggested Time: 1 Hour 25 minutes. **Allowed Time:** 2 Hours

Materials admitted: Pencil, eraser, straightedge, Mech 2 Approved Calculator, one hand-written 3x5 inch index card.

There are 5 Short Answer Questions and 2 Long Answer Problems on this Test. All questions must be answered.

Provide **all** work and solutions **on this test**. Scrap paper is available for your use, but **will not be collected or marked**. A bonus mark of up to 5% of the exam value is available for orderly presentation of work throughout the exam. **Illegible work, or answers that do not include supporting calculations and explanations will NOT BE MARKED**.

PLEASE WRITE YOUR NAME ON THE TOP OF ALL TEST PAGES

NAME:____Section___________

SIGNATURE:__________________________________

STUDENT NUMBER:__________________________

Question	Mark Received	Maximum Mark
SA 1		5
SA 2		5
SA 3		5
SA 4		5
SA 5		5
Prob 1		25
Prob 2		25
Bonus		

Name:______________________________ Section:__________________________

SA-1: Consider the differential equation

$$\frac{dy}{dt} = \frac{Ay}{B+y}$$

where $A > 0$ and $B > 0$ are given constants. Scale y and t in this general problem to make the equation as simple as possible.

$$y = \hat{y} Y \qquad t = \hat{t} T$$

$$\frac{Y}{T} \frac{d\hat{y}}{d\hat{t}} = \frac{AY\hat{y}}{B + Y\hat{y}}$$

$$\frac{d\hat{y}}{d\tilde{t}} = \frac{AT\tilde{y}}{B\left(1 + \frac{Y}{B}\tilde{y}\right)}$$

simple $\quad \frac{d\hat{y}}{d\tilde{t}} = \frac{\hat{y}}{1+\hat{y}}$

if $\quad \underbrace{\frac{AT}{B} = 1}_{T = \frac{B}{A}}, \qquad \underbrace{\frac{Y}{B} = 1}_{Y = B}.$

Name:______________________________ Section:__________________________

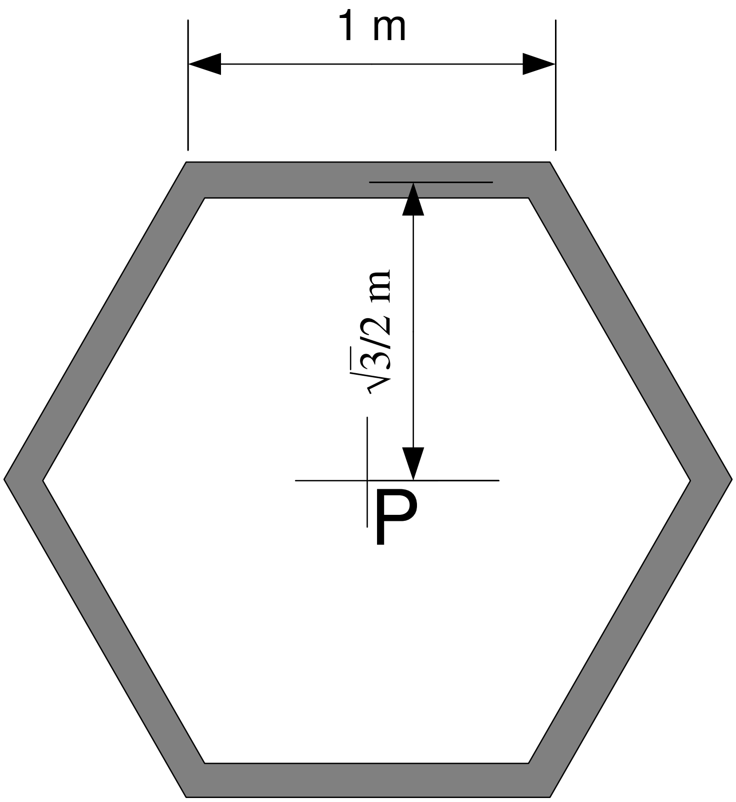
<u>SA-2:</u> Classify the following first order differential equations for the given functions and independent variables as linear or nonlinear. For linear equations, indicate which can be solved using the method of undetermined coefficients. For nonlinear equations, indicate which ones are separable.

a) $x(t)$ solves $dx/dt - 3x = e^x$
b) $x(t)$ solves $dx/dt - 3x = e^t$
c) $y(x)$ solves $\frac{dy}{dx} = e^x y + \arctan(x)$
d) $s(\theta)$ solves $s' = \theta^2 \cdot s$
e) $b(a)$ solves $\frac{db}{da} = a + b^3$

a) nonlinear, separable
b) linear, MUC
c) linear, not MUC
d) linear, not MUC
e) nonlinear, not separable.

Name:_______________________________ Section:___________________________

SA-3: The ***symmetrical*** hexagonal frame shown is made from uniform slender rods of mass 12 kg and length 1 m. Find the moment of inertia of the frame about an axis coming OUT OF THE PAGE at the center point P. Show your work.



$$I_{bar} = \frac{1}{12} m l^2 = \frac{1}{12}(12kg)(1m)^2 = 1 kg m^2$$

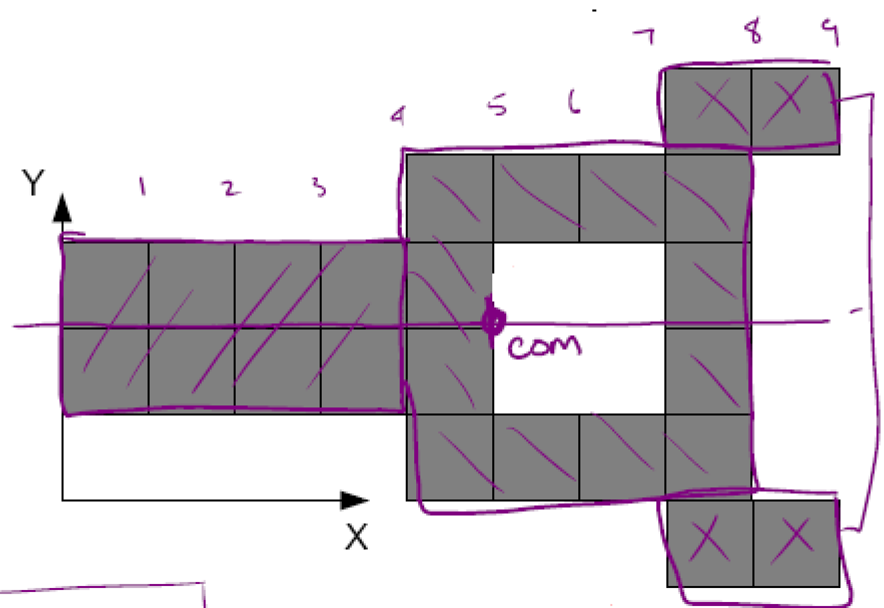
$$I_P = 6[I_{bar} + md^2)$$

$$= 6\left(1 kgm^2 + 12kg\left(\frac{\sqrt{3}}{2}m\right)^2\right)$$

$$I_P = 6(1+9) = 60 \text{ kg-m}^2$$

Name:______________________________ Section:__________________________

SA-4: Find the centroid of the shape with respect to the axis shown ***and*** *locate it on the diagram*. Show/explain how you found this location. Each box is dimension 1x1



$\boxed{\therefore Y_{com} = 2}$ by symmetry

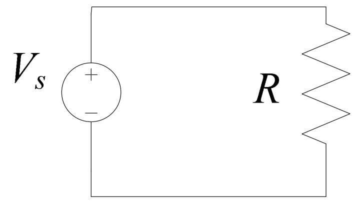
$$X_{com} = \frac{8 \times 2 + 12 \times 6 + 4 \times 8}{8 + 12 + 4}$$

$$= \frac{16 + 72 + 32}{24} = \frac{120}{24} = 5$$

$$\boxed{X_{com} = 5}$$

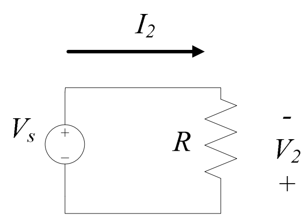
Name:______________________________ Section:__________________________

SA-5 We would like to find the voltage across the resistor and the current in the circuit below. Here, $V_s = 1^V$ and $R = 1^{k\Omega}$.

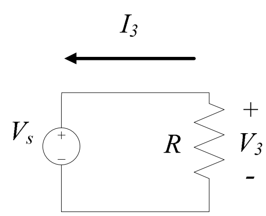


Two students have come up with the following approaches for solving the above circuit:

- Student *A* has chosen the direction of the current and the polarity of the voltage as follows:



- Student *B* has chosen the direction of the current and the polarity of the voltage as follows:



Is any of the above approaches correct? Justify your answer.

***Solution*:**

Both approaches are correct. Note that as long as the students apply KVL, KCL and Ohm's laws correctly and consistently, the direction of the current and the polarity of the voltage that they have chosen do not affect the final results. If the value that they calculate for the voltage/current is positive, it means that the chosen voltage polarity/current direction is indeed correct. If the value turns out to be negative, it means that the amplitude is correct. However, the voltage polarity/current direction is the opposite of the chosen polarity/direction. See the document at the end of Chapter 2 Notes for more information.

Name:______________________________ Section:__________________________

<u>Problem1</u>. Below there are three different first order Initial Value Problems (Differential Equations with Initial Conditions) for $x(t)$. For the three problems, solve for $x(1)$or approximate it using the method specified in the question.

(a) [8 points] $dx/dt = (1+t)/(1+x)$ with $x(0) = -2$. Use the separation of variables technique to find an analytic answer for $x(1)$.
(b) [8] $dx/dt = t^2 \sin(x)$ with $x(0) = 1$. Use 3 Forward Euler Time steps with step $h = 1/3$ to find an approximate answer for $x(1)$.
(c) [9] $dx/dt = \dfrac{t^2}{1+4x^3}$ with $x(0) = 1$. Use the separation of variables technique to find an implicit form $F(x) = G(t) + C$ with C determined by the initial condition. Use Newton's method on this form to find an approximation of $x(1)$ accurate to three decimal places.

a) $dx(1+x) = dt(1+t)$.

integrate $x + \frac{x^2}{2} = t + \frac{t^2}{2} + C$. $\quad x(0) = -2 \Rightarrow C = 0$.

$$x^2 + 2x - (2t + t^2) = 0.$$

$$x = \frac{-2 \pm \sqrt{4 + 4(2t + t^2)}}{2}$$

take - root above to match IC.

$$x(1) = \frac{-2 - \sqrt{16}}{2} = -3.$$

b) $\dot{x} = f(x,t) = t^2 \sin x$

$x_0 = 1$

$x_1 = x_0 + h f(x_0, 0) = 1 + \frac{1}{3}[0] = 1.$

$x_2 = x_1 + h f(x_1, \frac{1}{3}) = 1 + \frac{1}{3}\left[(\frac{1}{3})^2 \sin(1)\right]$

≈ 1.0312

$x_3 = x_2 + h f(x_2, 2/3) = x_2 + \frac{1}{3}\left[(\frac{2}{3})^2 \sin(x_2)\right]$

≈ 1.1583

c) $dx\,(1+4x^3) = t^2\,dt$

$$x + x^4 = \frac{t^3}{3} + C$$

$x(0) = 1 \Rightarrow C = 2.$

when $t = 1$, $x = x(1)$ satisfies

$$g(x) = x^4 + x - \frac{7}{3} = 0$$

$$g'(x) = 4x^3 + 1$$

Take initial guess $x_0 = 1$ for Newton's method

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = 1 - \frac{-1/3}{5} \approx 1.0667$$

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} \approx 1.0619$$

$$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} \approx 1.0619$$

So $x(1) \approx 1.0619$

Note: use ANS value in the formula and the back arrow to do this quickly.

Name:______________________________ Section:__________________________

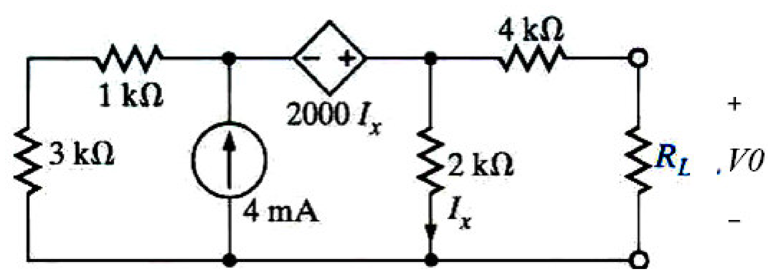
Problem2. In the circuit shown below,

a) [5 marks] Determine the number of sources, nodes, loops, branches, and meshes.

b) [5 marks] Assume that the value of resistor R_L can vary linearly between $1^{k\Omega}$ and $10^{k\Omega}$ with the steps of $1^{k\Omega}$ (i.e., its value can be $1^{k\Omega}$ or $2^{k\Omega}$ or … or $10^{k\Omega}$). We are interested in calculating the output voltage V_0 *for all of these values.* You have the option of choosing an analysis method from 1)Nodal Analysis, 2)Mesh Analysis and 3)Thevenin Equivalent. Which one is more computationally efficient to find V_0 *for all values of* R_L *mentioned above*? Justify your selection.

c) [15 marks] Using the method that you have chosen, roughly plot V_0 as a function of R_L for the range of resistor values specified above (the details of the calculations *must* be included).

d) [2 bonus marks] Is the function that you have plotted in part (c) linear? If not, does this mean that the Ohm's law is violated here?



Solution:

a)

The number of sources: 2

The number of nodes: 5

The number of loops: 6

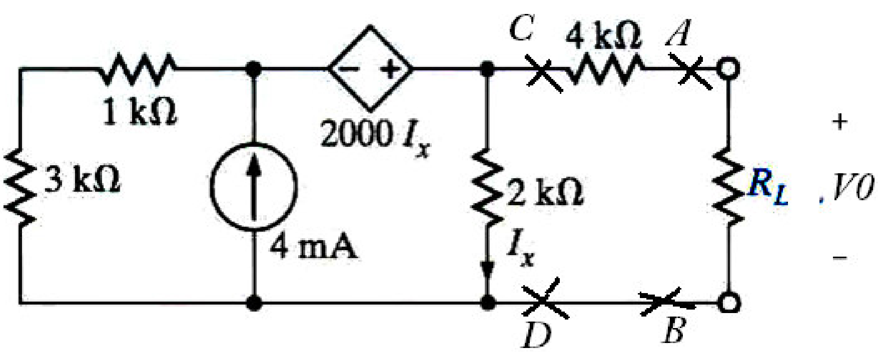
The number of branches:7

The number of meshes:3

b) Because we would like to find the output voltage when the resistor value is changing, the Thevenin equivalent approach is the best solution. The reason is that no matter what happens to the load (i.e., resistor R_L in this example), we can model the rest of the circuit with a voltage source in series with R_{th}. The effect of changing the values of R_L can then be easily determined in the resulting simple circuit. On the other hand, this is not the case with Nodal analysis or Mesh analysis. Changing one component in the circuit results in a new mesh or nodal equation related to that component. As a result, we need to solve the linear set of equations again, even if the value of only one component has changed.

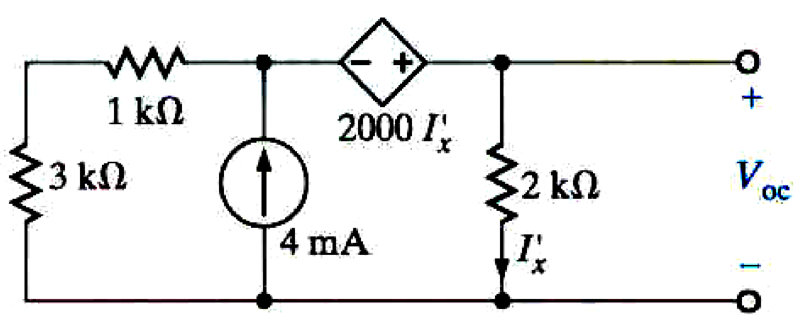
c) Here, we are dealing with a circuit that has both dependent and independent sources. As a result, in order to calculate the Thevenin equivalent, we have to 1) calculate Voc; 2) Calculate isc; 3) Rth=Voc/isc.

We can derive the Thevenin theorem at two points in the circuit below (either A-B or C-D).



Here, we derive the Thevenin equivalent at C-D, although the other solution is correct as well.

Calculating Voc:



If we add the two resistors in series (3K and 1K) and write the KCL for the supernode that contains the dependent voltage source, we have:

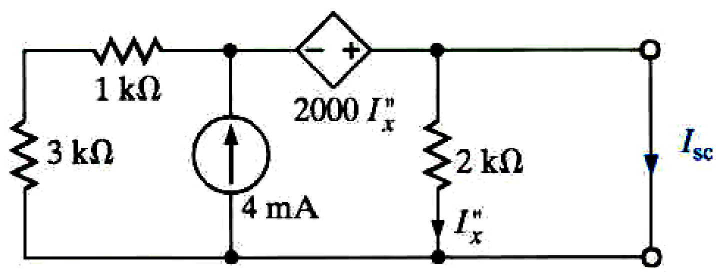
$$\frac{V_{oc} - 2000 I_x^{'}}{1^k + 3^k} + \frac{V_{oc}}{2^k} - 4^{mA} = 0 \quad (1)$$

The second equation is related to the controlling variable:

$$I'_x = \frac{V_{oc}}{2^k} \quad (2)$$

By replacing (2)in (1), we can solve for Voc: $\Rightarrow V_{oc} = 8^V$

Calculating isc:



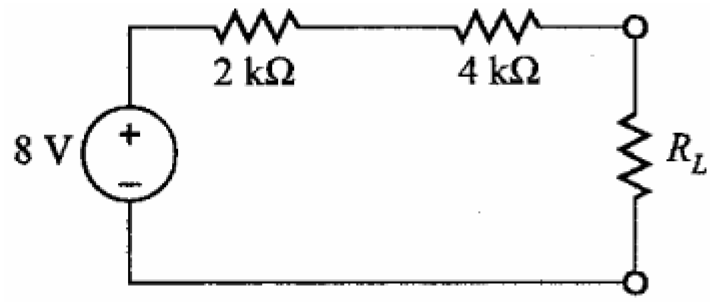
Because i_{sc} is in parallel with $I_x^{''}, I_x^{''} = 0$. This can be easily shown using current division formulae: Since $G_1 = \frac{1}{2^k}$ and $G_2 \rightarrow \infty$, then $I_x^{''} = 0$. As a result, the circuit can be simplified as the dependent current source will also have a zero value because $I''_x = 0$. The resulting circuit has a current source in parallel with a short circuit and in parallel with 4k resistor. As a result all the current from the 4mA current source will pass through the short circuit and thus:

$$i_{sc} = 4^{mA}$$

Calculating Rth:

$$R_{TH} = \frac{V_{oc}}{i_{sc}} = \frac{8^V}{4^{mA}} = 2^{k\Omega}$$

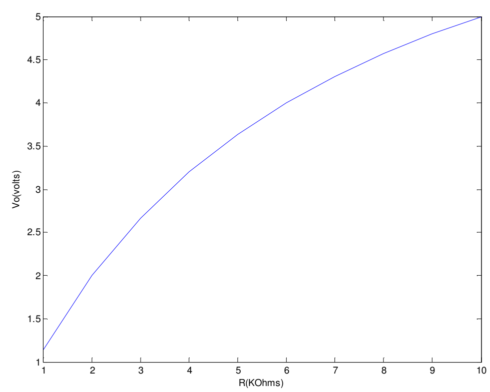
As a result, the Thevenin network will be as follows:



$$V_o = \frac{RL}{RL+6} \times 8^V$$

$R = 1^{k\Omega} \rightarrow V_o = 1.14^V$; $R = 2^{k\Omega} \rightarrow V_o = 2^V$; $R = 3^{k\Omega} \rightarrow V_o = 2.67^V$; $R = 4^{k\Omega} \rightarrow V_o = 3.2^V$

$R = 5^{k\Omega} \rightarrow V_o = 3.64^V$; $R = 6^{k\Omega} \rightarrow V_o = 4^V$; $R = 7^{k\Omega} \rightarrow V_o = 4.31^V$;
$R = 8^{k\Omega} \rightarrow V_o = 4.58^V$; $R = 9^{k\Omega} \rightarrow V_o = 4.8^V$; $R = 10^{k\Omega} \rightarrow V_o = 5^V$



d) No, the function is not linear. But it does not violate Ohm's law. Remember that Ohm's law is related to the relationship between the voltage across the resistor and the current that is flowing inside a linear resistor. It does not make any statement regarding the relationship between the voltage and the resistor's value.