Name: Sketch Student # _____

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Economics 303B
Midterm Exam #2
November 4, 2015

Instructions:

1. Do not start the exam until instructed to do so.
2. Answer *all* questions.
3. If you need more space for your answer, write on the back of the exam page and *clearly indicate* that your answer is continued on the back of the page.
4. No electronic devices (calculators, personal data assistants, notebook computers, etc.)
5. Leave knapsacks and other bags at the front of the exam room.
6. No papers or books of any sort.
7. No hats.
8. Write legibly. Use pen.
9. No cheating.

Question	Points	Score
1	30	
2	30	
3	34	
4	3	
5	3	
Total:	100	

1. Recall the consumer's problem from Chapter 4: (30)

$$\max U(C, \ell) \text{ subject to } C = w(h - \ell) + \pi - T.$$

Suppose that $U(C, \ell) = \ln C + \alpha \ln \ell$. Derive explicit expressions for the optimal choices of consumption and leisure in terms of w, h, π, T and α. (Note: You will have to use calculus.)

$$\mathcal{L} = \max_{C,\ell} \{ U(C,\ell) + \lambda [w(h-\ell) + \pi - T - C] \}$$

FOC:

C: $\frac{1}{C} - \lambda = 0$

ℓ: $\frac{\alpha}{\ell} - \lambda w = 0$

$\Rightarrow \quad w\ell = \alpha C$

Solution satisfies

$$\alpha C - w\ell = 0$$

and

$$C - w(h-\ell) - \pi + T = 0$$

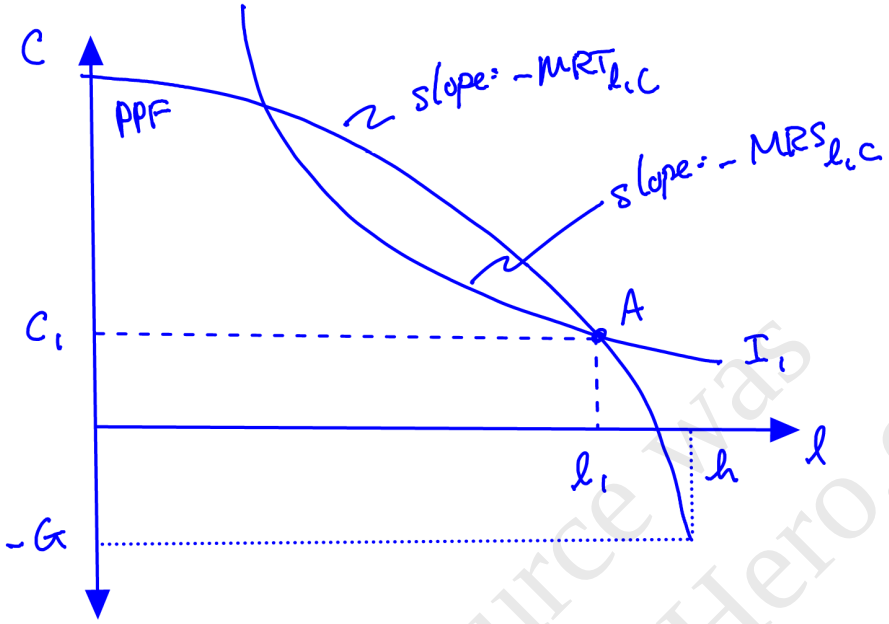
$$\begin{bmatrix} \alpha & -w \\ 1 & w \end{bmatrix} \begin{bmatrix} C \\ \ell \end{bmatrix} = \begin{bmatrix} 0 \\ wh + \pi - T \end{bmatrix}$$

$$\Rightarrow \quad C = \frac{wh + \pi - T}{1 + \alpha}$$

$$\ell = \frac{\alpha(wh + \pi - T)}{w(1+\alpha)}$$

2. Consider a bundle (C, ℓ) that is on the production possibilities frontier such that $\text{MRS}_{\ell,C} < \text{MRT}_{\ell,C}$.

(a) Depict this situation using a figure. Label the figure as completely as you can. (10)



(b) Use an arbitrage argument so show that this bundle does not solve the planner's problem (that is, the bundle is not an optimal choice). As part of your answer, indicate how the planner should adjust the bundle. (Hint: It may help to remember that $\text{MRS}_{\ell,C} = U_\ell(C, \ell)/U_C(C, \ell)$) (20)

Rewrite inequality as $U_\ell(C,\ell) < MP_N \times U_C(C,\ell)$

Deviation: work 1 extra unit of time, consume proceeds

Marginal cost: give up 1 unit of leisure, valued by its marginal utility, $U_\ell(C,\ell)$

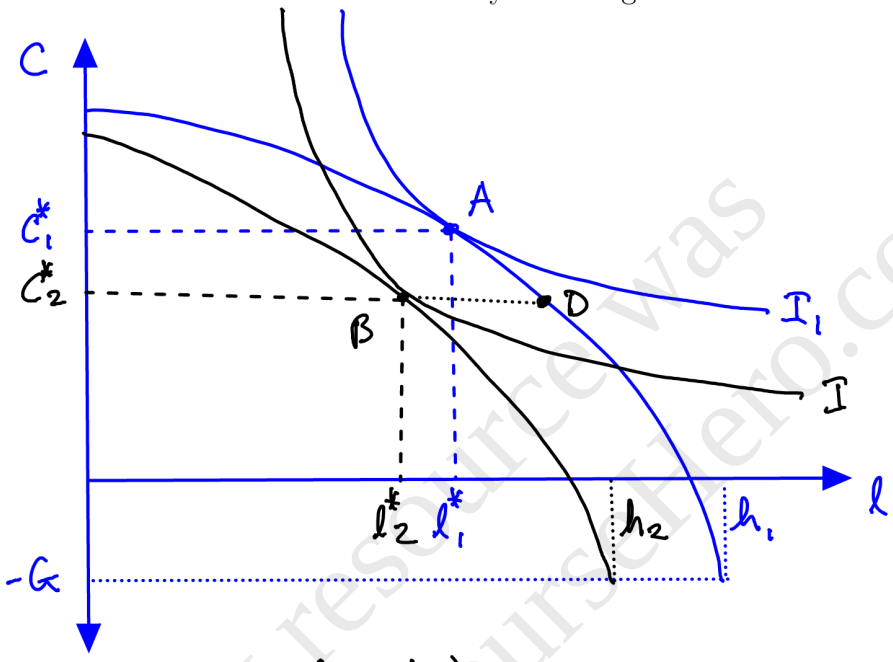
Marginal benefit: produce MP_N extra units of goods, each valued by marginal utility of consumption, $U_C(C,\ell)$

$\Rightarrow MP_N \times U_C(C,\ell)$

Since MB > MC, planner should in fact reduce leisure.

3. Most modern, developed countries impose mandatory schooling up to some particular age (typically 16 in North America). One way to model mandatory schooling is as a reduction in the time endowment, h, of the representative household.

(a) Use the one period model from Chapter 5 to illustrate the effects of mandatory schooling (that is, draw a figure). What effect does mandatory schooling have on the equilibrium values for C, N, w, Y and utility (hint: in the same figure, show the allocation when there is no mandatory schooling. (30)



$(C, l)\downarrow$ income effect (mostly)

$\because Y = C + G$, $Y\downarrow \Rightarrow N\downarrow$ since $Y = zF(K, N)$

$MRT^B = MRT^D > MRT^A$ $\therefore w\uparrow$

As shown above, utility falls

(b) What do you think is missing from this model of schooling? (4)

Schooling is an investment, partly in future productiveness

4. Business cycle facts. Circle the correct answer. (3)
   (a) Employment is (countercyclical / acyclical / procyclical).
   (b) Investment is (less / as / more) volatile than output.
   (c) Consumption (leads / is coincident with / lags) the cycle.

5. Define: Pareto optimal. (3)

See text, p. 137