FACULTY OF ENGINEERING AND COMPUTER SCIENCE
FINAL EXAMINATION FOR APPLIED DIFFERENTIAL EQUATIONS
ENGR 213 - FALL 2010

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Material Allowed: Calculators (non-programmable)

DO ALL THE PROBLEMS`

Problem No. 1. (10 MARKS) Solve the following equations using separation of variables:

(a) $$\frac{dy}{dx} = \frac{4-2x}{3y^2-5}$$

(b) $$x^2 \frac{dy}{dx} = y - xy$$

Problem No. 2. (10 MARKS) Solve the following equations using the exact differentials method:

(a) $$(2x+3y)\,dx + (3x+2y)\,dy = 0$$

(b) $$\left(1 + \ln x + \frac{y}{x}\right) dx = (1 - \ln x)\,dy$$

Problem No. 3. (10 MARKS) Solve the following Bernoulli equation

$$\frac{dy}{dx} - y = e^x y^2$$

subject to the boundary condition: $x = 0$, $y = 1$.

Problem No. 4. (10 MARKS) Solve the following linear differential equations using the integrating factor method:

(a) $$\frac{dy}{dx} - \frac{y}{x} = 1$$

(b) $x\dfrac{dy}{dx} + (3x+1)y = e^{-3x}$

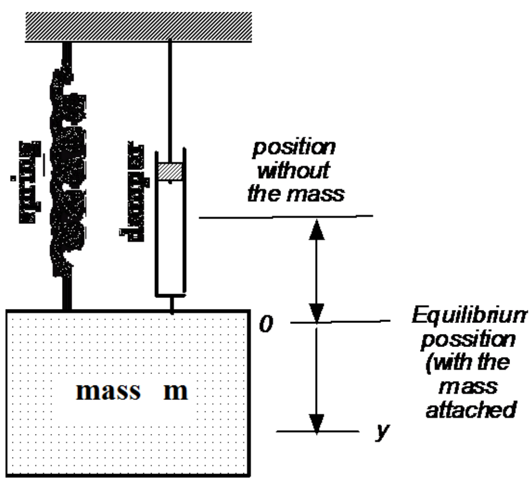
Problem No. 5. (10 MARKS) Give the general solutions of the following differential equations:

(a) $$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$$

(b) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

Problem No. 6. (10 MARKS) The equation describing the motion of the Mass - Spring-Damper system shown underneath is:

$$\frac{d^2y}{dt^2} + \frac{c}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$



where *k (*spring constant*)* = 7/4 N/m, *m (mass)* = 1 kg, and *c(damper coefficient)* = 4Ns / m.

(a) (2 MARKS) Based on the auxiliary equation characterize the system (underdamped/critically damped/overdamped).

(b) (8 MARKS). Originally the system is stretched downwards by y_{in} = 0.2 m, it is in static equilibrium, and then it is let go.

The initial conditions are:

i. $t = 0, y = y_{in} = 0.2$ m

ii. $t = 0, V_{in} = dy/dt = 0$

Find the position as function of time.

Problem No. 7. (10 MARKS) Solve the following differential equation by the method of undetermined coefficients:

$$y'' + 3y' + 4y = 3x + 2$$

Problem No. 8. (10 MARKS) Solve the following differential equation by variation of parameters:

$$y'' + 4y' + 4y = (1 + x)\, e^{3x}$$

Problem No. 9. (10 MARKS). Solve the following system of differential equations:

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = -x + 3y$$

Problem No. 10. (10 MARKS). Find the first 3 non-vanishing terms of power series solution of the equation:

$$\left(x^2 + 1\right) y'' + 2\, x\, y' = 0$$

around point $x = 0$.

USEFULL FORMULA: $$\int \frac{dx}{ax^2 - b} = \frac{1}{2\sqrt{ab}} \ln \left| \frac{\sqrt{a}\, x - \sqrt{b}}{\sqrt{a}\, x + \sqrt{b}} \right|$$