Test 2 MATH 1104 C, Fall, 2018

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1. [4 marks]: Let A be a 5×6 matrix such that $\dim(\mathrm{Col}A = 4)$. What is $\dim(\mathrm{Nul}\ A)$, (dimension of the solution space for $A\mathbf{x} = 0$)?

B

(a) 1 (b) 2 ✓ (c) 3 (d) 4 (e) 5 (f) None

4/4

35/40

2. [4 marks]: Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \text{ Find } T\left(\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}\right).$$

B

4/4

(a) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$ (b) $\begin{bmatrix} 9 \\ -6 \end{bmatrix}$ ✓ (c) $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} -11 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$

$\Rightarrow T\left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

$T\left(\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$

$+$ $-T\left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right) = -\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$T\left(\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ -4 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

3. [6 marks]: Find a value of k such that the given vectors are linearly **dependent**

6/6

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ k \\ -2 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 4 \\ 4 \\ k \end{bmatrix}$$

when $|A| = 0$, A is linearly dependent

$\therefore A = \begin{bmatrix} 1 & 0 & 4 \\ k & 2 & 4 \\ -2 & -2 & k \end{bmatrix}$

$|A| = 2k + 0 - 8k + 16 + 8 - 0 = 0$

$24 = 6k$

$k = 4$ ✓

4. [6 marks]: Find the coordinate vector for $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ relative to the basis $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ where $\mathbf{u}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$.

6/6

$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ -4 & 8 & 1 \end{array}\right]$

$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 14 & 3 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & \frac{3}{14} \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{3}{2} & \frac{9}{28} \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & \frac{5}{28} \\ 0 & \frac{3}{2} & \frac{9}{28} \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{5}{28} \\ 0 & 1 & \frac{3}{14} \end{array}\right]$

$\begin{bmatrix} \frac{5}{28} \\ \frac{3}{14} \end{bmatrix}$ ✓

5. [10: 2+2+3+1+2 marks]: Let the matrix A and its reduced row echelon form R be given by the following.

5/10

$$A = \begin{bmatrix} 1 & -2 & 2 & 1 & -3 \\ -3 & 1 & -2 & -4 & 0 \\ 4 & -1 & 4 & 7 & -1 \end{bmatrix} \sim \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \end{array} = R.$$

a: Find a basis for the column space of A

b: Find a basis for the row space of A

c: Find a basis for the null space of A

d: What is the rank A

e: Verify that the dimension of the column space of A plus the dimension of the null space of A is equal to the number of columns of A.

a)? $\begin{bmatrix} 1 & -2 & 2 \\ -3 & 1 & -2 \\ 4 & -1 & 4 \end{bmatrix}$ 1/2 $A_{colspace}: \left\{ \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\}$

d. Rank $|A| = 3$ 1/1

b)? $\begin{bmatrix} 1 & -2 & 2 & 1 & -3 \\ -3 & 1 & -2 & -4 & 0 \\ 4 & -1 & 4 & 7 & 1 \end{bmatrix}$ 0/2

$B_{rowspace} \{[1\,0\,0\,1\,1], [0\,1\,0\,1\,1], [0\,0\,1\,1\,-1]\}$

c)? $\begin{bmatrix} 1 & -3 \\ -4 & 0 \\ 7 & -1 \end{bmatrix}$ 1/3

Let $x_4 = a$ $x_5 = b$

$x_1 = -a - b$

$x_2 = -a - b$

$x_3 = b - a$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = a \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ null space$A$ = $\left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\dim(\text{col}\,A) = 3$ 2/2

e)? dim ~~null (col~~ (null A) = 2

$3+2 = n = 5$

10/10

6. [10: 2+2+2+2+2 marks]: Let $T : R^3 \longrightarrow R^3$ is defined by : $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y+z \\ x+2y+z \end{bmatrix}$.

a: Find the standard matrix of T.

b: Find a basis for the im T.

c: Find a basis for the ker T.

d: Is T one-to-one? Why?

d: Is T onto? Why?

a. $T(e_1)$, $T(e_2)$, $T(e_3)$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ 2/2

e $\because \dim(\text{img}(T)) = 2 \neq 3 = m$ 2/2

$\therefore$ Not onto.

d $\because$ null$(T) = 1 \neq \emptyset$ 2/2

$\therefore$ Not one to one

b. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ free

$\text{im}\,T = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$ 2/2

c $1 \cdot a + 1 \cdot b + 0 = 0$

$0 + 1 \cdot b + 1 c = 0$

$b = -c$

$a = c$

$c = a$

$\therefore \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} c$ 2/2

$\ker T = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$