[redacted]

~~[illegible]~~
1/11/18

TEST 2

15/15

MATH 1004, Fall 2018
Total marks: 15

1. Sketch the curve $y = \dfrac{1}{x^2-1}$ (show all your work) [15 marks].

1. Interception:

y: $f(0) = -1$ $\therefore \boxed{(0,-1)}$

x: $f(x) = 0$ $\because$ undefine $\therefore$ no x-interception.

2. V.A.:

$f(x)$ is undefine at $x = \pm 1$

$\therefore \lim\limits_{x\to 1^+} \dfrac{1}{x^2-1} = \dfrac{1}{0^+} = +\infty$

$\lim\limits_{x\to 1^-} \dfrac{1}{x^2-1} = \dfrac{1}{0^-} = -\infty$ $\therefore \boxed{\text{V.A. at } x = +1}$

$\lim\limits_{x\to -1^+} \dfrac{1}{x^2-1} = \dfrac{1}{0^-} = -\infty$

$\lim\limits_{x\to -1^-} \dfrac{1}{x^2-1} = \dfrac{1}{0^+} = +\infty$ $\therefore \boxed{\text{V.A at } x = -1}$

2. H.A.

$\lim\limits_{x\to \infty} \dfrac{1}{x^2-1} = 0$

$\lim\limits_{x\to -\infty} \dfrac{1}{x^2-1} = 0$ $\therefore \boxed{\text{H.A at } y = 0}$

3. C.P.: $f'(x) = \dfrac{0-2x}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2}$

$f'(x)$ 0/undefine when $x = 0, \pm 1$. $\therefore$ C.P. at $f(0) = -1$ $\boxed{(0,-1)}$

~~$f(\pm 1)$: undefine~~

4. I.P.: $f(x)'' = \dfrac{-2(x^2-1)^{\cancel{2}}[4x\cancel{(x^2-1)}\cdot(-2x)]}{(x^2-1)^{\cancel{4}\,3}}$

$= \dfrac{-2x^2+2+8x^2}{(x^2-1)^3}$

$f(x)'' = \dfrac{6x^2+2}{(x^2-1)^3}$

$f''(x) = 0$/undefine when $x^2 = \pm 1$ $\because f(\pm 1)$ = undefine $\therefore$ No I.P.

5. SDT

①

	(x^2-1)	$f(x)$
$(-\infty,-1)$	+	+
$(-1,1)$	−	−
$(1,\infty)$	+	+

②

	$-2x$	$(x^2-1)^2$	$f'(x)$
$(-\infty,-1)$	+	+	+
$(-1,0)$	+	+	+
$(0,1)$	−	+	−
$(1,\infty)$	−	+	−

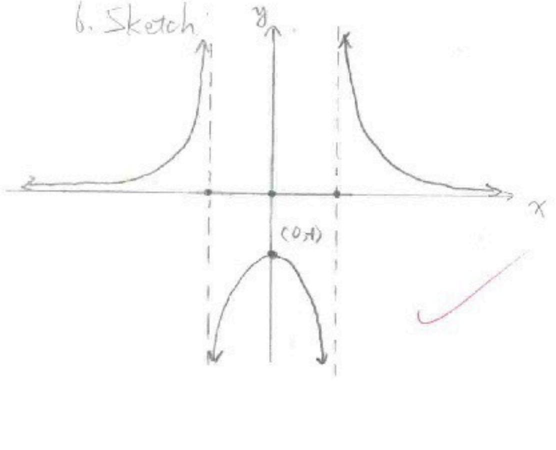
↑ ↓

③

	$(6x^2+2)$	$(x^2-1)^3$	$f''(x)$
$(-\infty,-1)$	+	+	+
$(-1,1)$	+	−	−
$(1,\infty)$	+	+	+

U ∩ U

6. Sketch



$\because f'(0) = 0$

$f''(0) = -2 < 0$

$\therefore f(0) = -1$; $(0,-1)$ is local max.

$f''(\pm 1)$ = undefine.

$\therefore f(x)$ at $x = \pm 1$ change concavety

work: 10/10

sketch: 5/5