# PHYS-271 HOMEWORK #1

*Due on Monday Feb 3rd at 11h25 in class. Late homework will not be graded.*

## 1. On the Planck Constant and Planck Scales

Planck realized the great importance of his constant $h$ beyond a simple fitting parameter in the quantum theory of light. In fact, he proposed that using the fundamental constant $h$, $c$ (the speed of light) and $G$ (Newton's gravitational constant) one can construct a natural set of units, or scales, related to length, time and mass. Discuss the scales that you found here in relation to other scales that you know.

a) Show that the expressions $(\frac{hG}{c^3})^{\frac{1}{2}}$, $(\frac{hG}{c^5})^{\frac{1}{2}}$, and $(\frac{hc}{G})^{\frac{1}{2}}$ have dimensions of length, time and mass and compute their numerical values. These quantities are known as Planck length, Planck time, and Planck mass.

b) Assume now that $h$ is much larger, and given by $h = 1\ J \cdot s$. Re-calculate the Planck length, time and mass. What do you conclude?

## 2. Light Quantization and the Photoelectric Effect

a) Calculate the energy of a photon whose frequency is a) 1 Hz, b) 100 GHz and c) 511 keV X-ray. Express your answer in electron volts.

b) Determine the corresponding wavelengths for the photons given in a).

c) An FM radio transmitter has a power output of 100 kW and operates at 94 MHz. How many photons per second does the transmitter emit?

d) The average power generated by the Sun is $10^{26}$ W. Assuming the average wavelength is 600 nm, find the number of photons emitted in 1 s.

e) The photocurrent of a photocell is cut off by a retarding potential of 2.92 V for radiation at 250 nm. Find the work function for the material.

f) Light of wavelength 500 nm is incident on a metallic surface. If the stopping potential for the photoelectric effect is 0.45V, find i) the maximum energy of the emitted electrons, ii) the work function and iii) the cutoff wavelength.

3. **The de Broglie Waves of Matter**

a) Calculate the de Broglie wavelength for a proton moving with a speed of $10^5$ m/s.

b) Calculate the de Broglie wavelength for an electron with kinetic energy i) 1 eV and ii) 1 MeV.

c) Calculate the de Broglie wavelength of a 74 kg person running at a speed of 5 m/s. Will this person "interfere" with a door 1 meter wide?

d) Calculate what would $h$ need to be for this person to "interfere" with the 1m wide door.

e) An electron has a de Broglie wavelength equal to the diameter of the hydrogen atom. What is the kinetic energy of the electron? Compare it with the the ground state energy of the hydrogen atom, =13.6 eV.

f) For an electron to be confined to to a nucleus, its de Broglie wavelength should be less than $10^{-14}$m. What would be the kinetic energy of an electron confined to that region? On the basis of this result, would you expect to find an electron in the a nucleus? Explain your answer.

g) Through what potential difference would an electron have to be accelerated to give it a de Broglie wavelength of $10^{-10}$m?

# PHYS-271 HOMEWORK #1 Solutions

**1. <u>On the Planck Constant and Planck Scales</u>**

a) Fundamental constants $h$, $c$, and $G$ are given by

$h = 6.63 \times 10^{-34} J \cdot s$ ,

$c = 3 \times 10^{8}\, m/s,$

$G = 6.67 \times 10^{-11}\, m^3/kg \cdot s^2$.

Using unit conversion,

$1J = 1N \cdot m = 1kg \cdot m^2/s^2$

we can easily verify that the following expressions have dimensions of length, time, and mass.

$$\left(\frac{hG}{c^3}\right)^{\frac{1}{2}} = \left[\frac{(6.63 \times 10^{-34} J \cdot s) \cdot (6.67 \times 10^{-11}\, m^3/kg \cdot s^2)}{(3 \times 10^8\, m/s)^3}\right]^{\frac{1}{2}} = 4.05 \times 10^{-35} m$$

$$\left(\frac{hG}{c^5}\right)^{\frac{1}{2}} = \left[\frac{(6.63 \times 10^{-34} J \cdot s) \cdot (6.67 \times 10^{-11}\, m^3/kg \cdot s^2)}{(3 \times 10^8\, m/s)^5}\right]^{\frac{1}{2}} = 1.35 \times 10^{-43} s$$

$$\left(\frac{hc}{G}\right)^{\frac{1}{2}} = \left[\frac{(6.63 \times 10^{-34} J \cdot s) \cdot (3 \times 10^8\, m/s)}{6.67 \times 10^{-11}\, m^3/kg \cdot s^2}\right]^{\frac{1}{2}} = 5.46 \times 10^{-8} kg$$

,which are the Planck mass, the Planck, time, and the Planck mass, respectively.

*The known values are slightly different because the reduced Planck constant $\hbar = h/2\pi$ was used instead of $h$.

b) Assuming that $h = 1J \cdot s$, the re-calculated Planck units are

$$\left(\frac{hG}{c^3}\right)^{\frac{1}{2}} = 1.57 \times 10^{-18} m$$

$$\left(\frac{hG}{c^5}\right)^{\frac{1}{2}} = 5.24 \times 10^{-27} s$$

$$\left(\frac{hc}{G}\right)^{\frac{1}{2}} = 2.12 \times 10^{9} kg$$

,which are obviously much larger than the original Planck units.

## 2. <u>Light Quantization and the Photoelectric Effect</u>

a) Using $E = h\nu$ and $1eV = 1.6 \times 10^{-19}J$,

$$a)\ E = (6.63 \times 10^{-34}J \cdot s) \cdot (1s^{-1}) \cdot \frac{1eV}{1.6 \times 10^{-19}J} = 4.14 \times 10^{-15}eV$$

$$b)\ E = (6.63 \times 10^{-34}J \cdot s) \cdot (1 \times 10^{11}s^{-1}) \cdot \frac{1eV}{1.6 \times 10^{-19}J} = 4.14 \times 10^{-4}eV$$

$$c)\ E = 511 \times 10^{3}eV$$

b) Using $\lambda = c/\nu$, and $\lambda = hc/E$

a) $\lambda = (3 \times 10^8\ m/s)/(1s^{-1}) = 3 \times 10^8 m$

b) $\lambda = (3 \times 10^8\ m/s)/(1 \times 10^{11}s^{-1}) = 3 \times 10^{-3}m$

c) $\lambda = (6.63 \times 10^{-34}J \cdot s) \cdot (3 \times 10^8\ m/s)/(511 \times 10^3 eV) \cdot \dfrac{1eV}{1.6 \times 10^{-19}J} = 2.43 \times 10^{-12}m$

c) Since the total energy is $E = nh\nu$, and the transmitted Energy in a time $t$ is $Pt$, where power $P = 100kW$, the total number of transmitted photon in a unit time is given by

$$\frac{n}{t} = \frac{P}{h\nu} = \frac{10^5\ J/s}{(6.63 \times 10^{-34}J \cdot s) \cdot (94 \times 10^6 s^{-1})} = 1.60 \times 10^{30}\ \ photons/s$$

d) Following the similar calculation as in Problem 2.c), one obtains

$$n = \frac{Pt}{h\nu} = \frac{Pt\lambda}{hc} = \frac{(10^{26}\ J/s) \cdot (1s) \cdot (600 \times 10^{-9}s^{-1})}{(6.63 \times 10^{-34}J \cdot s) \cdot (3 \times 10^8\ m/s)} = 3.01 \times 10^{44}\ \ photons$$

e) The energy of photoelectrons in the photocell $K$ is given by

$$K = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

where $W_0$ is the work function. Given the retarding potential of $2.92V$, and the wavelength of $250nm$, One can obtain the work function $W_0$,

$$W_0 = \frac{hc}{\lambda} - K = \frac{(6.63 \times 10^{-34}J \cdot s) \cdot (3 \times 10^8\ m/s)}{250 \times 10^{-9}m} \cdot \frac{1eV}{1.6 \times 10^{-19}J} - 2.92eV = 2.05eV$$

f) Following the similar calculation as in Problem 2.e), with the wavelength of $500nm$ and the stopping potential $V_S = 0.45V$, one can obtain,

i) $K_{max} = eV_S = 0.45eV$

ii) $W_0 = \frac{hc}{\lambda} - K_{max} = \frac{(6.63 \times 10^{-34} J \cdot s) \cdot (3 \times 10^8\, m/s)}{500 \times 10^{-9} m} \cdot \frac{1eV}{1.6 \times 10^{-19} J} - 0.45eV$

$= 2.04eV$

iii) $\lambda_{cutoff} = \frac{hc}{W_0} = \frac{(6.63 \times 10^{-34} J \cdot s) \cdot (3 \times 10^8\, m/s)}{2.04eV} \cdot \frac{1eV}{1.6 \times 10^{-19} J} = 609nm$

## 3. The de Broglie Waves of Matter

a) From the de Broglie wavelength relation $p = h/\lambda$,

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} J \cdot s}{(1.67 \times 10^{-27} kg) \cdot (10^5 m/s)} = 3.97 \times 10^{-12} m$$

b) To obtain the momentum of the electron from the given kinetic energies, we should figure out which limit case is proper for each. First let's see the relation between momentum and kinetic energy. From the Einstein's energy-momentum relation,

$$E^2 = p^2c^2 + m^2c^4 = (K + mc^2)^2 = K^2 + 2Kmc^2 + m^2c^4$$

$$p = \sqrt{K^2 + 2Kmc^2}/c = \frac{K\sqrt{1 + 2mc^2/K}}{c}$$

If $1 \ll 2mc^2/K$, the relativistic expression of momentum reduces to

$$p \approx \frac{K\sqrt{2mc^2/K}}{c} = K\sqrt{\frac{2m}{K}} = \sqrt{2mK}$$

,which is the classical momentum. Otherwise, we keep the relativistic one. Here, the rest mass energy of electrons is $mc^2 = 0.511MeV$.

i) $K = 1eV$

Since $K \ll 2mc^2$, taking the classical limit is a good approximation.

$$p = \sqrt{2mK} = [2 \times 0.511MeV/c^2 \times 1eV]^{\frac{1}{2}} = 1.01 \times 10^3 eV/c$$

$$= \frac{(1.01 \times 10^3 eV)}{3 \times 10^8\, m/s} \cdot \frac{1.6 \times 10^{-19} J}{1eV} = 5.40 \times 10^{-25} kg \cdot m/s$$

Thus the de Broglie wavelength of the electron with kinetic energy of $1eV$ can be obtained by,

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} J \cdot s}{5.40 \times 10^{-25} kg \cdot m/s} = 1.23 \times 10^{-9} m$$

ii) $K = 1MeV$

Since $K \approx 2mc^2$, we need the calculation in the relativistic limit.

$$p = \frac{K\sqrt{1 + 2mc^2/K}}{c} = \frac{1MeV\sqrt{1 + 2 \times \frac{0.511MeV}{1MeV}}}{c} = 2.02 \times 10^6 eV/c$$

$$= \frac{(2.02 \times 10^6 eV)}{3 \times 10^8\, m/s} \cdot \frac{1.6 \times 10^{-19} J}{1eV} = 1.08 \times 10^{-21} kg \cdot m/s$$

$$\therefore \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} J \cdot s}{1.08 \times 10^{-21} kg \cdot m/s} = 6.14 \times 10^{-13} m$$

c) With the mass of $74kg$ and the speed of $5m/s$, simply

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} J \cdot s}{74kg \cdot 5m/s} = 1.79 \times 10^{-36} m$$

Obviously, the wavelength of the running person is too short that we can simply ignore the wave property.

d) To observe the interference, the wavelength should be a roughly scale of the object that it interferes with. Thus enough value of $h$ that can make it possible to interfere with the door of $1m$ wide is roughly,

$$\lambda = \frac{h}{mv} = \frac{h}{74kg \cdot 5m/s} \sim 1m$$

$$h \sim 74kg \cdot 5m/s \cdot 1m = 370 J \cdot s$$

e) The diameter of the hydrogen atom is given by twice of the Bohr radius $a_0$, where $a_0 = 5.29 \times 10^{-11} m$. The kinetic energy of the electron is obtained by,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} J \cdot s}{2 \times 5.29 \times 10^{-11} m} = 6.27 \times 10^{-24} kg \cdot m/s$$

$$K = \frac{p^2}{2m} = \frac{(6.27 \times 10^{-24} kg \cdot m/s)^2}{2 \times 9.1 \times 10^{-31} kg} \cdot \frac{1eV}{1.6 \times 10^{-19} J} = 135eV$$

This is much larger than the absolute value of the ground state energy of the hydrogen atom, $E_g = 13.6eV$. Also the classical limit approximation is valid for this calculation since

$135eV \ll 2mc^2$ .

f) The wavelength of the electron confined to the a nucleus is less then $10^{-14}m$

$$pc > \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} J \cdot s) \cdot (3 \times 10^8\, m/s)}{10^{-14} m} \cdot \frac{1eV}{1.6 \times 10^{-19} J} = 1.24 \times 10^6 eV$$

$$= 124MeV \gg 2mc^2$$

In relativistic case,

$$\mathrm{K} = E - mc^2 = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 > 123MeV$$

The kinetic energy of the electron whose wavelength is less than the size of a nucleus is much larger than the scale of the bound-state energy of electrons; it cannot be confined to a region of the size of the nucleus.

g) Given the de Broglie wavelength of $10^{-10}m$, the kinetic energy can be obtained by,

$$pc = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} J \cdot s) \cdot (3 \times 10^8\, m/s)}{10^{-10} m} \cdot \frac{1eV}{1.6 \times 10^{-19} J} = 12.4keV$$

$$\mathrm{K} = E - mc^2 = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 = 150eV$$

Therefore the potential difference $V_e$ for the electron to have the wavelength of $10^{-10}m$ is

$$V_e = 150V$$