

ECO3152 Macroeconomic Theory III (Fall 2018)

Assignment 1: Answer Key

Question I (30 points)

Read the article “Canada’s GDP growth soars to 2.9% in second quarter – exports hit 4-year record” available at <https://globalnews.ca/news/4418317/canadas-gdp-q2-june-2018/>. Answer the following questions, based on your reading. Be precise. Use full sentences. Make sure to structure your responses in a way that demonstrates your understanding of the definitions of the key macroeconomic variables.

1. **(5 points)** What was the real GDP growth in the second quarter of 2018? How is this growth rate compared with the real GDP growth rate in the first quarter?

Answer key

Variable	Growth rate in 2018:Q1	Growth rate in 2018:Q2
Real GDP	1.4 %	2.9%.

The goal of this question is to make sure that students can read the information from the table. Typically, the media uses the growth rate of real GDP in discussing the growth performance of the economy.

There are different ways of describing the change in the growth rate series. One can use statements like this:

- “Compared with the first quarter of 2018, the growth rate of real GDP in the second quarter increased from 1.4% to 2.9%”
- “Compared with the first quarter of 2018, the growth rate of real GDP in the second quarter increased by 1.5 percentage points.”
- What is not correct is to say that “the growth rate of real GDP increased by 1.5%.”

Mark allocations: each statistics is 1.5 points (3 in total). The correct statement is 2 points.

2. **(5 points)** Was the realized GDP growth in line with economists’ predictions? Explain why or why not.

Answer key

According to the article, “Economists had expected an annualized pace of 3.0 per cent for the second quarter.” The actual growth rate is reported at 2.9%. Therefore, the growth rate was below the predicted values. However, the difference between the actual and the predicted values was rather small.

Marking scheme: exceptional (5), excellent (4-4.5), very good (3.5-4), good (3-3.5), passable (2.5-3). The grade will reflect the quality of writing.

3. (5 points) Complete the following table

Answer key

Table 1 The growth rate of aggregate demand components

Components of aggregate demand	Growth rate in 2018:Q2
Exports	2.9 %
Imports	1.6 %
Household spending	0.6 %
Housing investment	0.3 %
Business capital investment	0.4 %

Marking: 1 point for each correct number.

4. (5 points) Which of the components of Canadian GDP listed in the table above has grown at the *fastest rate*? Which was the *slowest*?

Answer key

Based on the information in the table, exports grew at the fastest rate of 2.9%. The article further states that energy exports's growth was the fastest (at 5.6%). Note, however, that the question was related to the variables in the table, so energy exports would not be the correct response.

Housing investment had the lowest growth of 0.3%.

5. (5 points) In Canada, consumption, investment, exports and imports are *procyclical*. Is the behaviour of these macroeconomic variables in the second quarter of 2018 (described in the article) consistent with their historical business cycle properties? Explain your response.

Answer key

A procyclical variable increase during expansions and declines during recessions. In the second quarter of 2018, the growth of consumption, investment, exports and imports was positive, at the time when the growth rate of GDP was positive. Therefore, we can conclude that all economic variables were behaviour as they typically do during economic expansions.

Marking scheme: exceptional (5), excellent (4-4.5), very good (3.5-4), good (3-3.5), passable (2.5-3). The grade will reflect the quality of writing.

6. (5 points) In your opinion, has Canada entered a recession in the second quarter of 2018? Explain your response. You can bring any additional evidence outside this article to justify your response. Any additional sources of data must be properly cited.

Answer key

Based on this article, the growth of the Canadian economy was rather strong. The growth of all major components of aggregate demand was positive. Further, the growth was largely driven by

the export sector, despite the uncertainty of NAFTA negotiations. There is virtually no other evidence that Canada was in a recession.

Marking scheme: exceptional (5), excellent (4-4.5), very good (3.5-4), good (3-3.5), passable (2.5-3). The grade will reflect the quality of writing.

An exceptional response should discuss (correctly!) the economic evidence outside the article.

Question II (10 points)

Consider a profit maximization problem of a firm that operates a Cobb-Douglas production function $Y = z K^a N^{(1-a)}$. The firm owns the stock of capital K and hires the labour input N on a competitive labour market at the real wage w per unit of labour.

1. Consider the case when the firm can only increase its labour input by one worker. Assume that $z=1$, $K=10$, $a = 0.35$ and $w=1$.

- a. **(5 points)** Compute the value of total output and the marginal product for each worker by filling out Table 2. I recommend using mathematical formulas functions in Excel to perform your calculations. Report your numbers with two decimal points approximations.

Answer key – see the table.

Marking: Each column is worth 2.5 (Y and MPN)

- b. **(5 points)** Graph the marginal product of labour for the number of workers from 1 to 10, and include this graph in your response. Is the marginal production of labour positive and diminishing? Justify your response.



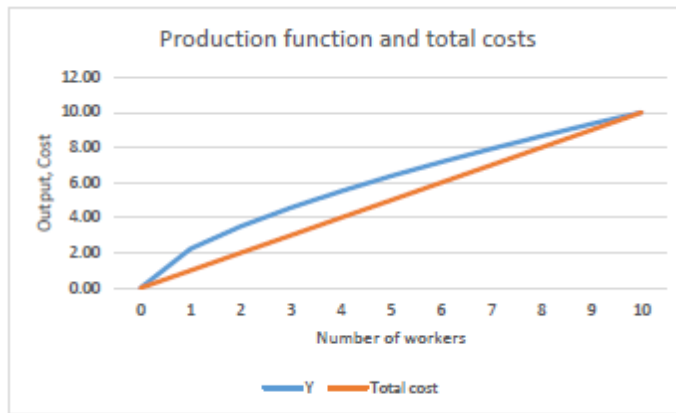
The MPN is positive and diminishing.

- c. (5 points) How many workers should the firm hire? Justify your response.

Answer key: The firm's objective is to maximise profits. In the example, the $MC = 1$, while the $MR=MPN$ is given in the table. When the firm can only hire a "discrete" number of workers, the firm will continue hiring as long as $MR=MPN > MC$. This occurs when $N=3$. One can also compute the profit function ($\pi=Y-wN$). At $N=4$, the profit is at the maximum value (1.57).

- d. (5 points) Graph the production function and the total cost function for the number of workers from 0 to 10. Clearly label the curves and the axes. Indicate the firm's optimal choice of the graph. Relate your response to part (c) of this subquestion.

Answer key: Total costs = $w * N$ = variable labour cost. The area between the output (revenue) and cost curves represent profits. The maximum profits are attained by $N=3$.



- e. (5 points) How much output will the firm produce? What will the firm's profits be?

Answer key: Output = 4.57, Profits = 1.57 (see explanation above)

2. (10 points) Consider the case when the firm can only increase its labour input by one worker. Assume that $z=1$, $K=10$ and $a = 0.35$. However, the wage rate increases to $w=1.5$. How should the firm adjust its labour input? Justify your response.

Answer key: When the wage rate increase, the MC increases. Now when the number of workers is 3, the $MPN=1.06 < MC=1.5$. The firm must decrease the number of workers. For $N=2$, $MPN=1.27 < MC$ but with $N=1$, $MPN = 2.24 > 1.5$. Thus, the firm will hire one worker (maximum profits are 0.74).

3. Consider again the case when the firm can only increase its labour input by one worker. Assume that $K=10$ and $a = 0.35$. However, now the value of TFP increases to $z=1.4$ and the real wage remains at $w=1.5$.

- a. **(5 points)** Re-compute the value of total output and the marginal product for each worker.

Answer key: See the table.

- b. **(5 points)** Graph the production function. Explain how the production function changes from part (2).

Answer key: The production function rotates up. With the higher level of TFP, the firm can produce more output for each worker.

- c. **(5 points)** How many workers should the firm hire? Explain.

Answer key: The firm should hire $N=2$ workers to maximize profits. The explanation can rely on the comparison of MPN and MC, on the profit function calculations (here $\pi = 1.92$) or on the graph of the profit and cost functions.

4. Consider the case when the labour input can take any value, so that the firm can make infinitesimal adjustments of the amount of labour input hired.

- a. **(5 points)** Derive the mathematical expression for the *marginal product of labour, MPN*. Is the marginal product positive and diminishing? Justify your response.

Answer key: $MPN = (1-a) z K^a N^{-a}$. Since $z > 0$, $K > 0$, $N > 0$ and $0 < a < 1$, $MPN > 0$. Further,

$\partial MPN / \partial z = -a(1-a) z K^a N^{-a-1} < 0$. The results are consistent with the graph in part (1) of this question.

- b. **(5 points)** Derive the mathematical expression for the *labour demand function*.

Answer key: The profit maximizing firm will choose labour input so that $MPN = w$. This expression defines the labour demand. With the Cobb-Douglas production function, the labour demand can be found explicitly as

$$N^d = [(1-a)z]^{1/a} K w^{-1/a}$$

- c. **(5 points)** How does the labour demand depend on the real wage? In particular, would the firm want to increase, decrease or leave unchanged its labour input? Answer this question by computing the derivative $\partial N^d / \partial w$. Is your response consistent with your findings in part (2) of the question?

Answer key:

$\partial N^d/\partial w = (-1/a) [(1-a)z]^{1/a} K w^{-1/a-1} < 0$. As the wage rate increases, the marginal cost of production rises as well. At the original level of employment, $MPN < w$. Since the MPN is decreasing in labour, the firm will decrease the amount of labour input demanded until the condition $MPN=w$ is restored. The results are consistent with part (2) of the question.

- d. (5 points) How does the labour demand depend on the value of TFP z ? In particular, would the firm want to increase, decrease or leave unchanged its labour input? Answer this question by computing the derivative $\partial N^d/\partial z$. Is your response consistent with your findings in part (3) of the question?

Answer key:

$\partial N^d/\partial z = (1/a) [(1-a)z]^{1/a} K w^{-1/a} > 0$. An increase in TFP increases the marginal product of labour, and hence the marginal revenue for any level of labour input. At the original level of employment, $MPN > w$. Since the MPN is decreasing in labour, the firm will increase the amount of labour input demanded until the condition $MPN=w$ is restored. The results are consistent with part (2) of the question.

Table 2 Profit maximization calculations

Number of workers	Output Y (z=1)	Marginal product (z=1)	Output Y (z=1.4)	Marginal product (z=1.4)
0	0.00		0.00	
1	2.24	2.24	3.13	3.13
2	3.51	1.27	4.92	1.78
3	4.57	1.06	6.40	1.48
4	5.51	0.94	7.72	1.32
5	6.37	0.86	8.92	1.20
6	7.17	0.80	10.04	1.12
7	7.93	0.76	11.10	1.06
8	8.65	0.72	12.11	1.01
9	9.34	0.69	13.07	0.96
10	10.00	0.66	14.00	0.93