

Key version A



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Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 1308 A – Midterm I Professor: S. Molladavoudi.
October 10, 2018 Time: 80 minutes.

Last name: _____ First name: _____ Student #: _____

INSTRUCTIONS

Please, read the following instructions carefully:

- Questions 1 to 4 are multiple choice. These questions are worth 1 point each and no partial marks are possible.
- Questions 5 to 7 are long answer questions. Questions 5 and 7 are worth 6 marks each, and question 6 is worth 4 marks, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the last page if necessary.
- Only basic, scientific and non programmable calculators are allowed, including Texas Instruments TI-30, TI-34, Casio fx-260 and fx-300.
- This is a closed book exam, and no notes of any kind are allowed. Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a zero for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature: _____

THIS SPACE IS RESERVED FOR THE MARKER

Question	1	2	3	4	5	6	7	Total
Mark								
Out of	1	1	1	1	6	4	6	20

Question 1. Find the slope of the tangent line to the graph $y = 3\sqrt{2x - x^3}$ at the point $x = 1$.
(1 point)

Cross (X) the correct answer:

A $-\frac{1}{2}$

B 3

C $\frac{3}{2}$

D $\frac{1}{2}$

E $-\frac{3}{2}$

F -3

$$y' = 3 \left(\frac{1}{2} \right) (2x - x^3)^{-1/2} \cdot (2 - 3x^2)$$

$$y'(x=1) = \frac{3}{2} (1) (-1) = -\frac{3}{2}$$

Question 2. Calculate

(1 point)

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

Cross (X) the correct answer:

A 0

B $\frac{1}{2}$

C $\frac{1}{4}$

D $\frac{1}{6}$

E 1

F It does not exist.

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{(x-2)(x-1)}{(x-2)(x+2)} =$$

$$\lim_{x \rightarrow 2} \frac{x-1}{x+2} = \frac{2-1}{2+2} = \frac{1}{4}$$

Question 3. Solve the following logarithmic function for x :

(1 point)

$$\ln(2x) - \ln(x+1) = -1$$

Cross (X) the correct answer:

A $\frac{2e+1}{e}$

B $\frac{2}{e-1}$

C $\frac{e+1}{2e-1}$

D $\frac{2}{2e+1}$

E $\frac{e-1}{2e-1}$

F $\frac{1}{2e-1}$

$$\ln(2x) - \ln(x+1) = -1 \Rightarrow \ln \frac{2x}{x+1} = -1$$

$$\Rightarrow \frac{2x}{x+1} = e^{-1} \Rightarrow 2x = e^{-1}x + e^{-1} \Rightarrow 2x - e^{-1}x = e^{-1}$$

$$\Rightarrow x(2 - e^{-1}) = e^{-1} \Rightarrow x = \frac{e^{-1}}{2 - e^{-1}} = \frac{1/e}{2e - 1/e}$$

$$\Rightarrow x = \frac{1}{2e - 1}$$

Question 4. Find the equation of the tangent line to the function $f(x) = \frac{x+3}{x-1}$ at the point $x = 3$.

(1 point)

Cross (X) the correct answer:

A $y = -x + 6$

B $y = 2x + 1$

C $y = 2x + 4$

D $y = 2x - 1$

E $y = -x + 2$

F $y = -x + 3$

$$f'(x) = \frac{(x-1) - (x+3)}{(x-1)^2} = \frac{-4}{(x-1)^2}$$

$$f'(x=3) = m = \frac{-4}{(3-1)^2} = \frac{-4}{4} = -1$$

$$\text{if } x=3 \Rightarrow y = f(x) = \frac{3+3}{3-1} = 3$$

Equation of the tangent line: $y = mx + b$, $m = -1$ & if $x = 3$

$$\Rightarrow y = 3 + b = 3 \Rightarrow b = 6 \Rightarrow \underline{y = -x + 6}$$

Question 5. The cost and revenue functions (in dollars) for producing x units of a certain product are $C(x) = 600 + 5x$ and $R(x) = 500 \ln(1.2x + 10)$.

(a) Find the marginal cost if 20 units are produced.

(2 points)

$$C'(x) = 5$$

$$C'(x=20) = 5 \text{ marginal cost.}$$

(b) Find the marginal revenue if 20 units are produced.

(2 points)

$$R'(x) = 500 \frac{1.2}{1.2x + 10} = \frac{600}{1.2x + 10}$$

$$R'(x=20) = \frac{600}{(1.2)(20) + 10} = \frac{600}{34} \approx 17.65$$

marginal Revenue.

(c) Find the marginal profit if 20 units are produced.

(2 points)

$$P'(x) = R'(x) - C'(x)$$

$$P'(x) = \frac{600}{1.2x + 10} - 5 \text{ and if } x=20, \text{ then}$$

$$P'(x=20) = \frac{600}{34} - 5 = 17.65 - 5 = 12.65$$

marginal Profit.

Question 6. The value of a new car depreciates with age. Suppose the value of a car (in dollars) after t years is given by $V(t) = 20,000e^{-0.1t}$. At what rate is the value of the car changing three years after purchase? (4 points)

$$\hat{V}(t) = -(0.1)(20000) e^{-0.1t}$$

$$\hat{V}(t) = -2000 e^{-0.1t}$$

$$\hat{\hat{V}}(t) = -(0.1)(-2000) e^{-0.1t}$$

$$\hat{\hat{V}}(t) = 200 e^{-0.1t}$$

This is the rate at which the value of the car is changing t years after purchase.

If $t=3$:

$$\hat{\hat{V}}(t=3) = 200 e^{-0.3} \approx 148.16 \$$$

Question 7. Assume that \$3,000 is invested at %8 annual interest rate that is compounded quarterly.

$$\frac{P_0}{n=4}$$

$$\frac{r}{4}$$

(a) Find the value of investment after two years.

(2 points)

$$A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt} = 3000 \left(1 + \frac{0.08}{4}\right)^{4t}$$

$$A(t) = 3000 (1.02)^{4t} : \text{value function after } t\text{-years.}$$

$$A(t=2) = 3000 (1.02)^8 \approx \underline{3,514.97 \$}$$

(b) How long does it take for the value of the investment to triple?

(2 points)

$$3000 (1.02)^{4t} = 3(3000) \Rightarrow (1.02)^{4t} = 3$$

$$\Rightarrow 4t = \log_{1.02} 3 = \frac{\ln 3}{\ln 1.02}$$

$$\Rightarrow t = \frac{\ln 3}{4 \ln 1.02} \approx \underline{13.86 \text{ years}}$$

(c) If interest is compounded continuously, how long does it take for the value to double?

(2 points)

If interest is compounded continuously:

$$A(t) = P_0 e^{rt} = 3000 e^{0.08t} = 2(3000)$$

$$\Rightarrow e^{0.08t} = 2 \Rightarrow 0.08t = \ln 2$$

$$\Rightarrow t \approx \underline{8.66 \text{ years}}$$