Final Exam Phys 1003/ BIT 1203 Fall 2013 and Solutions

Question 1

Object “C”, of mass 0.150 kg collides with a stationary target “T” in an elastic collision. After the collision, the object continues to move at 1/6 of the original speed after collision. You may assume that no external forces act during the collision.

a) What properties of the object-target system are conserved? *(2marks)*
b) Calculate the mass of the target *(10 marks)*
c) Calculate the speed of the two body centre of mass, if the initial speed of object “C” was 2.05 m/s *(4 marks)*
d) Calculate the speed of the target after collision, if the initial speed of object “C” was 2.05 m/s *(4 marks)*

Solution

(a) Momentum and Kinetic Energy

(b)

Using the Principle of Conservation of Momentum:

$$p_i = p_f$$

$$m_c v_c = m_c v_{c,f} + m_t v_t$$

Given the final speed of the car is 1/6 of the original

$$m_c v_c = m_c \frac{v_c}{6} + m_t v_t$$

$$m_c \frac{5v_c}{6} = m_t v_t$$

$$\frac{5m_c}{6m_t} = \frac{v_t}{v_c}$$

We can't solve this yet, as we have two unknowns, the speed of the car and the mass of the truck

Using the Principle of Conservation of Kinetic Energy

$$K_i = K_f$$

$$\frac{1}{2}m_c v_c^2 = \frac{1}{2}m_c v_{c,f}^2 + \frac{1}{2}m_t v_t^2$$

$$m_c v_c^2 = m_c \frac{v_c^2}{36} + m_t v_t^2$$

$$m_c \frac{35}{36} = m_t \frac{v_t^2}{v_c^2}$$

And substituting in from the conservation of momentum result

$$m_c \frac{35}{36} = m_t \left(\frac{5m_c}{6m_t}\right)^2$$

$$m_c \frac{35}{36} = m_t \left(\frac{5m_c}{6m_t}\right)^2$$

$$\frac{35}{36} = \frac{25m_c}{36m_t}$$

$$\frac{35}{25} = \frac{m_c}{m_t}$$

$$m_t = \frac{25 \times 0.150\ kg}{35} = 0.107\ kg\ to\ 3\ sf$$

(c) Momentum is conserved for the two bodies after collision so

$$m_c v_c = m_{com} v_{com}$$

$$\frac{m_c v_c}{m_{com}} = v_{com}$$

$$\frac{m_c v_c}{m_T + m_C} = v_{com}$$

$$\frac{(0.150\ kg)(2.05\frac{m}{s})}{0.150\ kg\left(1+\frac{5}{7}\right)} = v_{com} = 1.20\ m/s\ \ to\ 3\ sf$$

(d) The speed of the target after collision is given by

$$\frac{5m_c}{6m_t} = \frac{v_t}{v_c}$$

$$\frac{5 \times 0.150kg}{6 \times 0.107\ kg}(2.05\ m/s) = v_t = 2.39\ m/s$$

Question 2

An astronaut measures her body mass when in freefall (no gravity) by strapping herself into a chair of mass 0.840 kg which is attached to a spring of force constant 2.62×10^{3} N/m. The chair and astronaut then oscillate with simple harmonic motion. The period of oscillation when she is strapped into the chair is 0.850 seconds and the amplitude of vibration is 5.00×10^{-2} m.

(a) Find the frequency of oscillation *(2 marks)*
(b) Find the mass of the astronaut. *(6 marks)*
(c) Find the maximum elastic potential energy stored in the system *(4 marks)*
(d) Find the maximum speed of the astronaut-chair system *(4 marks)*

Solution

(a) Use $f = \frac{1}{T} = \frac{1}{0.850\ s} = 1.2\ Hz$

(b) For the simple mass-spring system:

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$$

$$m = \frac{kT^2}{4\pi^2}$$

Remember that the mass in the equation above is the combined mass of the chair+ astronaut.

$$m_{total} = \frac{\left(2.62\times10^{3}\ \text{N/m}\right)\times\left(0.850\ \text{s}\right)^2}{4\pi^2} = 47.6\,kg$$

Mass of astronaut = 47.6 – 0.840 kg =46.7 kg

(c) The maximum potential energy sotred by the system is when the springs are at the maximum compression or elongation, i.e. at the amplitude of vibration

$$U_{max} = \frac{1}{2}$$

(d) You could do this by conservation of total mechanical energy

The maximum kinetic energy

v_{max} = Aω

$$\omega = \frac{2\pi}{T}$$

$$v_{\max} = \frac{2\pi A}{T} = \frac{2\pi\left(5.00\times10^{-2}\ \mathrm{m}\right)}{\left(0.850s\right)} = 0.370\ \mathrm{m/s}$$

Question 3

A mass attached to a spring with a spring constant of 109 N/m is moving in simple harmonic motion. The period of motion is 4.50 seconds, and the amplitude of oscillation is 14.0 cm. At time t = 10.0 seconds, the spring is in the equilibrium position. You may assume that the mass of the spring is negligible

(a) What is the equation for the position x as a function of time, assuming no damping? *(10 marks)*

(b) What is the equation for the velocity as a function of time, assuming no damping? (*4 marks)*

(c) After 1.00×10^3 seconds, you notice that the amplitude of oscillation is now only 10.0 cm. Assuming that the oscillation behaves as an exponentially damped system, with a position function

$$x(t) = x_m e^{-\alpha t}\cos(\omega t + \varphi)$$

calculate the damping coefficient α *(4 marks)*

(d) What would be the value of the damping factor $e^{-\alpha t}$ at t = 10.0 seconds.? *(2marks)*

Solution

The general equation is $x(t) = \; x_m \cos(\omega t + \varphi)$

From the information given, we can see that the amplitude of vibration x_m = 14.0 cm = 0.140 m

We need to calculate the angular frequency and we know that the period of motion is 4.50 s

Hence $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.5\ s} = \frac{4\pi}{9} = 1.396\ rad/s$

$$x(t) = (14.0\ cm)\cos\left(\frac{4\pi}{9}t + \varphi\right)$$

and at t = 10.0 seconds, x(10.0) = 0, so we use this to solve for the phase angle

$$0 = (14.0\ cm)\cos\left(\frac{4\pi}{9}10.0 + \varphi\right)$$

$$\cos^{-1} 0 = \left(\frac{4\pi}{9}10.0 + \varphi\right)$$

$$\frac{\pi}{2} = \left(\frac{4\pi}{9}10.0 + \varphi\right)$$

$$\frac{\pi}{2} - \frac{40\pi}{9} = \varphi = \; -12.4\ radians$$

Note that the same solution will result for any value of ϕ where 2π radians is added or subtracted from this value.

$$\varphi = +0.175\ radians\ is\ also\ a\ solution$$

$$x(t) = (14.0\ cm)\cos\left(\frac{4\pi}{9}t - 12.4\right)$$

(b) Now we can differentiate x(t) to find v(t)

$$v(t) = -(14.0 \times \frac{4\pi}{9}\ cm/s)\sin\left(\frac{4\pi}{9}t - 12.4\right)$$

$$v(t) = -(19.5\ cm/s)\sin\left(\frac{4\pi}{9}t - 12.4\right)$$

(c)

You don't actually have the value of x_m because this is the value of the amplitude at t = 0

But we know the ratio$\frac{x(1000}{x(10)}$

$$\frac{x(1000}{x(10)} = \frac{10.0}{14.0} = \frac{e^{-1000\alpha}}{e^{-10\alpha}}$$

$$\frac{10.0}{14.0} = \frac{e^{-1000\alpha+10\alpha}}{1}$$

The cosine part = 1 when at the maximum amplitude.

$$\frac{10.0}{14.0} = e^{-990\alpha}$$

$$0.714 = e^{-990\alpha}$$

$$\ln(0.714) = -990\alpha$$

So $\alpha = 3.40\times10^{-4}\ s^{-1}$

(d) The damping factor will be $e^{-\alpha t}$ at t = 10.0 seconds

$$e^{-3.40\times10^{-4}(10.0\,s)} = 0.997$$

Question 4

You are given the following data for a cylinder of an unknown metal. You may assume that reading errors on the measurement instruments are negligible.

Mass (g)	Length (m)	Diameter (cm)
511.2	0.601	1.203
509.9	0.599	1.215
510.3	0.603	1.142
511.0	0.597	1.102
510.5	0.604	1.174

a) Calculate the density of the cylinder in kg/m^3. *(8 marks)*

b) Calculate the statistical errors on all three parameters, using inefficient statistics *(3 marks)*

c) Now assume that the mass has the dominant error term in it, and derive the error propagation equation for the density, assuming that ONLY the mass variable has an error *(6 marks)*

d) From your equation in (c) calculate the error on the density, and quote the final density with error to the appropriate number of significant figures *(3 marks)*

Solution

Calculate mean mass: 510.58 g or 0.51058 kg

Calculate mean length = 0.6008 m

Calculate mean diameter = 2.1672 cm = 1.1672×10^{-2} m

Volume of cylinder = $V = \frac{\pi}{4}LD^2$

Density = $\rho = \frac{M}{V} = \frac{4M}{\pi L D^2} = \frac{4\times(0.51058\,kg)}{\pi(0.6008\,kg)(1.1672\times10^{-2}m)} = 7942\ kg/m^3$ (iron)

(b)

Statistical error		
M (g)	L (m)	D (cm)
0.26	0.0014	0.0226

(c) Assuming that mass is the dominant error

$$\rho = \frac{M}{V} = \frac{4M}{\pi L D^2} = \frac{4}{\pi} M L^{-1} D^{-2}$$

$$\sigma_\rho^2 = \left(\frac{\partial \rho}{\partial M}\right)^2 \sigma_M^2 + \left(\frac{\partial \rho}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial \rho}{\partial D}\right)^2 \sigma_D^2$$

Hence the errors are

$$\sigma_\rho^2 = \left(\frac{\partial \rho}{\partial M}\right)^2 \sigma_M^2$$

$$\frac{\partial \rho}{\partial M} = \frac{4}{\pi} L^{-1} D^{-2}$$

$$\sigma_\rho^2 = \left(\frac{4}{\pi} L^{-1} D^{-2}\right)^2 \sigma_M^2$$

$$\sigma_\rho^2 = \left(\frac{\rho}{M}\right)^2 \sigma_M^2$$

$$\sigma_\rho = \frac{\rho}{M} \sigma_M$$

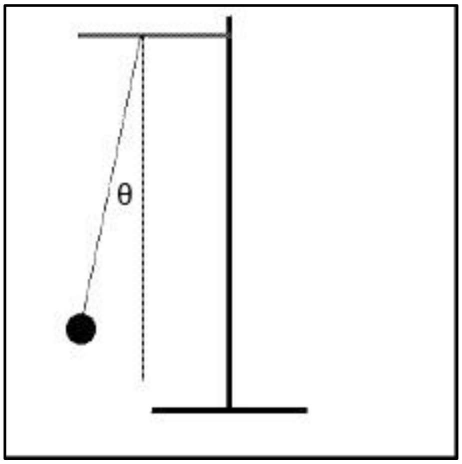
(d)

$$\sigma_\rho = \frac{7942 \frac{kg}{m^2}}{0.51058\, kg} (0.26 \times 10^{-3}\, kg) = 4\, kg/m^3$$

The density is thus = $\rho = 7942 \pm 4\, kg/m^3$

Question 5

A steel wire of circular cross –section, diameter 5.0 mm and length 2.0 m stretches 0.30 mm when a load of 60.0 kg is hung from it. Calculate



- *a)* The Stress. *(4 marks)*
- *b)* The Strain. (*4 marks)*
- *c)* Young's Modulus for the steel. *(4 marks)*
- *d)* If the mass and wire starts to swing, calculate the period, assuming that the system acts like a simple pendulum for small angles of θ. *(4 marks)*
- *e)* Draw a free body diagram of the ball when it is displaced by an angle θ from the vertical. Calculate the tension in the wire when θ = 5.4° (*4 marks)*

Solution

(a) The stress is defined as $S = \frac{Force}{Area}$

Given that the wire has a circular cross section, the area is $A = \pi r^2 = \frac{\pi d^2}{4}$

Hence $S = \frac{4F}{\pi d^2} = \frac{4 \times 60.0\ kg \times 9.81\ m/s^2}{\pi \times (5 \times 10^{-3} m)^2} = 2.99 \times 10^7\ Pa = 3.0 \times 10^7\ Pa\ to\ 2\ s.f.$

(b) The strain is defined as $\sigma = \frac{\Delta l}{l}$

$$\sigma = \frac{\Delta l}{l} = \frac{0.3\ mm}{2000\ mm} = 1.5 \times 10^{-4}$$

Remember there are no units on strain, because it is a ratio of two lengths.

(c) You could calculate the Young's Modulus from first principles:

$$Y = \frac{Stress}{Strain} = \frac{FL}{A\Delta L} = \frac{mgL}{\pi r^2 \Delta L}$$

$$Y = 4 \times \frac{(60.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(2.0\ \mathrm{m})}{\pi\,(5.0 \times 10^{-3}\ m)^2 (0.30 \times 10^{-3}\ m)} = 2.0 \times 10^{11}\ \mathrm{N/m^2}$$

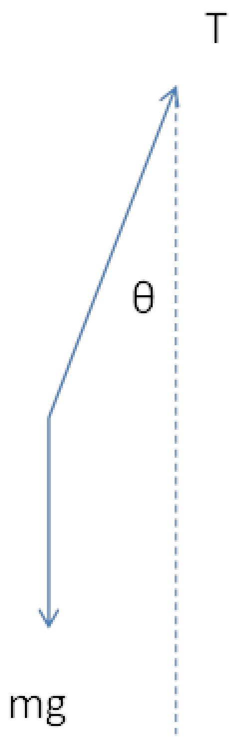
Or you could simply state that by definition, the Young's Modulus is

$$Y = \frac{stress}{strain} = \frac{3.0 \times 10^7\ Pa}{1.5 \times 10^{-4}} = 2.0 \times 10^{11}\ Pa$$

(d) For a simple pendulum use the formula

$$T = 2\pi \sqrt{\frac{l}{g}} = \sqrt{\frac{2.0 + 3.0 \times 10^{-4}\ m}{9.81\ m/s^2}} = 2.8\ seconds$$

(e) FBD



From the FBD, the tension T = mg cos θ so T at 5.4^0 = 60.0 kg ×9.81 m/s^2 cos 5.4^o = 585 N = 590 N to 2 sf.

Question 6

A hiker, of mass 80.0 kg, is rescued by a helicopter lowering a cable of length 15.0 m and mass 8.00 kg. The helicopter accelerates upwards with constant acceleration. The hiker signals the pilot by pulling on the cable, and the resulting wave takes 0.350 sec to travel up the cable

(a) Calculate the tension in the cable *(6 marks)*

(b) Draw a free body diagram of the forces acting on the hiker *(3 marks)*

(c) What is the acceleration of the helicopter? *(7 marks)*

Solution

For the tension in the cable, we can calculate the tension using

$$v = \sqrt{\frac{T}{\left({}^{M}\!/_{L}\right)}}$$

We know the time taken for the signal to move along the wave, so we can write

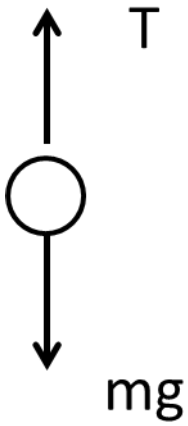
$$v = \frac{L}{t} = \sqrt{\frac{T}{\left({}^{M}\!/_{L}\right)}}$$

$$\frac{L^{2}}{t^{2}} = \frac{T}{\left({}^{M}\!/_{L}\right)}$$

$$\frac{L^{2}}{t^{2}}\left({}^{M}\!/_{L}\right) = T$$

$$T = \frac{ML}{t^2} = \frac{8.00\ kg\ \times 15.0\ m}{0.350\ s^2} = 979.6\ N\ or\ 9.80 \times 10^2 N\ to\ 3\ sf$$

(b) Free body diagram



From the free body diagram and Newton's Second Law (assume up is positive).

$$F_{net} = T - mg = ma$$

The mass m in this case is the total mass being lifted, the mass of the hiker and the rope.

$$\frac{T - mg}{m} = a$$

$$\frac{980\ N - (8.00 + 65.0\ kg)(9.81\ ms^{-2})}{(8.00 + 65.0\ kg)} = a = 3.61\ ms^{-2}$$

Question 7

A 0.350 kg rubber ball is dropped from a height of 2.00 m. It hits the floor and rebounds to a height of 1.80m. Neglect air resistance in any calculations.

a) Calculate the kinetic energy of the ball just as it hits the floor. (*6 marks)*

b) Calculate the momentum of the ball just as it hits the floor *(4 marks)*

c) What is the magnitude of the impulse the *floor* applies to the *ball*? *(7 marks)*

d) If the contact time of the ball on the floor was 0.100 s, calculate the magnitude of the average force on the ball exerted by the floor *(3 marks)*

Solution

(a) The ball is dropped (v = 0) from a height of 2.0 m

Final speed can be calculated using either $v^2 = v_0^2 + 2ax$ or using conservation of energy

$$v^2 = v_0^2 + 2ax$$

$$v^2 = 0 + 2ax$$

Defining down as negative

$$v^2 = 2 \times -9.81\, ms^{-2} \times -2.00\, m$$

$$v = 6.264\, m/s$$

(b) The momentum of the ball is a vector, so the direction depends on the chosen positive direction. In my choice, it will be a negative momentum

$$\vec{p} = -mv = 0.350\, kg\ \times 6.264 \frac{m}{s} = \ -2.19\, kg.m/s$$

Must have some sort of comment about the direction

The impulse applied to the ball is equal to the change in momentum

$$|J| = |\Delta p| = p_f - p_i = mv_f - mv_i$$

We need to know the initial and final speeds. We do not have these specified, but we do have the height that the ball is dropped from and the height it bounces up to, so we can apply the principle of Conservation of Energy

Assuming that there is no air resistance, the total mechanical energy of the ball is conserved when it is dropped

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

The ball is dropped from rest, so the initial kinetic energy is zero. It is dropped from a known height, 2.00 m. When it lands, it has kinetic energy and has lost gravitational potential energy. For convenience, set the floor level as h = 0

$$0 + mgh_0 = \frac{1}{2}mv^2 + 0$$

$$2gh_0 = v^2$$

This speed is the initial speed we use in the Impulse-Momentum Equation

$$|J| = |\Delta p| = p_f - p_i = mv_f - mv_i$$

So

$$\sqrt{2gh_0} = v_i$$

The speed v_f in the Momentum-Impulse Equation represents the speed at which the ball leaves the floor, before rising to the bounce height h_1. Again, assuming conservation of total mechanical energy:

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgh_1$$

$$v^2 = 2gh_1$$

So we can write in the Momentum-Impulse Equation

$$\sqrt{2gh_1} = v_f$$

Now, notice that we have to be careful to define the directions of the initial and final velocity, because the calculations for work-energy only give us the magnitudes. Assuming that down is negative, then

$$\vec{v_i} = -\sqrt{2gh_0}$$

$$\vec{v_f} = +\sqrt{2gh_1}$$

$$|J| = mv_f - mv_i = +m\sqrt{2gh_1} - -m\sqrt{2gh_0}$$

$$|J| = m\sqrt{2gh_1} + m\sqrt{2gh_0} = 4.27\ kg.\frac{m}{s}$$

Note that the unit kg.m/s (mass x velocity) can also be written as N.s (force x time).

(b) Having calculated the magnitude of the impulse, we can now calculate the average force since

$$|J| = F_{avg}\Delta t$$

$$F_{avg} = \frac{|J|}{\Delta t} = \frac{4.27\ kg.m/s}{0.100\ s} = 42.7\ N$$

Question 8

A longitudinal mechanical wave from an earthquake is propagating through the Earth. The following observations have been made by detections systems:

Moving in positive x direction
Amplitude = 35 .0 cm
Wavelength = 619 m
Frequency = 10.5 Hz

- *a)* Calculate the propagation speed of the wave *(4 marks)*
- *b)* Model this wave as a function of form $y(x,t) = y_m \sin(kx \pm \omega t + \varphi)$ by determining values of wavenumber *k*, angular frequency *ω*, amplitude y_m and phase constant *φ*. State the resultant equation for this particular wave *(10 marks).*
- *c)* What is the displacement y, at a point x = 90.3 km from the origin? (*6 marks)*

Solution

(a) Speed v = wavelength x frequency = 6.50 ×10^3 m/s

Wavelength = v/f = 6.50 ×10^3 m/s / 10.5 Hz = 619 m

Speed v = wavelength x frequency = 6.50 ×10^3 m/s

(b) The wavenumber is related to the wavelength by $k = \frac{2\pi}{\lambda}$

So $k = \frac{2\pi}{619\,m} = 1.02 \times 10^{-2}\,m^{-1}\ to\ 3\ sf$

The angular frequency is related to frequency by $\omega = 2\pi f = 2\pi \times 10.5\,Hz = 66.0\,rad/s$

So we can write

$$y(x,t) = y_m \sin(kx \pm \omega t + \varphi)$$

Negative sign indicated positive x direction

$$y(x,t) = (0.350\,m)\sin\left((1.02 \times 10^{-2}\,m^{-1} \times (1.30 \times 10^3 m) - (66.0\frac{rad}{s})t + \varphi\right)$$

At x = 0, y= 0, the amplitude is 0.350 m hence

$$\sin(\varphi) = 1$$

$$\varphi = \sin^{-1} 1 = \frac{\pi}{2}$$

$$y(x,t) = (0.350\ m)\sin\left((1.02 \times 10^{-2}\ m^{-1})x - (66.0\frac{rad}{s})t + \frac{\pi}{2}\right)$$

(c) Time taken for the wave to propagate 90.3×10^3 metres.

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v} = \frac{90.3 \times 10^3\ m}{6.50 \times 10^3} = 13.9\ s$$

Now substitute into the wave function:

$$y(x,t) = (0.350\ m)\sin\left((1.02 \times 10^{-2}\ m^{-1})(90.3 \times 10^3\ m) - (66.0\frac{rad}{s})(13.9\ s) + \frac{\pi}{2}\right)$$

$$y(x,t) = (0.350\ m)\sin(5.23) = -0.304\ m$$