1. Assume : A: an employee eats lunch at the company

B: an employee is female

then : $P(A) = 0.23$ $P(B) = 0.52$ $P(AB) = 0.11$

a) $P(\bar{A}B) + P(A\bar{B}) = P(A) + P(B) - PAB \times 2$

$= 0.23 + 0.52 - 0.11 \times 2$

$= 0.53$

b) No, A and B are not mutally exclusive events for $PAB \neq 0$

2. Assume : A: traveled internationally

B: Can speak a foreign language.

Then : $P(A) = 0.17$ $P(B) = 0.1$ $P(\bar{A}\bar{B}) = 0.81$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1 - P(\bar{A} \cup \bar{B})}{P(B)} = \frac{1 - (P\bar{A} + P\bar{B} - P(\bar{A}\bar{B}))}{P(B)}$$

$$= \frac{1 - (0.83 + 0.9 - 0.81)}{0.1} = \frac{0.08}{0.1} = 0.8$$

3. There are $5 \times 4 = 20$ kinds of combinations.

$P(\text{three are identical}) = (\frac{1}{20})^3$

4. Assume: A: applicant is drug user

B: applicant tests a positive result

Then: $P(B|A) = 0.98$ $P(B|\bar{A}) = 0.1$

$P(A) = 0.1$

$$P(\bar{A}|B) = \frac{P(\bar{A}B)}{P(B)} = \frac{P(B|\bar{A}) \cdot P\bar{A}}{P(B|A) \times P(A) + P(B|\bar{A})P\bar{A})}$$

$$= \frac{0.1 \times 0.9}{0.98 \times 0.1 + 0.1 \times 0.9} = \frac{0.09}{0.188} = 0.4787$$

5. A: person is female

B: supported capital punishment

then: $PB = 0.64$

$PA = 0.48$ $P(B|A) = 0.46$

a). $P(AB) = PA \cdot P(B|A)$

$= 0.48 \times 0.46 = 0.2208$

b). $P(A|B) = \frac{P(AB)}{P(B)}$

$= \frac{0.2208}{0.64} = 0.345$

c) $P(\bar{A}B) = PB - PAB$

$= 0.64 - 0.2208 = 0.4192$

d). $P(\bar{A}\bar{B}) = P\bar{A} - P(\bar{A}B)$

$= (1-0.48) - 0.4192$

$= 0.1008$

e). $P(A\bar{B}) = P(A) - PAB$

$$= 0.48 - 0.2208 = 0.2592$$

f). $P(\bar{A}|\bar{B}) = \dfrac{P\bar{A}\bar{B}}{P\bar{B}} = \dfrac{0.1008}{1-0.64} = 0.28$

g). $P(\bar{B}|A) = \dfrac{P(A\bar{B})}{PA}$

$$= \frac{0.2592}{0.48}$$

$$= 0.54$$

6. a): $XY \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 8 & 9 \\ 0.3 & 0.33 & 0.17 & 0.09 & 0.09 & 0.02 \end{pmatrix}$

$EXY = 0.3 + 2\times 0.33 + 3\times 0.17 + 4\times 0.09 + 6\times 0.09 + 9\times 0.02$

$= 2.55$

b).

$X \sim \begin{pmatrix} 1 & 2 & 3 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$ $\quad Y \sim \begin{pmatrix} 1 & 2 & 3 \\ 0.6 & 0.3 & 0.1 \end{pmatrix}$

$EX = 0.5 + 2\times 0.3 + 3\times 0.2 = 1.7$

$EY = 0.6 + 2\times 0.3 + 3\times 0.1 = 1.5$

$EXEY = 2.55 = EXY$

c) $\forall$ $P(X=i, Y=j)$.

have $P(X=i, Y=j) = P(X=i)$ $P(Y=j)$

$i\ j = 1, 2, 3$

Then $X : Y$ independent

d). $P(Y=2 \mid X=1) = \dfrac{P(Y=2, X=1)}{P(X=1)} = P(Y=2)$

$= 0.3$

e). $EX = 0.5 + 2\times 0.3 + 3\times 0.2 = 1.7$

$EY = 0.6 + 2\times 0.3 + 3\times 0.1 = 1.5$

f). $EX^2 = 0.5 + 4\times 0.3 + 9\times 0.2 = 3.5$

$EY^2 = 0.6 + 4\times 0.3 + 9\times 0.1 = 2.7$

$Var(X) = EX^2 - E^2X = 3.5 - 1.7^2 = 0.61$

$Var(Y) = EY^2 - E^2Y = 2.7 - 1.5^2 = 0.45$

g). $Cov(X,Y) = EXY - EXEY$

$= 2.55 - 1.7\times 1.5 = 0$

Yes, for X Y independent.

h) $X+Y \sim \begin{pmatrix} 2 & 3 & 4 & 5 & 6 \\ 0.3 & 0.33 & 0.26 & 0.09 & 0.02 \end{pmatrix}$

i) $E(X+Y) = 2\times 0.3 + 3\times 0.33 + 4\times 0.26 + 5\times 0.09 + 6\times 0.02 = 3.2$

$Var(X+Y) = E(X+Y)^2 - E^2(X+Y)$

$= 1.06$

j). $Var(X+Y) = 1.06 = 0.61 + 0.45 = Var(X) + Var(Y)$

Yes, for $Cov(X,Y) = 0$

7. a) $P(\text{at least } 3) = \sum_{i=3}^{20} C_{20}^{i} (0.1)^{i} (1-0.9)^{20-i}$

b) $P(\text{all can}) = (0.9)^{20}$

c) X : who ~~do~~ not recognize

Then $X \sim b(20, 0.1)$

~~Thus~~

$E(X) = np = 20 \times 0.1 = 2$

d) $var(X) = np(1-p) = 20 \times 0.1 \times 0.9 = 1.8$

e) $P(X \le 4) = \sum_{i=0}^{4} (0.1)^{i}\, 0.9^{20-i}\, C_{20}^{i}$

8. $X \sim N(75, 4^2)$

a) $P(X < X_p) = 0.9$

$P\left(\frac{X-75}{4} < \frac{X_p - 75}{4}\right) = 0.9$

$\frac{X_p - 75}{4} = 1.29 = U_{0.9}$

$X_p = U_{0.9} \times 4 + 75 = 1.29 \times 4 + 75$

$= 80.16$

8 b) $P(x < x_{0.95}) = 0.95$

Then $x_{0.95} = U_{0.95} \times 4 + 75$

$= 1.645 \times 4 + 75$

$= 81.58$

~~Comp~~

c) $P(x < x_{0.05}) = 0.05$

Then $x_{0.05} = U_{0.05} \times 4 + 75$

$= -2.57 \times 4 + 75$

$= 64.72$

9).

Alcoa Inc.	Reliant Energy	Sea Container
31.98286	13.07	10.55286
27.41879	54.08367	60.97572

	A	R	S
A	27.501		
R	18.99	49.35	
S	4.867	-23.785	52.264

$E(10A + 10R + 10S)$

$= 10[E(A) + E(R) + E(S)]$

$= 10(27.41879 + 54.08367 + 60.97)$

$= 556.05$

9

Var $E(10A + 10R + 10S$

$= 100\,(\mathrm{var}A + \mathrm{var}R + \mathrm{var}S + 200\,\mathrm{var}R) + 2\,\mathrm{cov}(A,S)$
$+ 2\,\mathrm{cov}(R,S)$

$= 14262.91$

(c)

	AB Volvo	Alcoa Inc	TEF	Pentair
mean	8.61	31.98	25.16	28.95
var.	~~...~~ 25.41	27.41	20.43	95.41

	AB	A	~~...~~ T	P
AB	21.78			
A	5.58	23.5		
T	26.75	4.68	81.78	
P	-3.7	-2.39	17.73	17.51

E (10 · Alcoa + 20 · AB + 10 TCF + 20 Pentair

= 1322.843

var (10 · Alcoa + 20 AB + 10 TCF + 20 Pentair

= 83752