Université d'Ottawa | University of Ottawa

GNG1505- Mécanique pour Ingénieurs

Chapitre 2: Statique des Particules

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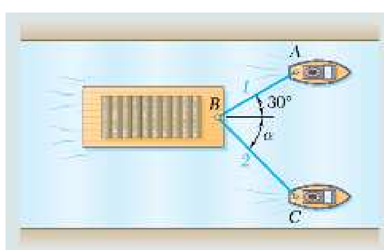
♦ EXERCICES SUPPLÉMENTAIRES

Un chaland est tiré par deux remorqueurs. Si la résultante des deux forces exercées par les remorqueurs est de 5000 N et dirigée parallèlement à l'axe du chaland, déterminez :

a) la tension dans chaque câble pour $\alpha = 45°$;

b) la valeur de α pour laquelle la tension dans le câble 2 est minimale.



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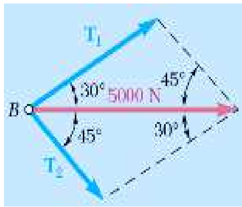
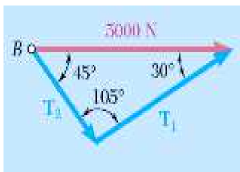
SOLUTION



a) Tension pour $\alpha = 45°$

Solution graphique Si on utilise la règle du parallélogramme, la diagonale (résultante) doit être égale à 5000 N et dirigée vers la droite (*voir la figure ci-contre*). Les côtés sont tracés parallèlement aux câbles. Si le dessin est fait à l'échelle, on trouve

$T_1 = 3700$ N $\quad T_2 = 2600$ N ◀



Solution trigonométrique En utilisant la méthode du triangle, on remarque que celui-ci représente la moitié du parallélogramme précédent (*voir la figure ci-contre*). Par trigonométrie, on a

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\text{ N}}{\sin 105°}$$

d'où $T_1 = 3660$ N et $T_2 = 2590$ N ◀

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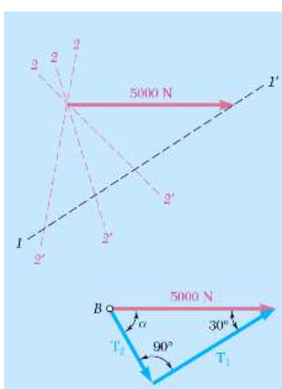
SOLUTION



b) Valeur de α pour laquelle T_2 est minimale La méthode du triangle est utilisée pour calculer la valeur de α. Le schéma ci-contre montre que la droite *1-1'* correspond à la direction connue de T_1. Des directions possibles de T_2 sont indiquées par les droites *2-2'*. On remarque que T_2 a une valeur minimale lorsqu'elle est perpendiculaire à T_1. On a alors

$$T_2 = (5000\text{ N})\sin 30° = 2500\text{ N}$$

Les valeurs correspondantes de T_1 et α sont

$$T_1 = (5000\text{ N})\cos 30° = 4330\text{ N}$$
$$\alpha = 90° - 30°$$

$\alpha = 60°$ ◀

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Une force **F** = (700 N)**i** + (1500 N)**j** est appliquée sur un boulon A. Nous devons déterminer la grandeur de la force et indiquer sa direction en donnant l'angle θ qu'elle forme avec l'horizontale.

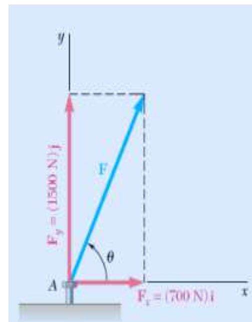
SOLUTION

Dessinons d'abord un schéma pour illustrer les composantes rectangulaires et l'angle θ (*voir la figure* 2.23). L'équation 2.9 donne

$$\tan\theta = \frac{F_y}{F_x} = \frac{1500\ \text{N}}{700\ \text{N}}$$

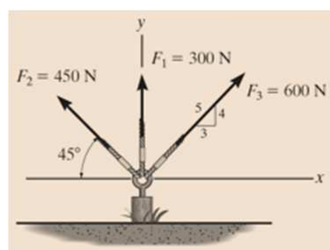
À l'aide d'une calculatrice[2], il nous reste à diviser 1500 N par 700 N; l'arc tangente du quotient donne θ = 65,0°. En isolant F de la seconde équation 2.8, nous obtenons

$$F = \frac{F_y}{\sin\theta} = \frac{1500\ \text{N}}{\sin 65{,}0°} = 1655\ \text{N}$$



♦ EXERCICES SUPPLÉMENTAIRES

Determiner la grandeur et l'angle de la force résultante.



SOLUTION

Plan:

a) Resolve the forces into their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

$\boldsymbol{F_1}$ = {0 $\boldsymbol{i}$ + 300 $\boldsymbol{j}$ } N

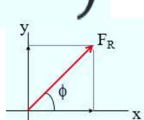
$\boldsymbol{F_2}$ = {– 450 cos (45°) $\boldsymbol{i}$ + 450 sin (45°) $\boldsymbol{j}$ } N
= {– 318.2 $\boldsymbol{i}$ + 318.2 $\boldsymbol{j}$ } N

$\boldsymbol{F_3}$ = { (3/5) 600 $\boldsymbol{i}$ + (4/5) 600 $\boldsymbol{j}$ } N
= { 360 $\boldsymbol{i}$ + 480 $\boldsymbol{j}$ } N

$\boldsymbol{F_R}$ = { (0 – 318.2 + 360) $\boldsymbol{i}$ + (300 + 318.2 + 480) $\boldsymbol{j}$ } N
= { 41.80 $\boldsymbol{i}$ + 1098 $\boldsymbol{j}$ } N

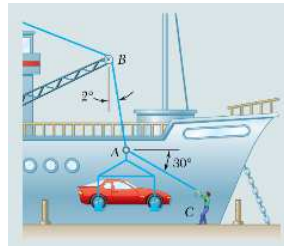
F_R = ((41.80)^2 + (1098)^2)$^{1/2}$ = 1099 N

ϕ = tan^{-1}(1098/41.80) = 87.8°



♦ EXERCICES SUPPLÉMENTAIRES

Lors du déchargement d'un cargo, on soulève une automobile de 1530 kg à l'aide d'un câble. Une corde, attachée au point *A*, est tirée de façon à centrer la voiture sur un point précis. L'angle entre le câble et la verticale est de 2°, tandis que celui formé par la corde et la ligne horizontale est de 30°. Déterminez l'effort de tension dans la corde.



SOLUTION

L'automobile a un poids de 1530 kg × 9,81 N/kg = 15 kN.

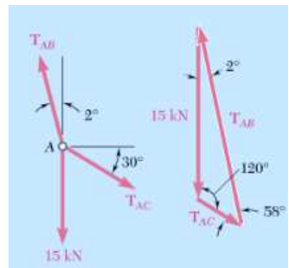
Diagramme du corps libre (DCL) On commence par isoler le point *A*, puis on trace le schéma du DCL: $\mathbf{T}_{AB}$ sera la tension dans le câble *AB*, et $\mathbf{T}_{AC}$, la tension dans la corde *AC*.

Condition d'équilibre Puisqu'on n'a que trois forces appliquées en *A*, on doit tracer le triangle de forces pour exprimer son équilibre. La loi du sinus donne alors

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{15 \text{ kN}}{\sin 58°}$$

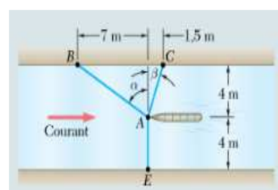
Avec une calculatrice, on calcule le dernier quotient et on l'envoie en mémoire. En multipliant successivement ce quotient par sin 120° et sin 2°, on obtient

$$T_{AB} = 15{,}3 \text{ kN} \qquad T_{AC} = 617 \text{ N}$$



♦ EXERCICES SUPPLÉMENTAIRES

L'équipe responsable de la conception d'un nouveau type de voilier veut connaître la force de traînée à une certaine vitesse. Pour ce faire, un prototype de la coque proposée est placé dans un bassin. L'équipe simule la situation en utilisant trois câbles pour stabiliser le bateau au centre du bassin. Des dynamomètres indiquent à une certaine vitesse les lectures suivantes: câble *AB*, 400 N ; câble *AE*, 600 N. Déterminez la force de traînée appliquée sur la coque et la tension dans le câble *AC*.



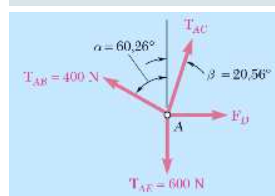
SOLUTION

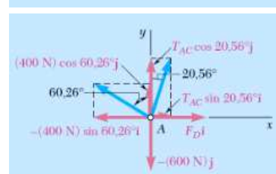
Évaluation des angles On doit commencer par déterminer les angles α et β, qui indiquent les directions des câbles *AB* et *AC*, respectivement. On peut écrire

$$\tan \alpha = \frac{7 \text{ m}}{4 \text{ m}} = 1{,}75 \qquad \tan \beta = \frac{1{,}5 \text{ m}}{4 \text{ m}} = 0{,}375$$

$$\text{donc } \alpha = 60{,}26° \qquad \text{donc } \beta = 20{,}56°$$

Diagramme du corps libre (DCL) On doit d'abord choisir la coque comme point d'équilibre et tracer le diagramme des forces comme illustré. On doit ensuite y inscrire les forces appliquées par les trois câbles ainsi que la force de traînée $\mathbf{F}_D$ exercée par l'eau.





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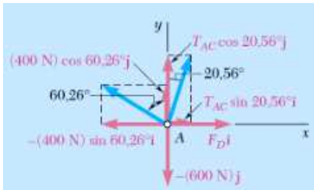
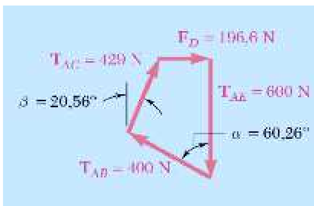
SOLUTION





Condition d'équilibre L'équation suivante exprime la condition d'équilibre de la coque du navire :

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AE} + \mathbf{F}_D = 0 \qquad (1)$$

Étant donné qu'il y a plus de trois forces en présence, on doit les décomposer selon leurs coordonnées x et y :

$$\mathbf{T}_{AB} = -(400\text{ N})\sin 60{,}26°\mathbf{i} + (400\text{ N})\cos 60{,}26°\mathbf{j}$$
$$= -(347{,}3\text{ N})\mathbf{i} + (198{,}4\text{ N})\mathbf{j}$$
$$\mathbf{T}_{AC} = T_{AC}\sin 20{,}56°\mathbf{i} + T_{AC}\cos 20{,}56°\mathbf{j}$$
$$= 0{,}3512T_{AC}\mathbf{i} + 0{,}9363T_{AC}\mathbf{j}$$
$$\mathbf{T}_{AE} = -(600\text{ N})\mathbf{j}$$
$$\mathbf{F}_D = F_D\mathbf{i}$$

En substituant ces expressions dans l'équation 1 et en mettant en facteurs les vecteurs unitaires **i** et **j**, on aura

$$(-347{,}3\text{ N} + 0{,}3512T_{AC} + F_D)\mathbf{i} + (198{,}4\text{ N} + 0{,}9363T_{AC} - 600\text{ N})\mathbf{j} = 0$$

Cette équation sera satisfaite si, et seulement si, les coefficients des vecteurs **i** et **j** sont nuls. On a donc deux conditions d'équilibre exprimées chacune par une équation (une pour chaque axe, x et y). La condition d'équilibre exige que les deux composantes soient nulles.

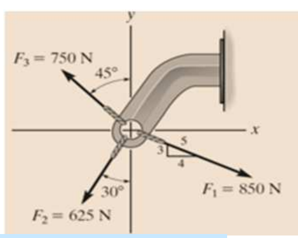
$$(\Sigma F_x = 0:) \qquad -347{,}3\text{ N} + 0{,}3512T_{AC} + F_D = 0 \qquad (2)$$
$$(\Sigma F_y = 0:) \qquad 198{,}4\text{ N} + 0{,}9363T_{AC} - 600\text{ N} = 0 \qquad (3)$$

La solution de l'équation 3 est $T_{AC} = +429\text{ N}$ ◀

En substituant cette valeur dans l'équation 2, on aura $F_D = +196{,}6\text{ N}$ ◀

En traçant le diagramme des forces, on a attribué arbitrairement une direction pour chacune des forces recherchées. Une valeur positive dans la réponse indique que le sens est correct selon l'hypothèse de départ. Le traçage du polygone des forces présentes permettra de valider les résultats.

♦ EXERCICES SUPPLÉMENTAIRES



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force. Show the resultant in a sketch.

SOLUTION

Plan:

a) Resolve the forces into their x and y-components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

F_1 = {850 (4/5) i – 850 (3/5) j } N
= { 680 i – 510 j } N

F_2 = {- 625 sin (30°) i – 625 cos (30°) j } N
= {- 312.5 i – 541.3 j } N

F_3 = {-750 sin (45°) i + 750 cos (45°) j } N
{- 530.3 i + 530.3 j } N

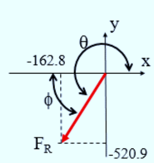
F_R = { (680 – 312.5 – 530.3) i + (–510 – 541.3 + 530.3) j } N
= {– 162.8 i – 520.9 j } N

Now find the magnitude and angle,

F_R = ((– 162.8)^2 + (– 520.9)^2)^{½} = 546 N

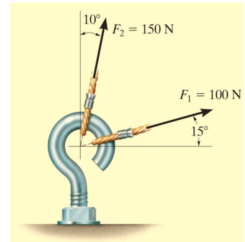
ϕ = tan^{-1}(520.9 / 162.8) = 72.6°

From the positive x-axis, θ = 253°



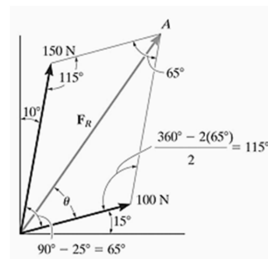
♦ EXERCICES SUPPLÉMENTAIRES

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point *A*, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).



♦ EXERCICES SUPPLÉMENTAIRES

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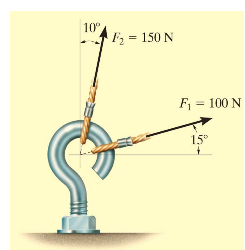


SOLUTION

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

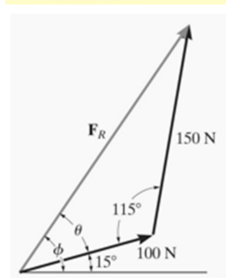
Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115°)$$
$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8° + 15.0° = 54.8° \qquad \textit{Ans.}$$

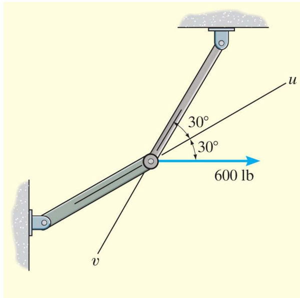
NOTE: The results seem reasonable, since Fig. 2–11*b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them.



(c)

♦ EXERCICES SUPPLÉMENTAIRES

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.



(a)

♦ EXERCICES SUPPLÉMENTAIRES

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the *v* axis until it intersects the *u* axis at point *B*, Fig. 2–12*b*. The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the *v* axis at point *C*, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

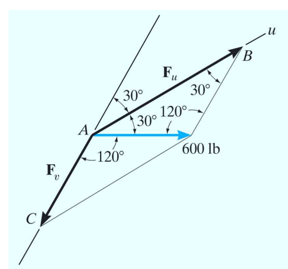
$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

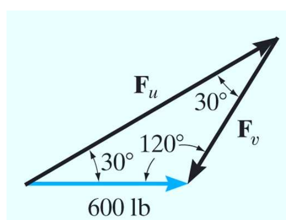
$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$

NOTE: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

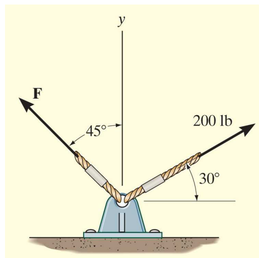


(b)



♦ EXERCICES SUPPLÉMENTAIRES

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.



(a)

♦ EXERCICES SUPPLÉMENTAIRES

SOLUTION

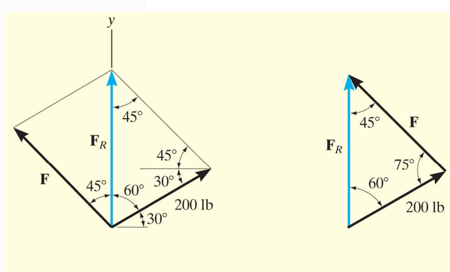
The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of $\mathbf{F}_R$ and **F** are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

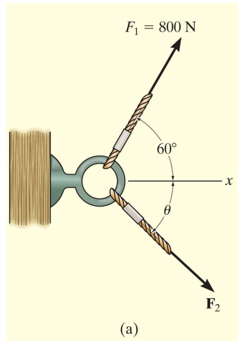
$$F_R = 273 \text{ lb}$$



(b) (c)

♦ EXERCICES SUPPLÉMENTAIRES

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive x axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.



(a)

♦ EXERCICES SUPPLÉMENTAIRES

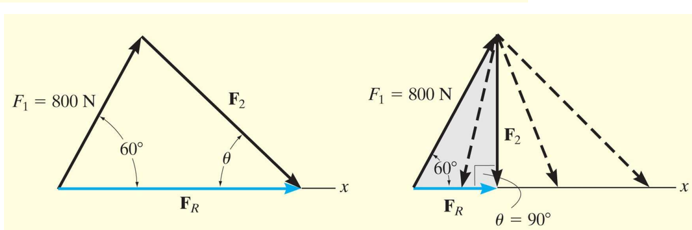
The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

$$\theta = 90^\circ \qquad \textit{Ans.}$$

Since the vector addition now forms the shaded right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800 \text{ N})\cos 60^\circ = 400 \text{ N} \qquad \textit{Ans.}$$

$$F_2 = (800 \text{ N})\sin 60^\circ = 693 \text{ N} \qquad \textit{Ans.}$$



♦ EXERCICES SUPPLÉMENTAIRES

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig. 2–18*a*. Express each force as a Cartesian vector.

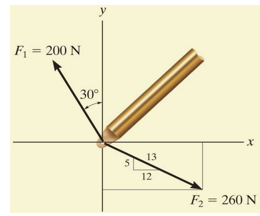
SOLUTION

SOLUTION

Scalar Notation. By the parallelogram law, $\mathbf{F}_1$ is resolved into x and y components, Fig. 2–18*b*. Since $\mathbf{F}_{1x}$ acts in the $-x$ direction, and $\mathbf{F}_{1y}$ acts in the $+y$ direction, we have

$$F_{1x} = -200 \sin 30° \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow \qquad \textit{Ans.}$$

$$F_{1y} = 200 \cos 30° \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow \qquad \textit{Ans.}$$

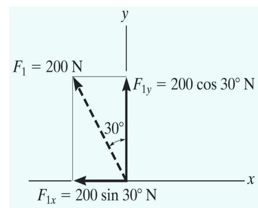


The force $\mathbf{F}_2$ is resolved into its x and y components, as shown in Fig. 2–18*c*. Here the *slope* of the line of action for the force is indicated. From this "slope triangle" we could obtain the angle θ, e.g., $\theta = \tan^{-1}\left(\frac{5}{12}\right)$, and then proceed to determine the magnitudes of the components in the same manner as for $\mathbf{F}_1$. The easier method, however, consists of using proportional parts of similar triangles, i.e.,

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

Similarly,

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$



♦ EXERCICES SUPPLÉMENTAIRES

SOLUTION

Notice how the magnitude of the *horizontal component*, $\mathbf{F}_{2x}$, was obtained by multiplying the force magnitude by the ratio of the *horizontal leg* of the slope triangle divided by the hypotenuse; whereas the magnitude of the *vertical component*, F_{2y}, was obtained by multiplying the force magnitude by the ratio of the *vertical leg* divided by the hypotenuse. Hence, using scalar notation to represent these components, we have

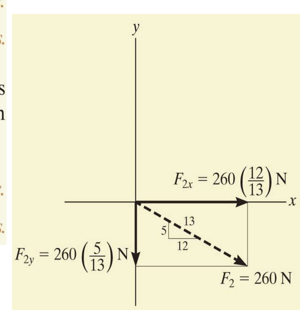
$$F_{2x} = 240 \text{ N} = 240 \text{ N} \rightarrow \qquad \textit{Ans.}$$

$$F_{2y} = -100 \text{ N} = 100 \text{ N} \downarrow \qquad \textit{Ans.}$$

Cartesian Vector Notation. Having determined the magnitudes and directions of the components of each force, we can express each force as a Cartesian vector.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$

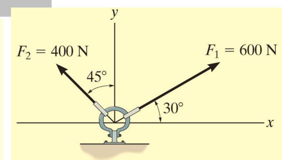
$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$



(c)

♦ EXERCICES SUPPLÉMENTAIRES

The link in Fig. 2–19*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a)

SOLUTION

Scalar Notation. First we resolve each force into its *x* and *y* components, Fig. 2–19*b*, then we sum these components algebraically.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N}$$
$$= 236.8 \text{ N} \rightarrow$$

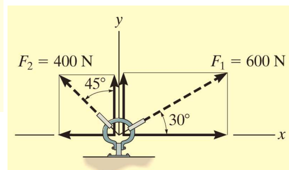
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N}$$
$$= 582.8 \text{ N} \uparrow$$

The resultant force, shown in Fig. 2–19*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2}$$
$$= 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition,

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9^\circ \qquad \textit{Ans.}$$



(b)

♦ EXERCICES SUPPLÉMENTAIRES

SOLUTION

SOLUTION II

Cartesian Vector Notation. From Fig. 2–19*b*, each force is first expressed as a Cartesian vector.

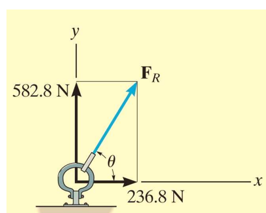
$$\mathbf{F}_1 = \{600 \cos 30^\circ \mathbf{i} + 600 \sin 30^\circ \mathbf{j}\} \text{ N}$$
$$\mathbf{F}_2 = \{-400 \sin 45^\circ \mathbf{i} + 400 \cos 45^\circ \mathbf{j}\} \text{ N}$$

Then,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N})\mathbf{i}$$
$$+ (600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N})\mathbf{j}$$
$$= \{236.8\mathbf{i} + 582.8\mathbf{j}\} \text{ N}$$

The magnitude and direction of $\mathbf{F}_R$ are determined in the same manner as before.

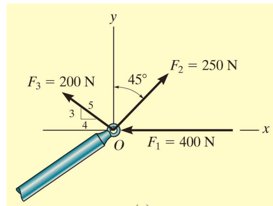
NOTE: Comparing the two methods of solution, notice that the use of scalar notation is more efficient since the components can be found *directly*, without first having to express each force as a Cartesian vector before adding the components. Later, however, we will show that Cartesian vector analysis is very beneficial for solving three-dimensional problems.



(c)

♦ EXERCICES SUPPLÉMENTAIRES

The end of the boom O in Fig. 2–20a is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



(a)

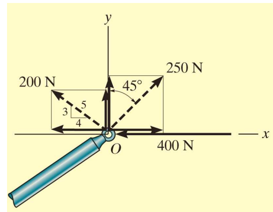
SOLUTION

Each force is resolved into its x and y components, Fig. 2–20b. Summing the x components, we have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = -400\text{ N} + 250 \sin 45^\circ \text{ N} - 200\left(\tfrac{4}{5}\right)\text{ N}$$
$$= -383.2\text{ N} = 383.2\text{ N} \leftarrow$$

The negative sign indicates that F_{Rx} acts to the left, i.e., in the negative x direction, as noted by the small arrow. Obviously, this occurs because F_1 and F_3 in Fig. 2–20b contribute a greater pull to the left than F_2 which pulls to the right. Summing the y components yields

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 250 \cos 45^\circ \text{ N} + 200\left(\tfrac{3}{5}\right)\text{ N}$$
$$= 296.8\text{ N}\uparrow$$



(b)

♦ EXERCICES SUPPLÉMENTAIRES

SOLUTION

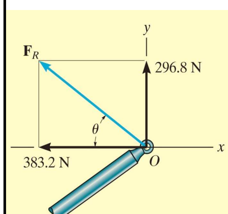
The resultant force, shown in Fig. 2–20c, has a *magnitude* of

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$
$$= 485\text{ N} \qquad \textit{Ans.}$$

From the vector addition in Fig. 2–20c, the direction angle θ is

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ \qquad \textit{Ans.}$$

NOTE: Application of this method is more convenient, compared to using two applications of the parallelogram law, first to add $\mathbf{F}_1$ and $\mathbf{F}_2$ then adding $\mathbf{F}_3$ to this resultant.



(c)

Fig. 2–20

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♦ EXERCICES SUPPLÉMENTAIRES

Express the force **F** shown in Fig. 2–30*a* as a Cartesian vector.

SOLUTION

The angles of 60° and 45° defining the direction of **F** are *not* coordinate direction angles. Two successive applications of the parallelogram law are needed to resolve **F** into its *x, y, z* components. First $\mathbf{F} = \mathbf{F}' + \mathbf{F}_z$, then $\mathbf{F}' = \mathbf{F}_x + \mathbf{F}_y$, Fig. 2–30*b*. By trigonometry, the magnitudes of the components are

$$F_z = 100 \sin 60° \text{ lb} = 86.6 \text{ lb}$$

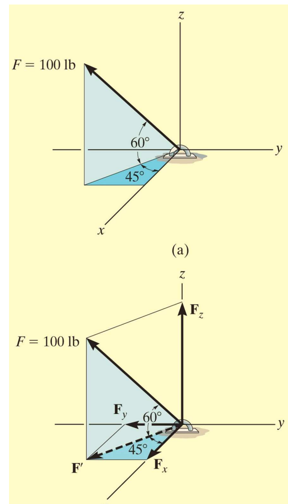
$$F' = 100 \cos 60° \text{ lb} = 50 \text{ lb}$$

$$F_x = F' \cos 45° = 50 \cos 45° \text{ lb} = 35.4 \text{ lb}$$

$$F_y = F' \sin 45° = 50 \sin 45° \text{ lb} = 35.4 \text{ lb}$$

Realizing that $\mathbf{F}_y$ has a direction defined by –**j**, we have

$$\mathbf{F} = \{35.4\mathbf{i} - 35.4\mathbf{j} + 86.6\mathbf{k}\} \text{ lb}$$ *Ans.*



♦ EXERCICES SUPPLÉMENTAIRES

To show that the magnitude of this vector is indeed 100 lb, apply Eq. 2–4,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2} = 100 \text{ lb}$$

If needed, the coordinate direction angles of **F** can be determined from the components of the unit vector acting in the direction of **F**. Hence,

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$= \frac{35.4}{100}\mathbf{i} - \frac{35.4}{100}\mathbf{j} + \frac{86.6}{100}\mathbf{k}$$

$$= 0.354\mathbf{i} - 0.354\mathbf{j} + 0.866\mathbf{k}$$

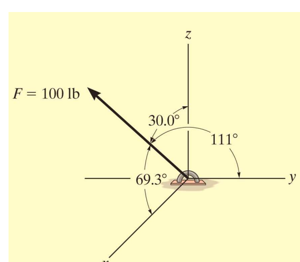
so that

$$\alpha = \cos^{-1}(0.354) = 69.3°$$

$$\beta = \cos^{-1}(-0.354) = 111°$$

$$\gamma = \cos^{-1}(0.866) = 30.0°$$

These results are shown in Fig. 2–30*c*.



(c)

Fig. 2–30

♦ EXERCICES SUPPLÉMENTAIRES

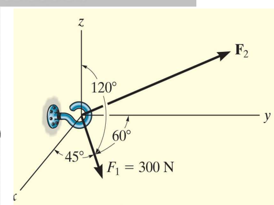
Two forces act on the hook shown in Fig. 2–31*a*. Specify the magnitude of $\mathbf{F}_2$ and its coordinate direction angles so that the resultant force $\mathbf{F}_R$ acts along the positive *y* axis and has a magnitude of 800 N.

SOLUTION

To solve this problem, the resultant force $\mathbf{F}_R$ and its two components, $\mathbf{F}_1$ and $\mathbf{F}_2$, will each be expressed in Cartesian vector form. Then, as shown in Fig. 2–31*b*, it is necessary that $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$.

Applying Eq. 2–9,

$$\begin{aligned}\mathbf{F}_1 &= F_1 \cos \alpha_1 \mathbf{i} + F_1 \cos \beta_1 \mathbf{j} + F_1 \cos \gamma_1 \mathbf{k}\\ &= 300 \cos 45^\circ\, \mathbf{i} + 300 \cos 60^\circ\, \mathbf{j} + 300 \cos 120^\circ\, \mathbf{k}\\ &= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\ \text{N}\\ \mathbf{F}_2 &= F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\end{aligned}$$



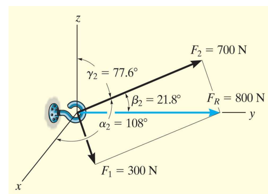
(a)

Since $\mathbf{F}_R$ has a magnitude of 800 N and acts in the $+\mathbf{j}$ direction,

$$\mathbf{F}_R = (800\ \text{N})(+\mathbf{j}) = \{800\mathbf{j}\}\ \text{N}$$

We require

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2\\ 800\mathbf{j} &= 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\\ 800\mathbf{j} &= (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}\end{aligned}$$



(b)

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♦ EXERCICES SUPPLÉMENTAIRES

To satisfy this equation the **i**, **j**, **k** components of $\mathbf{F}_R$ must be equal to the corresponding **i**, **j**, **k** components of $(\mathbf{F}_1 + \mathbf{F}_2)$. Hence,

$$0 = 212.1 + F_{2x} \qquad F_{2x} = -212.1\ \text{N}$$

$$800 = 150 + F_{2y} \qquad F_{2y} = 650\ \text{N}$$

$$0 = -150 + F_{2z} \qquad F_{2z} = 150\ \text{N}$$

The magnitude of $\mathbf{F}_2$ is thus

$$F_2 = \sqrt{(-212.1\ \text{N})^2 + (650\ \text{N})^2 + (150\ \text{N})^2}$$

$$= 700\ \text{N} \qquad \textit{Ans.}$$

We can use Eq. 2–9 to determine $\alpha_2, \beta_2, \gamma_2$.

$$\cos \alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108^\circ \qquad \textit{Ans.}$$

$$\cos \beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8^\circ \qquad \textit{Ans.}$$

$$\cos \gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6^\circ \qquad \textit{Ans.}$$

These results are shown in Fig. 2–31*b*.

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♦ EXERCICES SUPPLÉMENTAIRES

An elastic rubber band is attached to points A and B as shown in Fig. 2–35a. Determine its length and its direction measured from A toward B.

SOLUTION

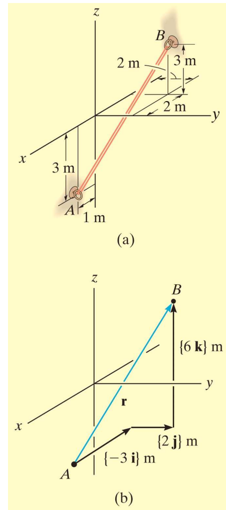
We first establish a position vector from A to B, Fig. 2–35b. In accordance with Eq. 2–11, the coordinates of the tail $A(1\text{ m}, 0, -3\text{ m})$ are subtracted from the coordinates of the head $B(-2\text{ m}, 2\text{ m}, 3\text{ m})$, which yields

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

These components of **r** can also be determined *directly* by realizing that they represent the direction and distance one must travel along each axis in order to move from A to B, i.e., along the x axis $\{-3\mathbf{i}\}$ m, along the y axis $\{2\mathbf{j}\}$ m, and finally along the z axis $\{6\mathbf{k}\}$ m.

The length of the rubber band is therefore

$$r = \sqrt{(-3\text{ m})^2 + (2\text{ m})^2 + (6\text{ m})^2} = 7\text{ m} \qquad \textit{Ans.}$$



♦ EXERCICES SUPPLÉMENTAIRES

Formulating a unit vector in the direction of **r**, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

The components of this unit vector give the coordinate direction angles

$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115° \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4° \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0° \qquad \textit{Ans.}$$

NOTE: These angles are measured from the *positive axes* of a localized coordinate system placed at the tail of **r**, as shown in Fig. 2–35c.

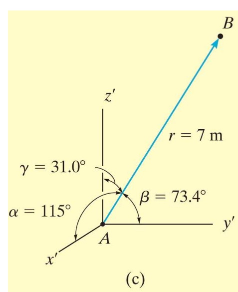


Fig. 2–35

♦ EXERCICES SUPPLÉMENTAIRES

The man shown in Fig. 2–37*a* pulls on the cord with a force of 70 lb. Represent this force acting on the support *A* as a Cartesian vector and determine its direction.

SOLUTION

Force **F** is shown in Fig. 2–37*b*. The *direction* of this vector, **u**, is determined from the position vector **r**, which extends from *A* to *B*. Rather than using the coordinates of the end points of the cord, **r** can be determined *directly* by noting in Fig. 2–37*a* that one must travel from *A* $\{-24\mathbf{k}\}$ ft, then $\{-8\mathbf{j}\}$ ft, and finally $\{12\mathbf{i}\}$ ft to get to *B*. Thus,

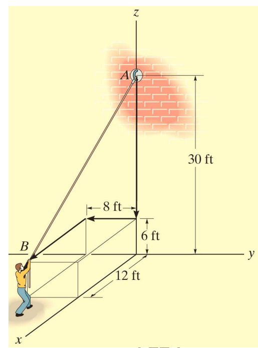
$$\mathbf{r} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ ft}$$

The magnitude of **r**, which represents the *length* of cord *AB*, is

$$r = \sqrt{(12\text{ ft})^2 + (-8\text{ ft})^2 + (-24\text{ ft})^2} = 28\text{ ft}$$

Forming the unit vector that defines the direction and sense of both **r** and **F**, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = \frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}$$



♦ EXERCICES SUPPLÉMENTAIRES

Since **F** has a *magnitude* of 70 lb and a *direction* specified by **u**, then

$$\mathbf{F} = F\mathbf{u} = 70\text{ lb}\left(\frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right)$$

$$= \{30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}\}\text{ lb} \qquad \textit{Ans.}$$

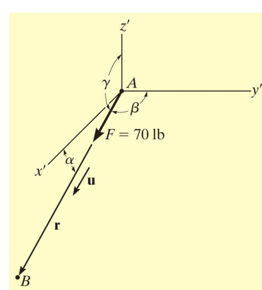
The coordinate direction angles are measured between **r** (or **F**) and the *positive axes* of a localized coordinate system with origin placed at *A*, Fig. 2–37*b*. From the components of the unit vector:

$$\alpha = \cos^{-1}\left(\frac{12}{28}\right) = 64.6° \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{-8}{28}\right) = 107° \qquad \textit{Ans.}$$

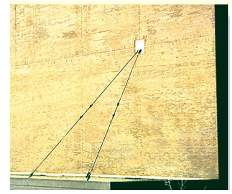
$$\gamma = \cos^{-1}\left(\frac{-24}{28}\right) = 149° \qquad \textit{Ans.}$$

NOTE: These results make sense when compared with the angles identified in Fig. 2–37*b*.



♦ EXERCICES SUPPLÉMENTAIRES

The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the wall hook at A as shown in Fig. 2–38*a*, determine the resultant force acting at A. Express the result as a Cartesian vector.



SOLUTION

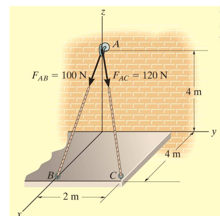
The resultant force $\mathbf{F}_R$ is shown graphically in Fig. 2–38*b*. We can express this force as a Cartesian vector by first formulating $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ as Cartesian vectors and then adding their components. The directions of $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ are specified by forming unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ along the cables. These unit vectors are obtained from the associated position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$. With reference to Fig. 2–38*a*, to go from A to B, we must travel $\{-4\mathbf{k}\}$ m, and then $\{4\mathbf{i}\}$ m. Thus,

$$\mathbf{r}_{AB} = \{4\mathbf{i} - 4\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(4\text{ m})^2 + (-4\text{ m})^2} = 5.66\text{ m}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = (100\text{ N})\left(\frac{4}{5.66}\mathbf{i} - \frac{4}{5.66}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\text{ N}$$



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♦ EXERCICES SUPPLÉMENTAIRES

To go from A to C, we must travel $\{-4\mathbf{k}\}$ m, then $\{2\mathbf{j}\}$ m, and finally $\{4\mathbf{i}\}$. Thus,

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(4\text{ m})^2 + (2\text{ m})^2 + (-4\text{ m})^2} = 6\text{ m}$$

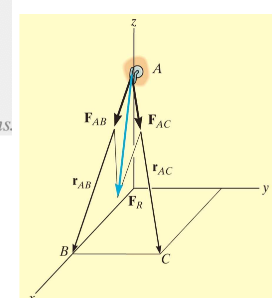
$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = (120\text{ N})\left(\frac{4}{6}\mathbf{i} + \frac{2}{6}\mathbf{j} - \frac{4}{6}\mathbf{k}\right)$$

$$= \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\text{ N}$$

The resultant force is therefore

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\text{ N} + \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\text{ N}$$

$$= \{151\mathbf{i} + 40\mathbf{j} - 151\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$



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♦ EXERCICES SUPPLÉMENTAIRES

The force in Fig. 2–39*a* acts on the hook. Express it as a Cartesian vector.

SOLUTION

As shown in Fig. 2–39*b*, the coordinates for points *A* and *B* are

$$A(2\text{ m}, 0, 2\text{ m})$$

and

$$B\left[-\left(\frac{4}{5}\right)5\sin 30^\circ\text{ m}, \left(\frac{4}{5}\right)5\cos 30^\circ\text{ m}, \left(\frac{3}{5}\right)5\text{ m}\right]$$

or

$$B(-2\text{ m}, 3.464\text{ m}, 3\text{ m})$$

Therefore, to go from *A* to *B*, one must travel $\{-4\mathbf{i}\}$ m, then $\{3.464\mathbf{j}\}$ m, and finally $\{1\mathbf{k}\}$ m. Thus,

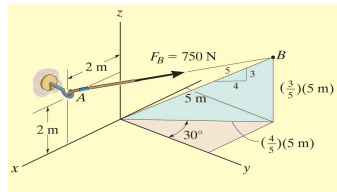
$$\mathbf{u}_B = \left(\frac{\mathbf{r}_B}{r_B}\right) = \frac{\{-4\mathbf{i} + 3.464\mathbf{j} + 1\mathbf{k}\}\text{ m}}{\sqrt{(-4\text{ m})^2 + (3.464\text{ m})^2 + (1\text{ m})^2}}$$

$$= -0.7428\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k}$$

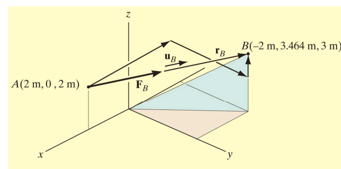
Force $\mathbf{F}_B$ expressed as a Cartesian vector becomes

$$\mathbf{F}_B = F_B\,\mathbf{u}_B = (750\text{ N})(-0.74281\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k})$$

$$= \{-557\mathbf{i} + 482\mathbf{j} + 139\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$



(a)



(b)

♦ EXERCICES SUPPLÉMENTAIRES

Determine the magnitudes of the projection of the force **F** in Fig. 2–42 onto the *u* and *v* axes.

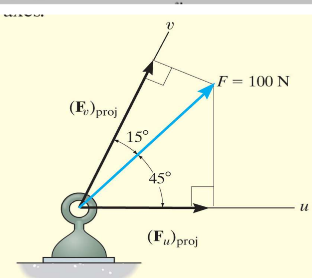


Fig. 2–42

SOLUTION

Projections of Force. The graphical representation of the *projections* is shown in Fig. 2–42. From this figure, the magnitudes of the projections of **F** onto the *u* and *v* axes can be obtained by trigonometry:

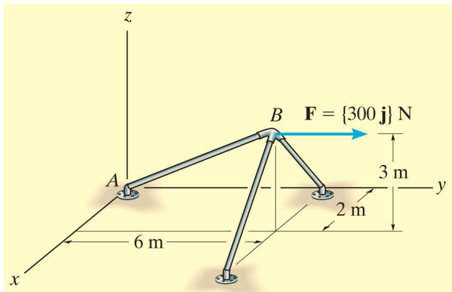
$$(F_u)_{\text{proj}} = (100\text{ N})\cos 45^\circ = 70.7\text{ N} \qquad \textit{Ans.}$$

$$(F_v)_{\text{proj}} = (100\text{ N})\cos 15^\circ = 96.6\text{ N} \qquad \textit{Ans.}$$

NOTE: These projections are not equal to the magnitudes of the components of force **F** along the *u* and *v* axes found from the parallelogram law. They will only be equal if the *u* and *v* axes are *perpendicular* to one another.

♦ EXERCICES SUPPLÉMENTAIRES

The frame shown in Fig. 2–43*a* is subjected to a horizontal force $\mathbf{F} = \{300\mathbf{j}\}$ N. Determine the magnitudes of the components of this force parallel and perpendicular to member AB.



♦ EXERCICES SUPPLÉMENTAIRES

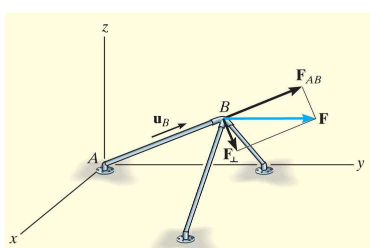
The magnitude of the component of **F** along AB is equal to the dot product of **F** and the unit vector $\mathbf{u}_B$, which defines the direction of AB, Fig. 2–43*b*. Since

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.286\,\mathbf{i} + 0.857\,\mathbf{j} + 0.429\,\mathbf{k}$$

then

$$\begin{aligned} F_{AB} &= F\cos\theta = \mathbf{F}\cdot\mathbf{u}_B = (300\mathbf{j})\cdot(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}) \\ &= (0)(0.286) + (300)(0.857) + (0)(0.429) \\ &= 257.1\ \text{N} \end{aligned}$$

Ans.



♦ EXERCICES SUPPLÉMENTAIRES

Since the result is a positive scalar, $\mathbf{F}_{AB}$ has the same sense of direction as $\mathbf{u}_B$, Fig. 2–43*b*.

Expressing $\mathbf{F}_{AB}$ in Cartesian vector form, we have

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_B = (257.1\text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k})$$
$$= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

The perpendicular component, Fig. 2–43*b*, is therefore

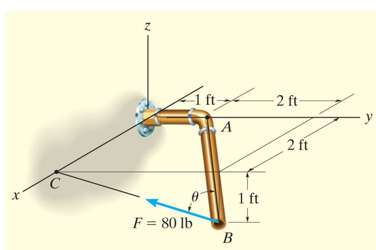
$$\mathbf{F}_\perp = \mathbf{F} - \mathbf{F}_{AB} = 300\mathbf{j} - (73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k})$$
$$= \{-73.5\mathbf{i} + 79.6\mathbf{j} - 110\mathbf{k}\}\text{ N}$$

Its magnitude can be determined either from this vector or by using the Pythagorean theorem, Fig. 2–43*b*:

$$F_\perp = \sqrt{F^2 - F_{AB}^2} = \sqrt{(300\text{ N})^2 - (257.1\text{ N})^2}$$
$$= 155\text{ N} \qquad \textit{Ans.}$$

♦ EXERCICES SUPPLÉMENTAIRES

The pipe in Fig. 2–44*a* is subjected to the force of $F = 80$ lb. Determine the angle θ between **F** and the pipe segment *BA* and the projection of **F** along this segment.



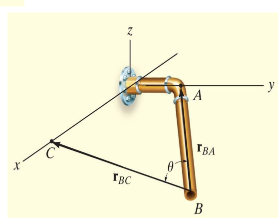
Solution

Angle θ. First we will establish position vectors from *B* to *A* and *B* to *C*; Fig. 2–44*b*. Then we will determine the angle θ between the tails of these two vectors.

$$\mathbf{r}_{BA} = \{-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}\text{ ft},\ r_{BA} = 3\text{ ft}$$
$$\mathbf{r}_{BC} = \{-3\mathbf{j} + 1\mathbf{k}\}\text{ ft},\ r_{BC} = \sqrt{10}\ ft$$

Thus,

$$\cos\theta = \frac{\mathbf{r}_{BA}\cdot\mathbf{r}_{BC}}{r_{BA}r_{BC}} = \frac{(-2)(0) + (-2)(-3) + (1)(1)}{3\sqrt{10}} = 0.7379$$
$$\theta = 42.5° \qquad \textit{Ans.}$$



♦ EXERCICES SUPPLÉMENTAIRES

Components of F. The component of **F** along *BA* is shown in Fig. 2–44*c*. We must first formulate the unit vector along *BA* and force **F** as Cartesian vectors.

$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{(-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k})}{3} = -\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$$

$$\mathbf{F} = 80\text{ lb}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = 80\left(\frac{-3\mathbf{j} + 1\mathbf{k}}{\sqrt{10}}\right) = -75.89\mathbf{j} + 25.30\mathbf{k}$$

Thus,

$$F_{BA} = \mathbf{F} \cdot \mathbf{u}_{BA} = (-75.89\mathbf{j} + 25.30\mathbf{k}) \cdot \left(-\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right)$$

$$= 0\left(-\frac{2}{3}\right) + (-75.89)\left(-\frac{2}{3}\right) + (25.30)\left(\frac{1}{3}\right)$$

$$= 59.0\text{ lb} \qquad \textit{Ans.}$$

NOTE: Since θ has been calculated, then also, $F_{BA} = F\cos\theta =$ 80 lb cos 42.5°