November 6

EXPONENTIAL & POLY

Example!

1 $\int xe^x dx$

u	v'
x	e^x
1	e^x
0	e^x

$= xe^x - e^x + C$

$= e^x(x-1) + C$

2 $\int x^2 e^{-2x} dx$

u	v'
x^2	e^{-2x}
$2x$	$-1/2\, e^{-2x}$
2	$1/4\, e^{-2x}$
0	$-1/8\, e^{-2x}$

$= C - \left(\frac{x^2}{2} + \frac{2x}{4} + \frac{2}{8}\right) e^{-2x}$

$= C - \frac{x^2 + x + 1/2}{2e^{2x}}$

3. $\int x^3 e^{5x} dx$

u	v'
x^3	e^{5x}
$3x^2$	$1/5\, e^{5x}$
$6x$	$1/25\, e^{5x}$
6	$1/625\, e^{5x}$
0	$1/625\, e^{5x}$

$= e^{5x}\left(\frac{x^3}{5} - \frac{3x^2}{25} + \frac{6x}{125} - \frac{6}{625}\right) + C$

LOG (or Inv. Trig) & POLY

Example

1. $\int x^2 \ln x \, dx$

u	v'
$\ln x$	x^2
$1/x$	$x^3/3$

$= \frac{x^3 \ln x}{3} - \int \frac{1}{x} \frac{x^3}{3} \, dx$

$= \frac{x^3 \ln x}{3} - \frac{1}{3} \int x^2 \, dx = \frac{x^3 \ln x}{3} - \frac{1}{9} x^3 + C$

2. $\int (x^3 + 2x^2) \ln x \, dx$

u	v'
$\ln x$	$x^3 + 2x^2$
$1/x$	$x^4/4 + 2x^3/3$

$= \left(\frac{x^4}{4} + \frac{2x^3}{3}\right) \ln x - \int \frac{1}{x} \left(\frac{x^4}{4} + \frac{2x^3}{3}\right) dx$

$= \left(\frac{x^4}{4} + \frac{2x^3}{3}\right) \ln x - \frac{1}{4} \int x^3 \, dx - \frac{2}{3} \int x^2 \, dx$

$= \left(\frac{x^4}{4} + \frac{2x^3}{3}\right) \ln x - \frac{x^4}{16} - \frac{2x^3}{9} + C$

3. $\int \ln x \, dx = \int (1) \ln x \, dx$

u	v'
$\ln x$	1
$1/x$	x

$= x \ln x - \int \frac{1}{x} \cdot x \, dx$

$= x \ln x - \int dx$

$= x \ln x - x + C$

$x\,x(\ln x - 1) + C$

EXPONENTIAL & WAVE (OR 2 WAVES)

1 $I = \int \sin(2x)\cos x\, dx$

u	v'
$\sin(2x)$	$\cos x$
$2\cos(2x)$	$\sin x$
$-4\sin(2x)$	$-\cos x$

$$I = \sin x \sin(2x) + 2\cos x \cos(2x) + 4\int \sin(2x)\cos x\, dx$$

$$I = \sin x \sin(2x) + 2\cos x \cos(2x) + 4I$$

$$I - 4I = \sin x \sin(2x) + 2\cos x \cos(2x)$$

$$-3I = \sin x \sin(2x) + 2\cos x \cos(2x)$$

$$I = C - \tfrac{1}{3}\sin x \sin(2x) - \tfrac{2}{3}\cos x \cos(2x)$$