Math/Comp 3808 Winter 2017 Test 2 Mar. 22, 4:35—5:25pm

Instruction: Provide clear and well justified answers, in good handwriting. You may lose marks if you don't justify your answers. You may use a non-programmable calculator and a one-sided formula sheet of size 8×11".

[7]1. Suppose you are dealt {J♦,Q♣,K♠} in the game Let It Ride, and each of your three bets is **$10**.

(a) Find the probabilities that your final hand is, respectively, straight, three of a kind, two pair, and high pair.

(b) Use part (a) and the payoff table below to compute your expected value of bet #1 if you let it ride. (Recall that your final hand consists of the three initial cards and two random cards from the remaining 49 cards)

Final Hand	Straight	Three of a kind	Two pair	High pair (10,10)-(A,A)
Payoff	5 to 1	3 to 1	2 to 1	1 to 1

Solution (a) The required probabilities are summarized in the following table. **[4]**

Final Hand	Probability
High pair	$\frac{3\times3\times40+2C(4,2)}{C(49,2)}=0.316$
Two pair	$\frac{C(3,2)\times3\times3}{C(49,2)}=0.023$
Triple	$\frac{C(3,2)\times3}{C(49,2)}=0.0076$
Straight	$\frac{2\times4\times4}{C(49,2)}=0.0272$

(b) Hence your losing probability (lower than pair 10) is

$1-(0.316+0.023+0.0076+0.0272)=0.612$. **[1]**

The player's expected value is (note the $10 bet)

$10(0.316\times1+0.023\times2+0.0076\times3+0.0272\times5-0.612\times1)=-1.05$ dollars. **[2]**

Deduct 1 mark if the factor 10 is missing.

[4]2. Recall that two poker hands are equivalent if there is a permutation of four suits which converts one hand to the other.

(a) Find all permutations of the four suits which fix the hand {2♠,4♠,Q♣,Q♠,Q♦}.

(b) Use part (a) to find the number of hands which are in the equivalent class of {2♠,4♠,Q♣,Q♠,Q♦}.

Solution (a) If a permutation of the four suits fixes the hand, it has to fix the spade suit and the heart suit, and it may swap club and diamond. Hence there are exactly two permutations of the suits which fix the hand, namely, the identity permutation

and the permutation which swaps club and diamond.
(b) Consequently, the number of hands which are in the equivalent class is equal to $24/2 = 12$. **[2]**

[7]3. This question deals with Three Card Poker. Recall that the dealer's hand does not qualify if his hand is lower than Queen-high. Suppose the player is dealt {2♠,4♠,Q♣}.
(a) Find the probability that the dealer's hand does not qualify.
(b) Find the probability that the dealer's hand qualifies and is lower than player's hand.
(c) Find the probability that the dealer's hand ties the player's hand.

Solution (a) **[5]**
An unqualified hand is of the form $\{x, y, z\}$, where x, y, z are distinct face values from the set {2,3,…,J}. Also it should not be a straight or a flush. We consider the following four cases depending on whether x, y, z are equal to 2 and/or 4.
Case 1: $x, y, z \neq 2{,}4$. The number of such hands is equal to (excluding straight and flush) $(C(8,3) - 5)\times(4\times4\times4 - 4) = 3060$.

Case 2: $x = 2, y, z \neq 2{,}4$. The number of such hands is equal to (excluding flush) $C(8,2)\times(4\times4\times3 - 3) = 1260$.

Case 3: $x = 4, y, z \neq 2{,}4$.
The number of such hands is equal to (excluding straight and flush)
$(C(8,2) - 2)\times(3\times4\times4 - 3) = 1170$.
Case 4: $x = 4, y = 2$. The number of such hands is equal to (excluding flush)
The number of such hands is equal to (excluding flush)

$$7\times(3\times3\times4 - 3) = 231.$$

Hence the required probability is equal to $\frac{3060+1260+1170+231}{C(49,3)} = 0.3105$.
(b) **[1]** The dealer's hand must be of the form (2,3,Q). The number of such hands is equal to $3\times4\times3 - 2 = 34$. Hence the required probability is equal to 34/C(49,3) = 0.002.
(c) **[1]** The tie probability is $\frac{3\times3\times3-2}{C(49,3)} = 0.001$.