Math/Comp 3808 Winter 2017 Test 1 Solution

Instruction: Provide clear and well justified answers, in good handwriting. You may lose marks if you don't justify your answers. You may use a non-programmable calculator and a one-sided formula sheet of size 8×11".

[5]1. Suppose 5 people are randomly selected from the street.
(i) Find the probability that no two of them were born in the same month.
(ii) Suppose you bet for the event that no two of them were born in the same month, what is the odds against you?
(iii) Suppose the payoff for your bet is k to 1. Find the smallest positive integer k such that your expected value is positive.

Solution (i) The required probability is $p = \frac{12\times11\times10\times9\times8}{12^5} = 55/144$. [2]
(ii) The odds against you is $\frac{1-p}{p} = \frac{89}{55} \approx 1.6$. [2]
(iii) The smallest value of k is 2. [1]

[5]2. Recall that a poker hand consists of 5 unordered cards from the standard deck of 52 cards. A *full house* poker hand contains a pair and a triple, that is, its face values are of the form x, x, y, y, y. Find the probability that a random poker hand is a full house. (no need to simplify your answer for this question, answers expressed in terms of factorials and binomial numbers etc. are acceptable.)

Solution The total number of poker hands is C(52,5). To find the number of full house poker hands, we first select a face value, in C(13,1) ways for the triple. Then we select a face value for the pairs and this is done in C(12,1) ways. [2] Finally we choose three suits for the x and two suits for the y. This is done in C(4,3)C(4,2) ways [2]. Thus the number of full house poker hands is equal to C(13,1)C(12,1)C(4,3)C(4,2), and the required probability is C(13,1)C(12,1)C(4,3)C(4,2)/C(52,5). [1]

[8]3. Recall that the pick-3 Keno ticket costs $1, and the player picks 3 numbers from 80 numbers which will be compared with 20 numbers randomly drawn from the 80 numbers. The paybacks are given in the following table.

# Matches	0 or 1	2	3
Payback	0	1	43

(i) Find the probabilities of winning various types of prizes.
(ii) Find the house edge of the pick-3 ticket (expressed in percentage).
(iii) Find the standard deviation of the pick-3 ticket (give numerical value).
(iv) Suppose a player played this game 10000 rounds, find the 95% confidence interval of the player's net loss. (The z value corresponding to 95% confidence

interval is 1.96).

Solution (i) The house net results are 1,0, and −42, with respective probabilities $p_1 = 1 - p_2 - p_3 \approx 0.84737, p_2 = \mathrm{C}(20,2)\mathrm{C}(60,1)/\mathrm{C}(80,3) \approx 0.13875, p_3 = \mathrm{C}(20,3)/\mathrm{C}(80,3) \approx 0.013875$. [3]

(ii) The house edge is $\mu = 1\times p_1 + 0\times p_2 - 42\times p_3 \approx 26.46\%$.
Or use $\mu = 1 - 1\times p_2 - 43\times p_3$. [2]

(iii) Hence the standard deviation is equal to
$\sqrt{p_1(1-\mu)^2 + p_2(0-\mu)^2 + p_3(-42-\mu)^2} \approx 5.03$ [2]

(iv) The margin of error is $100\sigma z \approx 985$. Hence the 95% confidence interval is
$[10000\mu - 985, 10000\mu + 985] = [1661,3631]$ [1]