CHAPTER 2

CLOSED ECONOMY

ONE PERIOD MODELS

OF THE MACROECONOMY

Objectives of this chapter

- To build and analyze a static macroeconomic model describing the behavior of consumers and firms
  - Consumers make choices concerning consumption and leisure
  - Firms make choices concerning production
  - The model considers only one time period

    $\rightarrow$ Decisions are static
- Optimization
  - Consumers: Utility maximization given a budget constraint
  - Firms: Profit maximization given a technological constraint
- Competitive behavior
  - Agents are price-takers
- Representative agents
  - Existence of many identical agents or of many heterogenous agents whose behavior may be perfectly aggregated

PART 1

THE REPRESENTATIVE CONSUMER

The representative consumer's preferences

- Two goods:
  - Consumption good C

    $\rightarrow$ Aggregation of all consumption goods in the economy
  - Leisure l

    $\rightarrow$ Time spent not working (recreational activities, domestic work)
- Consumption bundle: Specific combination of the two goods (C_1, l_1)
- Utility function

$$U = U(C, l)$$

  - It allows to rank the consumption bundles

$$\begin{aligned} U(C_1, l_1) &> U(C_2, l_2) \rightarrow \text{The consumer prefers } (C_1, l_1) \\ U(C_1, l_1) &< U(C_2, l_2) \rightarrow \text{The consumer prefers } (C_2, l_2) \\ U(C_1, l_1) &= U(C_2, l_2) \rightarrow \text{The consumer is indifferent} \end{aligned}$$

- Assumptions on the utility function:

  1) More is always preferred to less

     - For a given quantity of one good, if the quantity of the other good increases

       $\rightarrow$ Utility increases

  2) The consumer prefers diversity

     - Suppose that $U(C_1, l_1) = U(C_2, l_2)$

       with C_1 high and l_1 low, C_2 low and l_2 high

       $\rightarrow \quad U\left(\frac{C_1+C_2}{2}, \frac{l_1+l_2}{2}\right) > U(C_1, l_1)$

  3) Consumption and leisure are normal goods

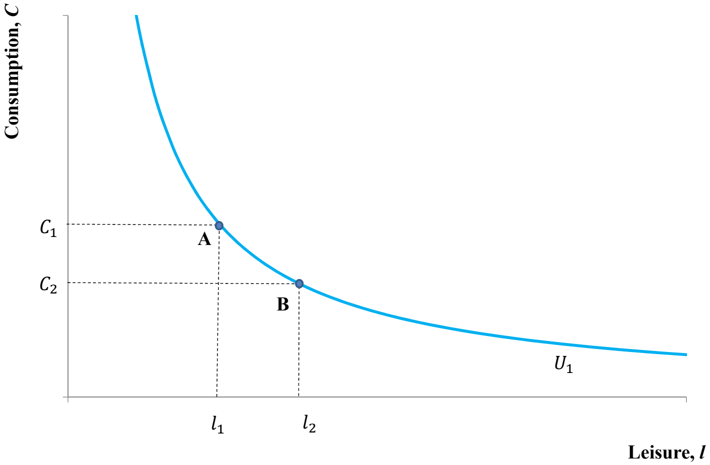
     - If income increases $\rightarrow$ Consumption and leisure increase

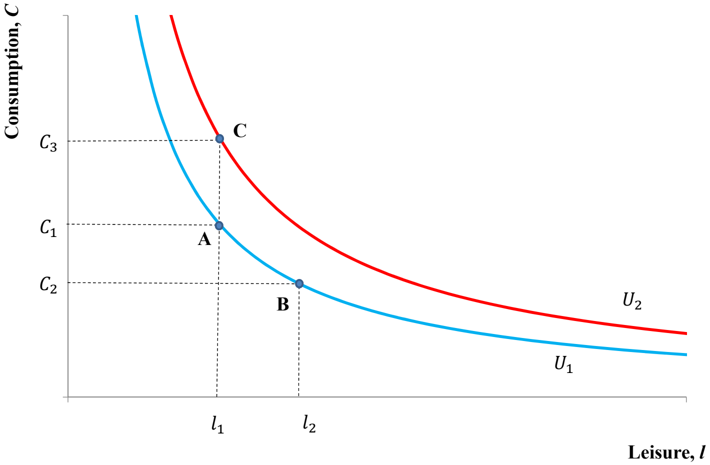
- Indifference curve
    - All the combinations of consumption and leisure giving the same utility level

      $\rightarrow$ The consumer is indifferent between all the bundles belonging to the same indifference curve

- Infinite number of indifference curves

- Properties
    - An indifference curve is decreasing
    - An indifference curve is convex
    - Indifference curves never cross


Consumption, C
C1
C2
A
B
U1
l1
l2
Leisure, l


Consumption, C
C
A
B
C_3
C_1
C_2
U_2
U_1
l_1
l_2
Leisure, l

- Differentiation of the utility function all along an indifference curve

$$
\begin{aligned}
\mathrm{d}U(C,l) &= \frac{\partial U}{\partial C}\cdot \mathrm{d}C + \frac{\partial U}{\partial l}\cdot \mathrm{d}l = 0 \\
\rightarrow \frac{\mathrm{d}C}{\mathrm{d}l} &= -\frac{\frac{\partial U}{\partial l}}{\frac{\partial U}{\partial C}}
\end{aligned}
$$

- The assumption that consumers prefer more to less implies that:

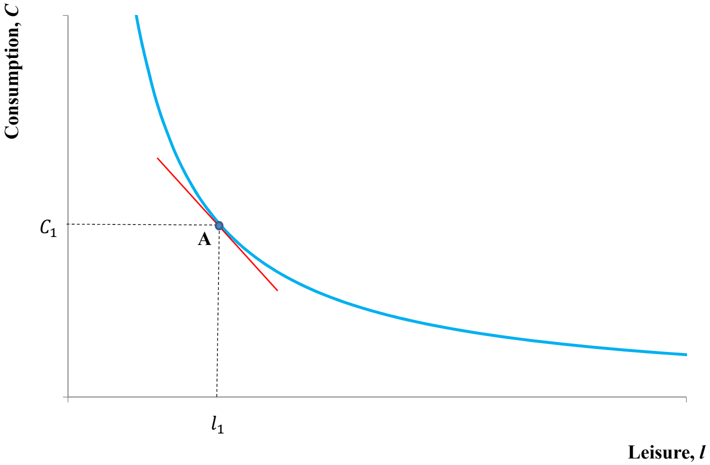
  i) $\frac{\partial U}{\partial C} > 0$ and $\frac{\partial U}{\partial l} > 0$

  ii) Indifference curves are decreasing

$$
\frac{\mathrm{d}C}{\mathrm{d}l} < 0
$$

- Marginal rate of substitution of leisure to consumption $(MRS_{l,C})$:
  - Quantity of consumption that the consumer has to give up in order to obtain one additional unit of leisure and to maintain the same level of utility

$$
MRS_{l,C} = -\frac{\mathrm{d}C}{\mathrm{d}l} = \frac{\frac{\partial U}{\partial l}}{\frac{\partial U}{\partial C}} > 0
$$



Consumption, C
C_1
A
l_1
Leisure, l

- The assumption that consumer prefers diversity implies that:

  i) If $U(C_1, l_1) = U(C_2, l_2)$

  $\rightarrow \quad U\left(C_3 = \frac{C_1+C_2}{2},\ l_3 = \frac{l_1+l_2}{2}\right) > U(C_1, l_1)$

  ii) Indifference curves are convex $\left(\frac{d^2C}{dl^2} > 0\right)$

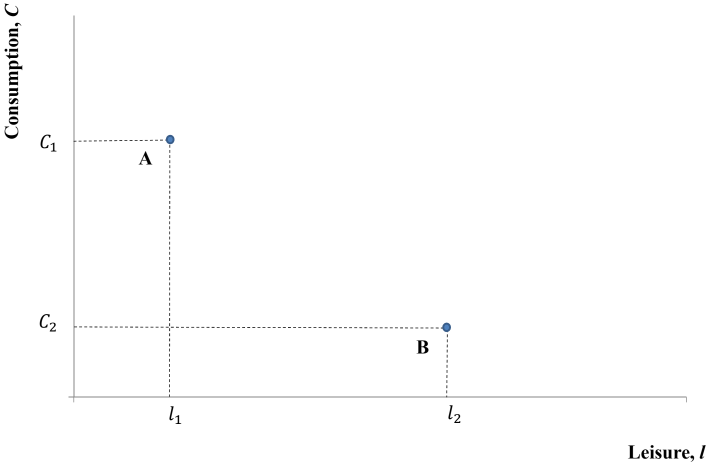
  iii) The $MRS_{l,C}$ is decreasing

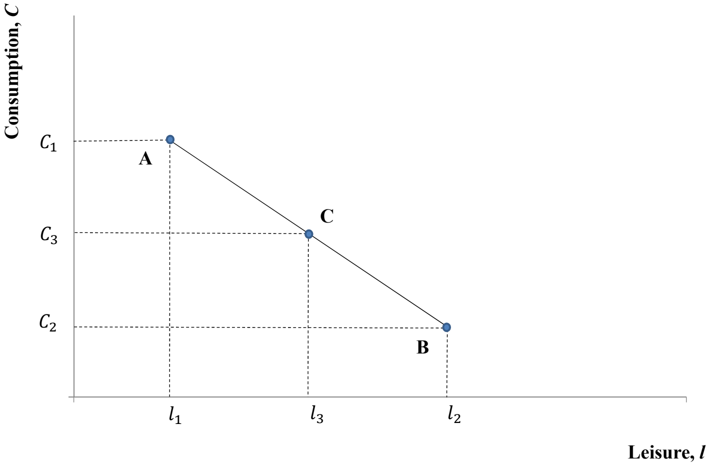
$$\frac{dMRS_{l,C}}{dl} = -\frac{d^2C}{dl^2} < 0$$

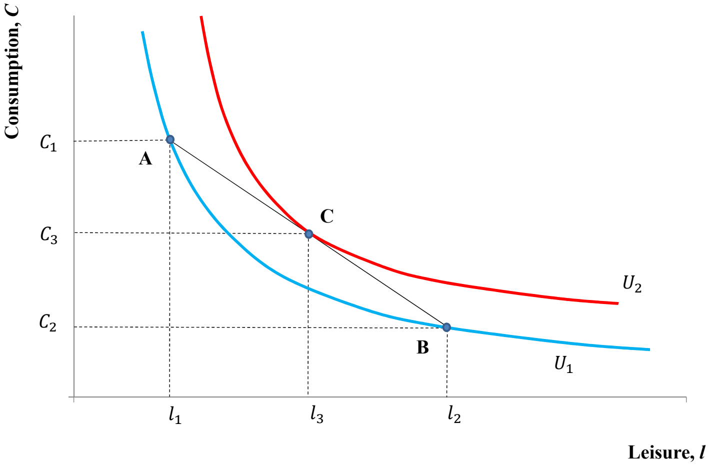
  - If C is high and l is low $\rightarrow$ MRS is high

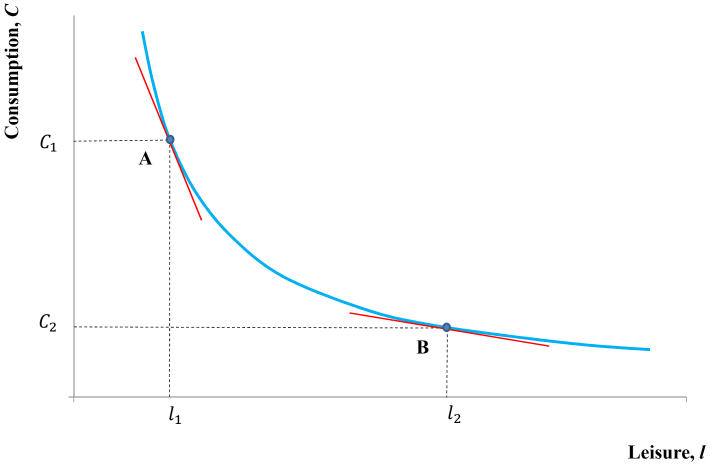
    $\rightarrow$ The consumer is willing to accept an important reduction in consumption in order to obtain 1 additional unit of leisure
  - If C is low and l is high $\rightarrow$ MRS is low

    $\rightarrow$ The consumer is willing to accept an important reduction in leisure in order to obtain 1 additional unit of consumption


Consumption, C
C_1
A
C_2
B
l_1
l_2
Leisure, l


Consumption, C
C_1
A
C
C_3
C_2
B
l_1
l_3
l_2
Leisure, l


Consumption, C
C_1
C_3
C_2
A
C
B
U_2
U_1
l_1
l_3
l_2
Leisure, l


Consumption, C
C_1
A
C_2
B
l_1
l_2
Leisure, l

The representative consumer's budget constraint

- The time budget constraint is:

$$l + N^s = h$$

h	hours of time available
l	time spent for leisure
N^s	time spent working

- The *real* disposable income is:

$$w \cdot N^s + \pi - T$$

w	real wage: units of good that can be bought with 1 unit of labor
$w \cdot N^s$	real labor income
π	dividends (in real terms)
T	lump-sum tax (in real terms)

- There is no money in the economy (goods are exchanged for other goods)
- The consumption good is the *numéraire* ($\rightarrow$ $P = 1$)

- The budget constraint is:

$$C \leq w \cdot N^s + \pi - T$$

- In this one-period model there is no reason to save
  - If consumption is lower than the disposable income

    $\rightarrow$ Utility is not maximized since more is always preferred to less

    $\rightarrow$ At the optimum, the budget constraint can be written as:

$$C = w \cdot N^s + \pi - T$$

- Given that $N^s = h - l$, the budget constraint can be written in two ways:

1) $C \leq (w \cdot h + \pi - T) - w \cdot l$

   - The real wage w represents the number of units of good that can be bought with one unit of labor

     $\rightarrow$ w is the opportunity cost of leisure

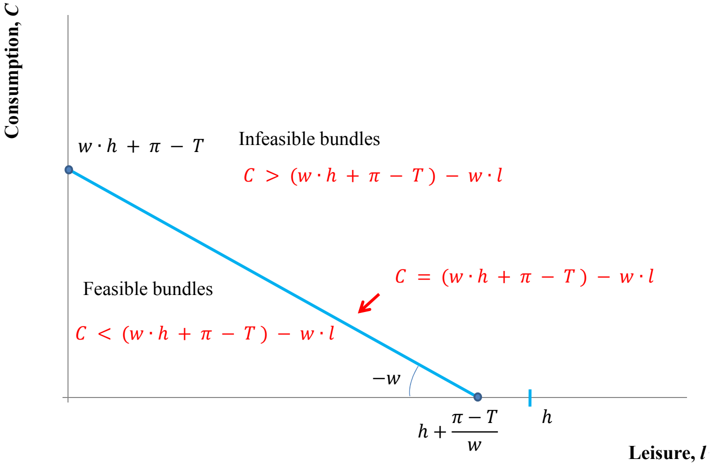
   - This equation is useful for the graphical representation

2) $C + w \cdot l \leq w \cdot h + \pi - T$

   - RHS: full disposable income
   - LHS: total expenditure to buy consumption goods and leisure

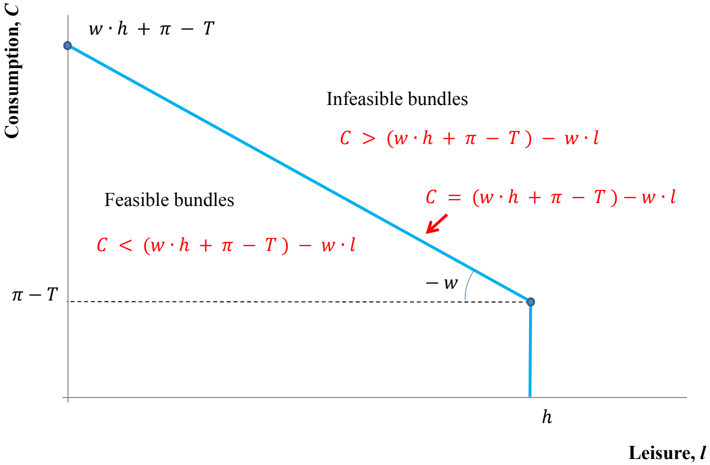
- Case 1) $\pi - T < 0$

$$
\begin{aligned}
C &= 0 &\rightarrow\quad l &= h + \frac{\pi - T}{w} < h \\
l &= 0 &\rightarrow\quad C &= w \cdot h + \pi - T \\
l &= h &\rightarrow\quad C &= \pi - T < 0 \quad \text{(infeasible)}
\end{aligned}
$$



- Case 2) $\pi - T > 0$

$$
\begin{aligned}
C &= 0 &\rightarrow\quad l &= h + \frac{\pi - T}{w} > h \quad \text{(infeasible)} \\
l &= 0 &\rightarrow\quad C &= w \cdot h + \pi - T \\
l &= h &\rightarrow\quad C &= \pi - T > 0
\end{aligned}
$$



The optimization problem

- The objective is to determine the optimal level of consumption and leisure that maximize utility given the budget constraint

→ The optimization problem is:

$$\begin{aligned} \text{Max} \quad & U = U(C, l) \\ \text{s.c.} \quad & C + w \cdot l \leq w \cdot h + \pi - T \end{aligned}$$

Analytical resolution of the optimization problem

- The Lagrangian function is:

$$\mathcal{L} \;=\; U(C,l) \;+\; \lambda \cdot [w \cdot h + \pi - T - C - w \cdot l]$$

- The FOCs are:

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial C} = 0 & \rightarrow & \frac{\partial U}{\partial C} & = & \lambda \\ \frac{\partial \mathcal{L}}{\partial l} = 0 & \rightarrow & \frac{\partial U}{\partial l} & = & \lambda \cdot w \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 & \rightarrow & C + w \cdot l & = & w \cdot h + \pi - T \end{cases}$$

- The optimality condition is:

$$\frac{\frac{\partial U}{\partial l}}{\frac{\partial U}{\partial C}} \;\equiv\; MRS_{l,C} \;=\; w$$

Economic interpretation

1) If $MRS_{l,C} > w$

$\rightarrow$ It is profitable to increase leisure by one unit

- The consumer has to give up $MRS_{l,C}$ units of consumption to maintain the same level of utility
- Income decreases by w and expenditure decreases by $MRS_{l,C}$

  $\rightarrow$ The difference can be used to increase consumption

  $\rightarrow$ Utility increases

2) If $MRS_{l,C} < w$

$\rightarrow$ It is profitable to reduce leisure by one unit

- The consumer can obtain $MRS_{l,C}$ units of consumption to maintain the same level of utility
- Income increases by w and expenditure increases by $MRS_{l,C}$

  $\rightarrow$ The difference can be used to increase consumption

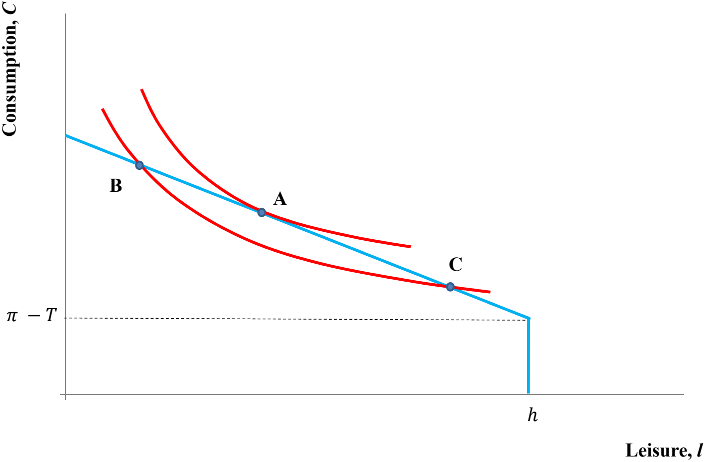
  $\rightarrow$ Utility increases

3) If $MRS_{l,C} = w$

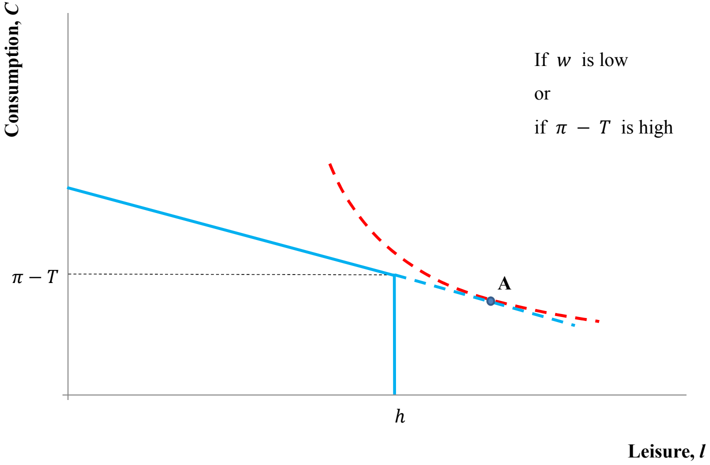
$\rightarrow$ No incentives to modify the demand of leisure and consumption

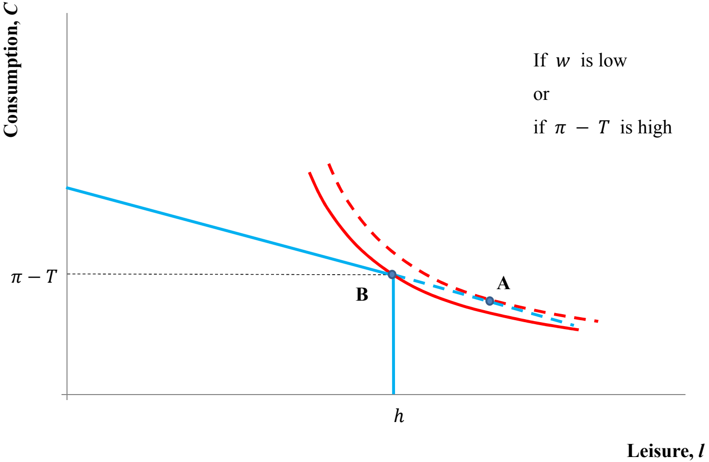
Graphical resolution of the optimization problem (i)

- The optimality condition implies the tangency between the highest indifference curve and the budget constraint



Graphical resolution of the optimization problem (ii)





Consumption, C
If w is low
or
if $\pi - T$ is high
$\pi - T$
A
B
h
Leisure, l

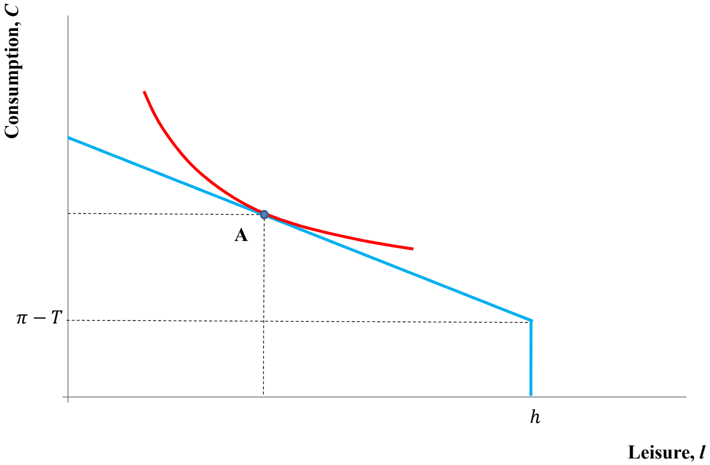
Change in $\pi - T$: Income effect

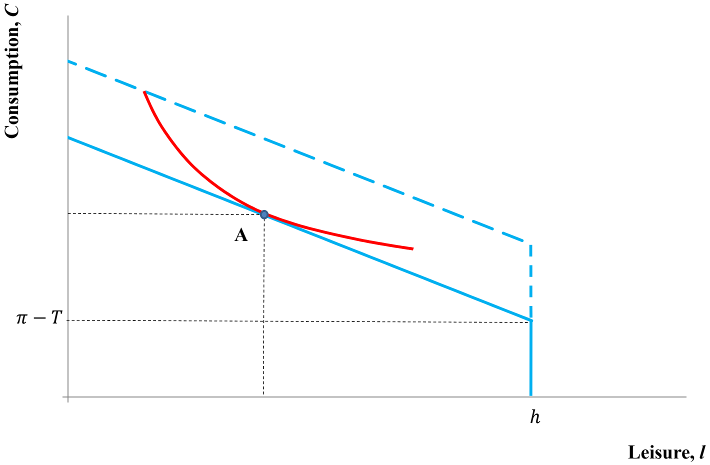
- Budget constraint

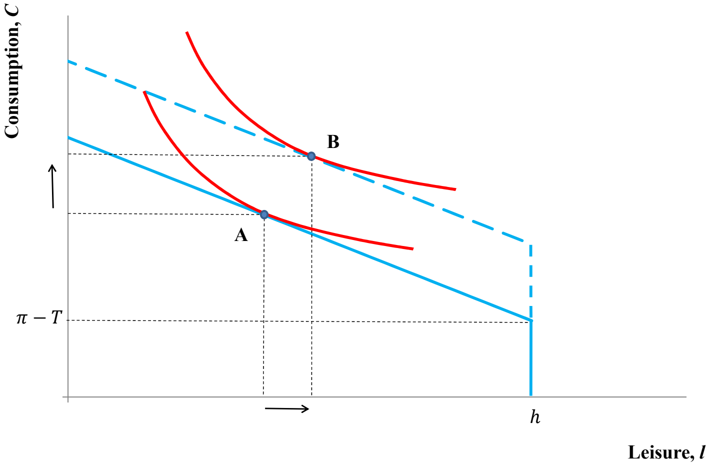
$$C \leq (w \cdot h + \pi - T) - w \cdot l$$

- An increase in $\pi - T$ provokes an increase in the full disposable income
  $\rightarrow$ The intercept of the budget constraint increases
- The real wage is not affected
  $\rightarrow$ The slope of the budget constraint remains unchanged

- At the optimum: Both consumption and leisure increase
  $\rightarrow$ Normal goods


Consumption, C
A
π − T
h
Leisure, l


Consumption, C
A
π − T
h
Leisure, l


Consumption, C
B
A
π − T
h
Leisure, l

Change in w: Income and substitution effects

- Budget constraint

$$C \leq (w \cdot h + \pi - T) - w \cdot l$$

- An increase in w

  $\rightarrow$ The intercept of the budget constraint increases

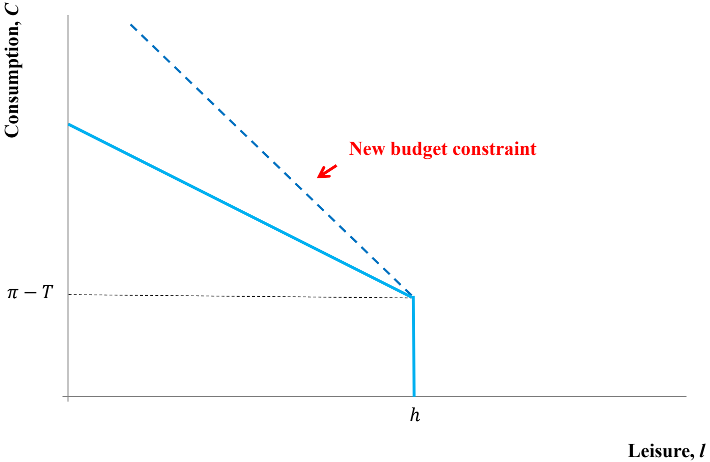
  $\rightarrow$ The slope of the budget constraint increases (in absolute value)

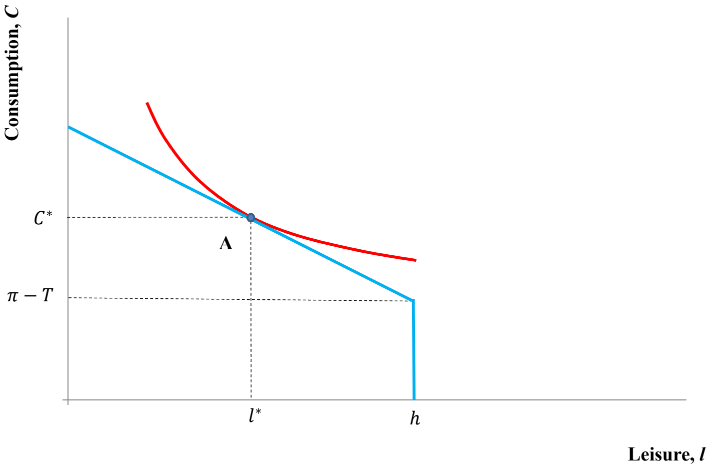
- Two effects:
  - Income effect: The full disposable income increases

    $\rightarrow$ Both consumption and leisure increase
  - Substitution effect: The opportunity cost of leisure increases

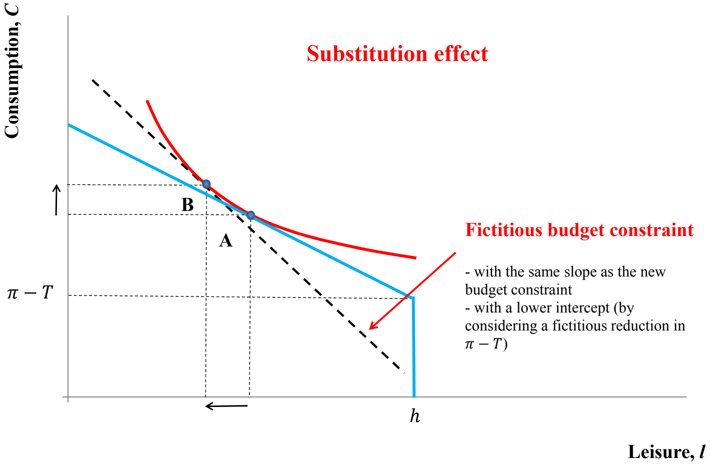
    $\rightarrow$ Leisure decreases and consumption increases

- Total effect:
  - Increase in consumption
  - Ambiguous effect on leisure

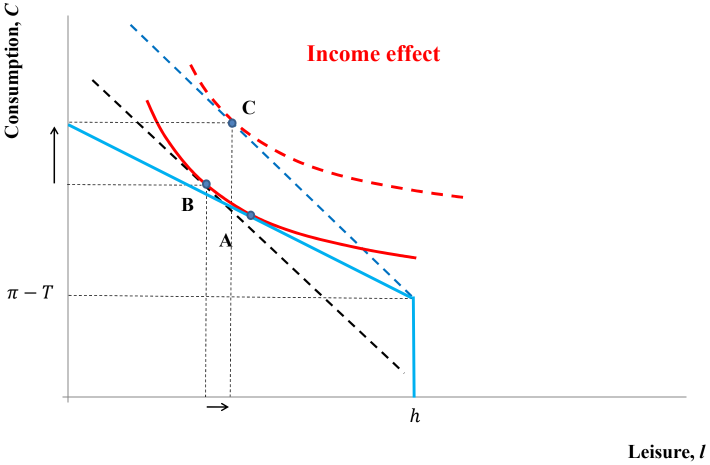

Consumption, C
New budget constraint
π − T
h
Leisure, l


Consumption, C
C*
A
π − T
l*
h
Leisure, l

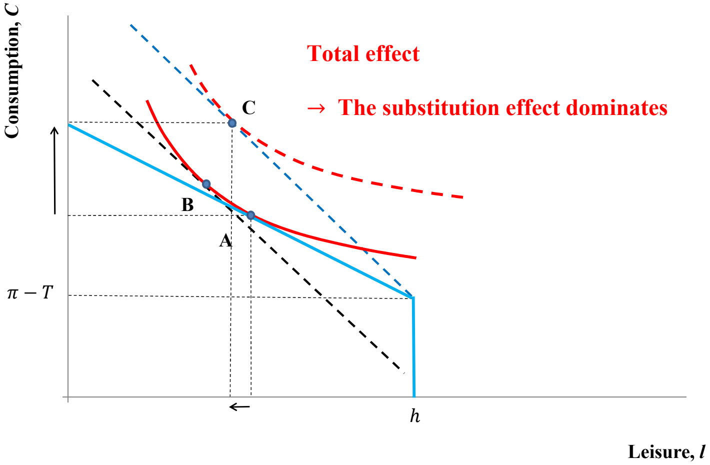
- Case 1: The substitution effect dominates



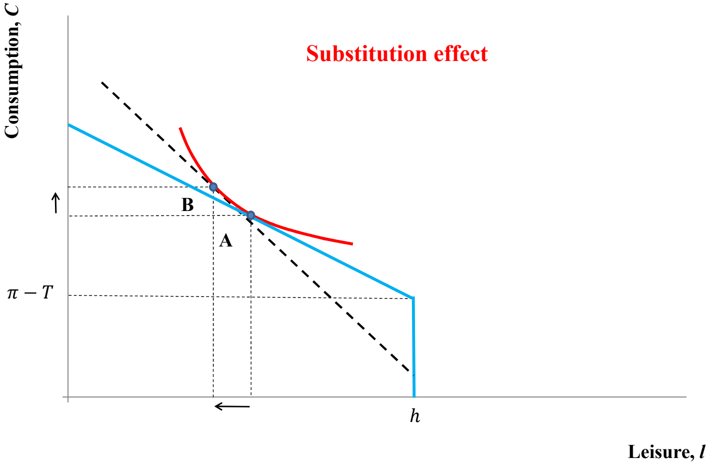
- Case 1: The substitution effect dominates



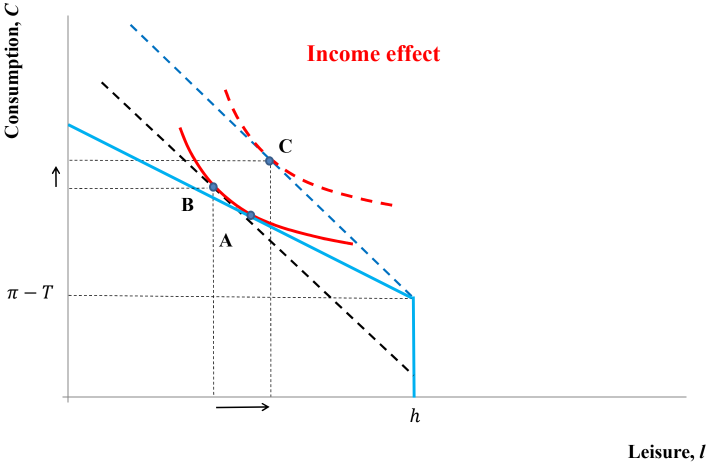
- Case 1: The substitution effect dominates



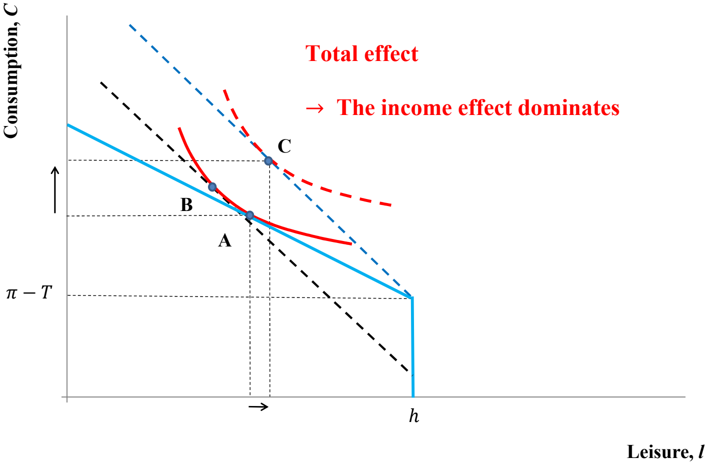
- Case 2: The income effect dominates



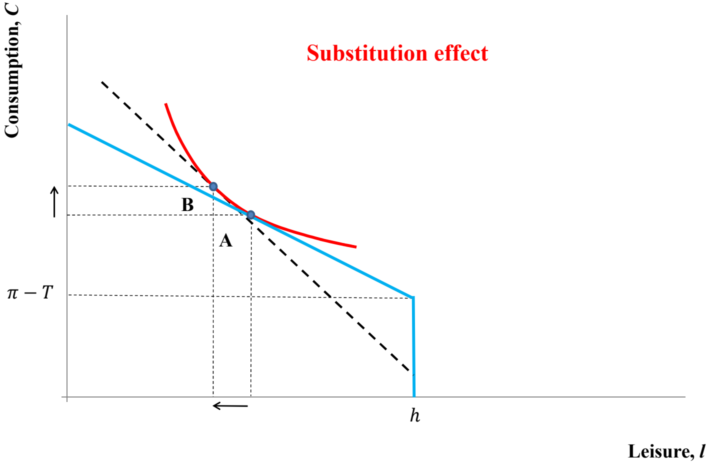
- Case 2: The income effect dominates



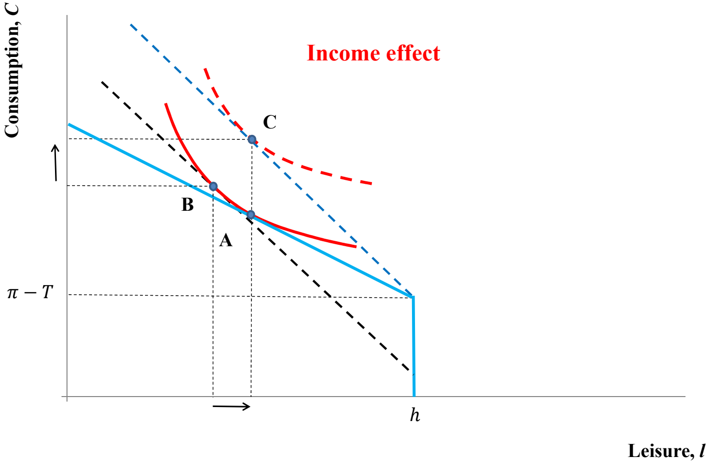
- Case 2: The income effect dominates



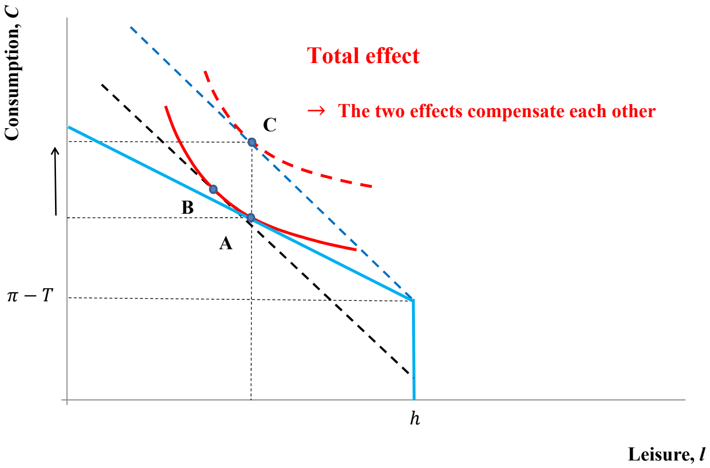
- Case 3: The income effect compensates the substitution effect



- Case 3: The income effect compensates the substitution effect



- Case 3: The income effect compensates the substitution effect



Example with a Cobb-Douglas utility function: $U = C^{\beta} \cdot l^{1-\beta}$, with $0 \leq \beta \leq 1$

- The optimality condition $\frac{\frac{\partial U}{\partial l}}{\frac{\partial U}{\partial C}} = w$ becomes:

$$\frac{(1-\beta) \cdot C^{\beta} \cdot l^{-\beta}}{\beta \cdot C^{\beta-1} \cdot l^{1-\beta}} = w \quad \rightarrow \quad \frac{C}{l} = \frac{\beta}{1-\beta} \cdot w \quad \rightarrow \quad C = \frac{\beta}{1-\beta} \cdot w \cdot l$$

- Budget constraint: $C = (w \cdot h + \pi - T) - w \cdot l \quad \rightarrow$

$$\frac{\beta}{1-\beta} \cdot w \cdot l = (w \cdot h + \pi - T) - w \cdot l \quad \rightarrow \quad \frac{1}{1-\beta} \cdot w \cdot l = w \cdot h + \pi - T$$

- The optimal solution is then:

$$l^* = (1-\beta) \cdot \left(h + \frac{\pi - T}{w}\right)$$
$$C^* = \beta \cdot (w \cdot h + \pi - T)$$

- Effect of a change in $\pi - T$:

$$\frac{\partial l^*}{\partial(\pi - T)} = \frac{1-\beta}{w} > 0$$

$$\frac{\partial C^*}{\partial(\pi - T)} = \beta > 0$$

- Effect of a change in w:

$$\frac{\partial l^*}{\partial w} = -\frac{(1-\beta)\cdot(\pi - T)}{w^2} < 0 \rightarrow \text{The substitution effect dominates}$$

$$\frac{\partial C^*}{\partial w} = \beta \cdot h > 0$$

Labor supply curve

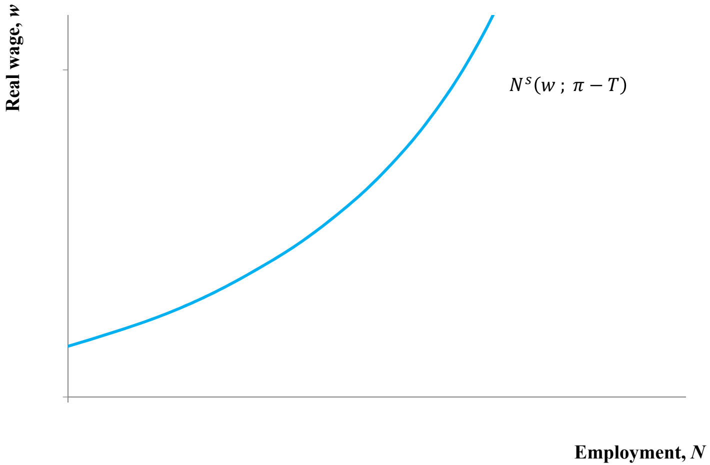
- For each level of the real wage it is possible to compute the optimal level of leisure l^*
  - Demand function for leisure: $l(w)$

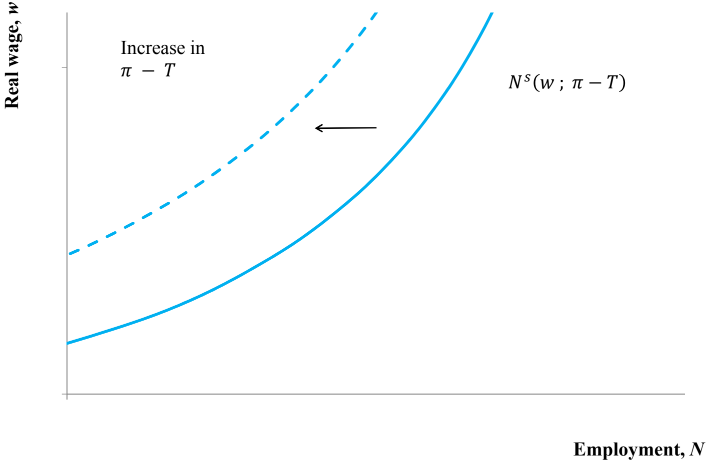
- Labor supply curve

$$N^s(w) = h - l(w)$$

- In the case in which the substitution effect dominates

$\rightarrow$ The labor supply curve is upward-sloping


Real wage, w
$N^s(w\,;\,\pi - T)$
Employment, N


Real wage, w
Increase in
π − T
N^s(w ; π − T)
Employment, N

PART 2

THE REPRESENTATIVE FIRM

The representative firm's technology

- The representative firm
  - Produces one good under perfect competition
  - Uses production factors (labor and capital) and the available technology
  - Owns productive capital (plant and equipments)
  - Hires the quantity of labor that maximizes its profit given the technological constraint

- Production function

$$Y \;=\; z \cdot F(K, N^d)$$

Y	Quantity produced by the representative firm $\rightarrow$ Real GDP
K	Quantity of capital (fixed at $\overline{K}$ since the model is static)
N^d	Number of hours worked by the employees
F	Ability of the firm to use the inputs in the productive activity
z	Total factor productivity

  - An increase in z implies that both factors are more productive

- Property: Constant returns to scale

$$z \cdot F(x \cdot K, x \cdot N^d) \;=\; x \cdot Y \qquad \text{with} \;\; x > 0$$

- Increasing returns to scale

  → Large firms are more efficient than small firms (Automobile industry)

- Decreasing returns to scale

  → Small firms are more efficient than large firms (High quality restaurant food)

- Constant returns to scale

  → Large firms are efficient as small firms

- At the aggregate level, the hypothesis of constant returns to scale is sufficiently reasonable

  → The economy behaves in the same way if there are many small firms or few large firms

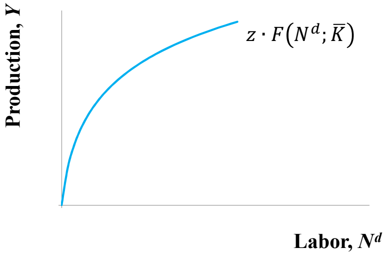
  → Representative firm

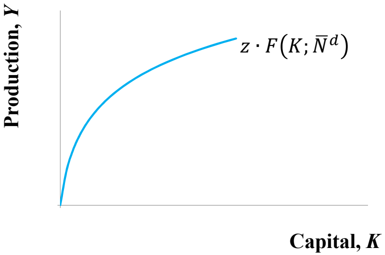
- For a given level of K if N^d increases $\rightarrow$ Y increases

$$\frac{\partial Y}{\partial N^d} > 0$$

- For a given level of N^d if K increases $\rightarrow$ Y increases

$$\frac{\partial Y}{\partial K} > 0$$





- Marginal product of labor

$$MP_N = \frac{\partial Y}{\partial N^d}$$

  - MP_N indicates the increase in the quantity produced due to the increase of one unit of labor (for a given quantity of capital)

- Marginal product of capital

$$MP_K = \frac{\partial Y}{\partial K}$$

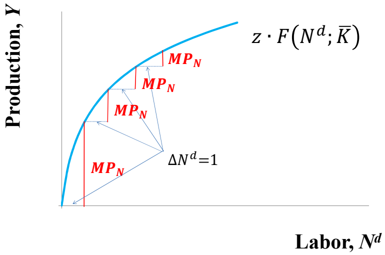
  - MP_N indicates the increase in the quantity produced due to the increase of one unit of capital (for a given quantity of labor)

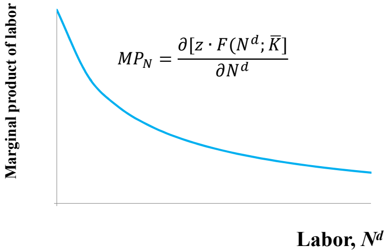
- For a given level of K
  - An increase in labor increases production

$$\frac{\partial Y}{\partial N^d} > 0 \quad \leftrightarrow \quad MP_N > 0$$

  - An increase in labor reduces the marginal product of labor

$$\frac{\partial MP_N}{\partial N^d} < 0 \quad \leftrightarrow \quad \frac{\partial^2 Y}{\partial (N^d)^2} < 0$$



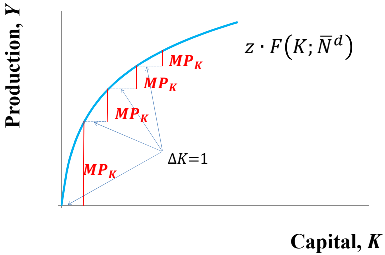


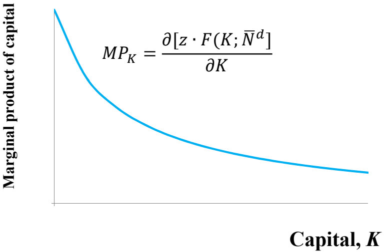
- For a given level of N^d
  - An increase in capital increases production

$$\frac{\partial Y}{\partial K} > 0 \quad \leftrightarrow \quad MP_K > 0$$

  - An increase in capital reduces the marginal product of capital

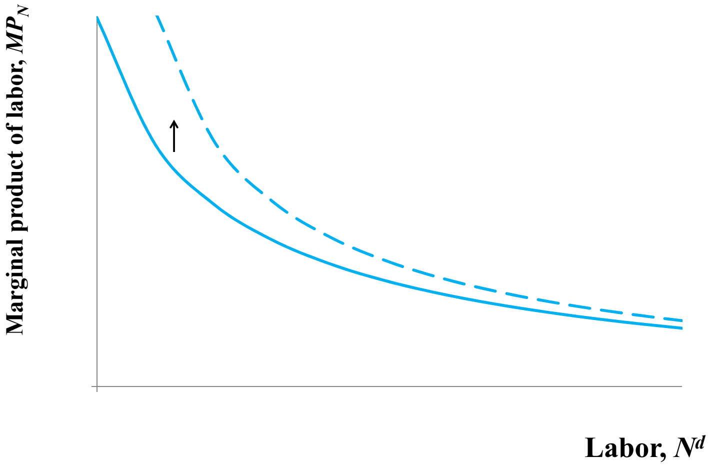
$$\frac{\partial MP_K}{\partial K} < 0 \quad \leftrightarrow \quad \frac{\partial^2 Y}{\partial K^2} < 0$$





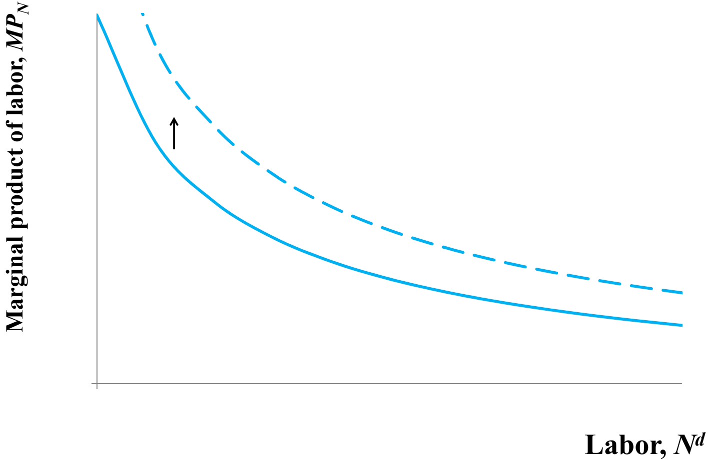
- An increase in capital increases the MP_N

$$\frac{\partial MP_N}{\partial K} > 0$$



- An increase in the total factor productivity increases the MP_N

$$\frac{\partial MP_N}{\partial z} > 0$$



The profit maximization problem

$$\begin{cases} \text{Max} & \pi = Y - w \cdot N^d \\ \text{s.t.} & Y = z \cdot F(K, N^d) \end{cases}$$

π	Profit (in real terms)
Y	Real GDP and total revenues (in real terms since $P = 1$)
$w \cdot N^d$	Total production cost (in real terms)

- The first order condition for profit maximization is:

$$\frac{\partial \pi}{\partial N^d} = \frac{\partial Y}{\partial N^d} - w = 0$$

- The optimality condition is then:

$$MP_N = w$$

Economic interpretation

1) If $MP_N > w$

$\rightarrow$ It is profitable to increase labor demand by one unit

- Production and total revenues (in real terms) increase by MP_N
- Production costs (in real terms) increase by w

  $\rightarrow$ Profits (in real terms) increase

2) If $MP_N < w$

$\rightarrow$ It is profitable to reduce labor demand by one unit

- Production and total revenues (in real terms) decrease by MP_N
- Production costs (in real terms) decrease by w

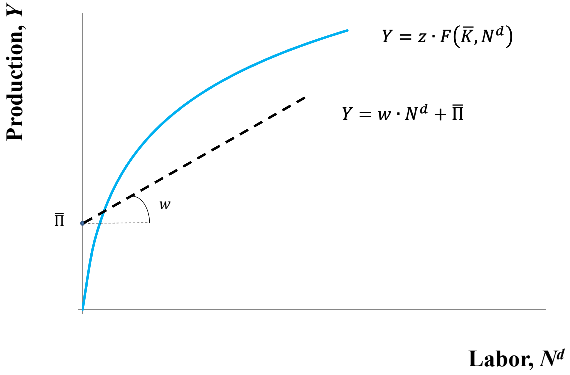
  $\rightarrow$ Profits (in real terms) increase

3) If $MP_N = w$

$\rightarrow$ The firm has no incentives to modify labor demand

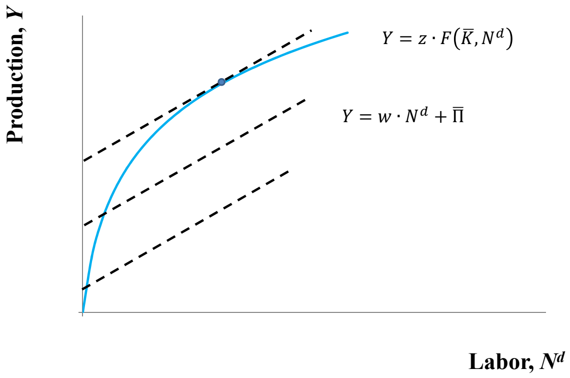
Graphical resolution of the profit maximization problem

- Iso-profit curve: $Y = w \cdot N^d + \overline{\Pi}$
  - All the combinations of Y and N^d such that the level of profits is the same, $\overline{\Pi}$

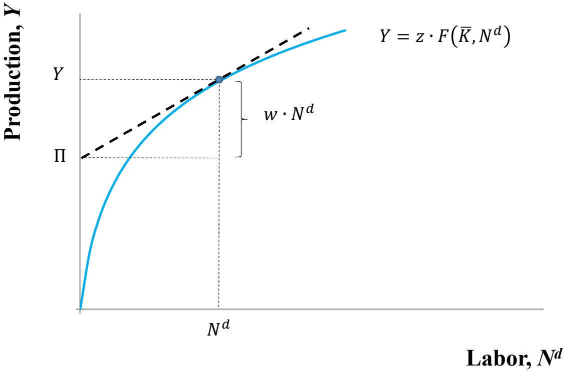


- Optimality condition: The highest possible iso-profit curve must be tangent to the production function

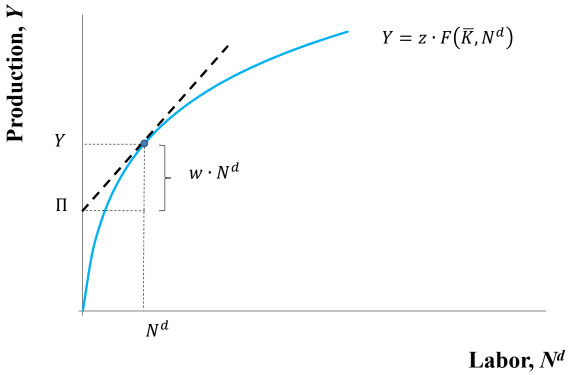
$\rightarrow \quad MP_N = w$



- Optimal choice for a given w:
  - N^d is determined such that $MP_N = w$
  - Y is determined by $Y = z \cdot F(K, N^d)$
  - Π is determined by $\Pi = Y - w \cdot N^d$



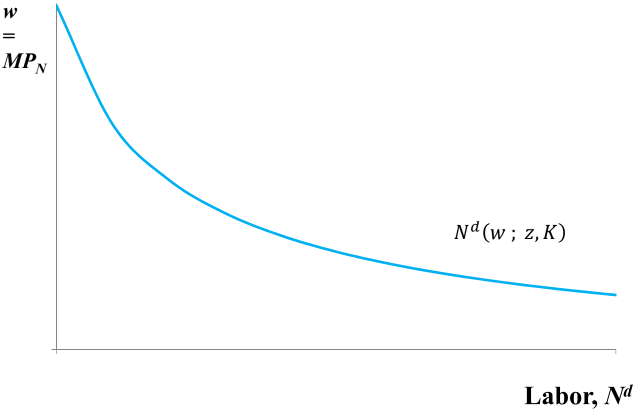
- Optimal choice for a higher w:
  - N^d is lower $\rightarrow$ Negative relationship between N^d and w
  - Y is lower
  - Π is lower



Labor demand curve, $N^d(w)$

- Optimality condition: $MP_N = w$

  $\rightarrow$ The labor demand curve is identical to the marginal product of labor



Example with a Cobb-Douglas production function $Y = z \cdot K^{\alpha} \cdot (N^d)^{1-\alpha}$

- The optimality condition $MP_N = w$ becomes:

$$(1-\alpha) \cdot z \cdot K^{\alpha} (N^d)^{-\alpha} = w \quad \rightarrow \quad \frac{(1-\alpha) \cdot z}{w} \cdot K^{\alpha} = (N^d)^{\alpha}$$

- The labor demand curve is:

$$N^d(w) = \left[\frac{(1-\alpha) \cdot z}{w} \right]^{\frac{1}{\alpha}} \cdot K$$

$$\frac{\partial N^d(w)}{\partial w} < 0 \quad \rightarrow \quad \text{The curve is downward sloping}$$

$$\frac{\partial N^d(w)}{\partial K} > 0 \quad \rightarrow \quad \text{The curve shifts to the right if } K \text{ increases}$$

$$\frac{\partial N^d(w)}{\partial z} > 0 \quad \rightarrow \quad \text{The curve shifts to the right if } z \text{ increases}$$

PART 3

COMPETITIVE EQUILIBRIUM

IN A ONE PERIOD CLOSED-ECONOMY

Simple Macroeconomic model

- The representative consumer chooses:
  - Labor supply N^s
  - Demand of goods C
- The representative firm chooses:
  - Labor demand N^d
  - Production of goods Y
- The government:
  - Demands goods G
  - Receives lump-sum taxes T

  The government budget constraint requires: $G = T$
- Closed economy and static model
- Competitive behavior of consumers and firms and perfect flexibility of prices
- Endogenous variables: $C, N^s, l, T, N^d, Y, \pi, w$
- Exogenous variables: G, z, K

Competitive equilibrium

- Set of endogenous variables that, given the exogenous variables, satisfy the following conditions:

  1) Y and N^d are chosen to maximize the firm's profit (given w, z, K)
  2) C and N^s are chosen to maximize the consumer's welfare (given w, T, π, where $\pi = Y - w \cdot N^d$)
  3) The government budget constraint is satisfied:
     $T = G$
  4) The labor market clears:
     $N^d = N^s$

- Property of the competitive equilibrium:

  $Y = C + G$ Income-expenditure identity (with $I = 0$ and $NX = 0$)

  - In fact:
    $$C = w \cdot N^s + \pi - T = w \cdot N^s + Y - w \cdot N^d - G = Y - G$$

Alternative definition of competitive equilibrium

- The competitive equilibrium is a set of endogenous variables that, given the exogenous variables, satisfy the following:

  1) Y and N^d are chosen to maximize the firm's profit (given w, z, K)

  2) C and N^s are chosen to maximize the consumer's welfare (given w, T, π, where $\pi = Y - w \cdot N^d$)

  3) The government budget constraint is satisfied
     $T = G$

  4) The labor market clears
     $N^d = N^s$

  5) The market of the goods clears
     $Y = C + G$

- Two markets $\rightarrow$ Two equilibrium equations $\rightarrow$ Two equilibrium prices
- Walras' Law $\rightarrow$ One equilibrium equation is redundant
- Numéraire $\rightarrow$ $P = 1$ and w is the real wage

Interpretation of the equilibrium

- For a given w
  - The representative firm chooses N^d

$$
\begin{array}{rrcl}
\rightarrow & Y & = & z \cdot F(\overline{K}, N^d) \\
\rightarrow & \pi & = & Y - w \cdot N^d \\
\rightarrow & C & = & Y - G
\end{array}
$$

- For the same level of real wage w and for the level of dividends π
  - The representative consumer chooses C and N^s
- Unique equilibrium w^* that guarantees the competitive equilibrium and, thus, the consistency of the choices:

$$
\begin{array}{llcll}
C & \text{(from production)} & = & C & \text{(chosen by the consumer)} \\
N^d & \text{(chosen by the firm)} & = & N^s & \text{(chosen by the consumer)}
\end{array}
$$

Graphical resolution of the model

1) The relationship between output and employment (production function, with $N = N^d = N^s$)

$$Y = z \cdot F(K, N)$$

2) The relationship between output and leisure ($l = h - N$)

$$Y = z \cdot F(K, h - l)$$

3) The relationship between consumption and leisure ($C = Y - G$)

$$C = z \cdot F(K, h - l) - G$$

$\rightarrow$ Production Possibilities Frontier

- Combinations of consumption and leisure that are technically possible

4) Competitive equilibrium

- The highest possible indifference curve is tangent to the budget constraint and the PPF

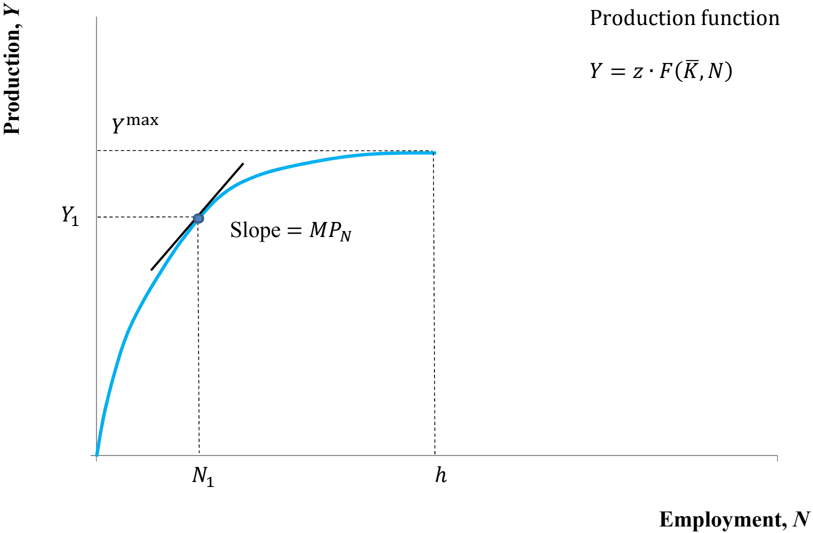
- The Marginal Rate of Transformation between consumption and leisure

$$MRT_{l,C} = -\text{ Slope of the } PPF$$

  - Quantity of consumption the consumer has to give up to obtain one additional unit of leisure given the technological constraint

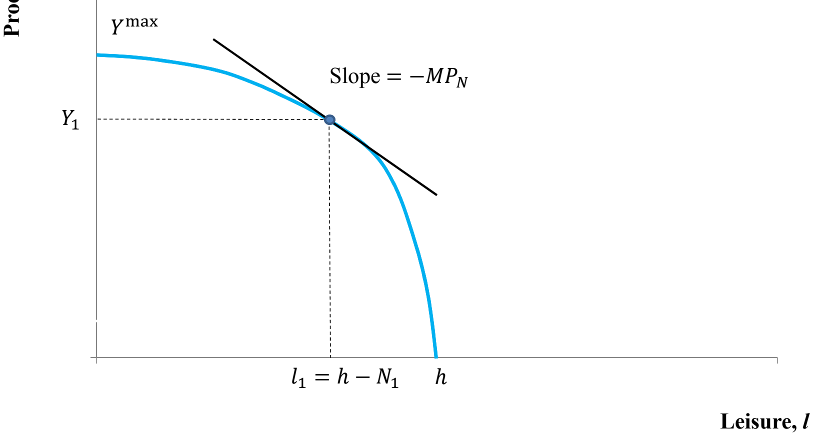
Production function

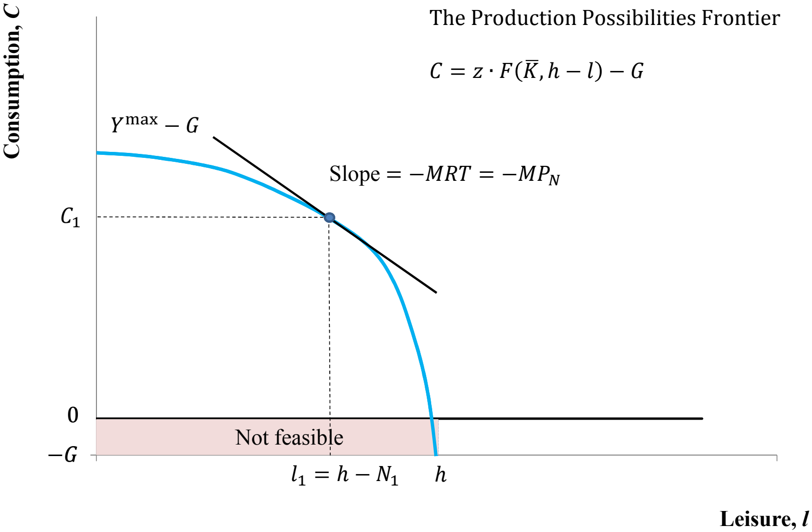
$Y = z \cdot F(\bar{K}, N)$

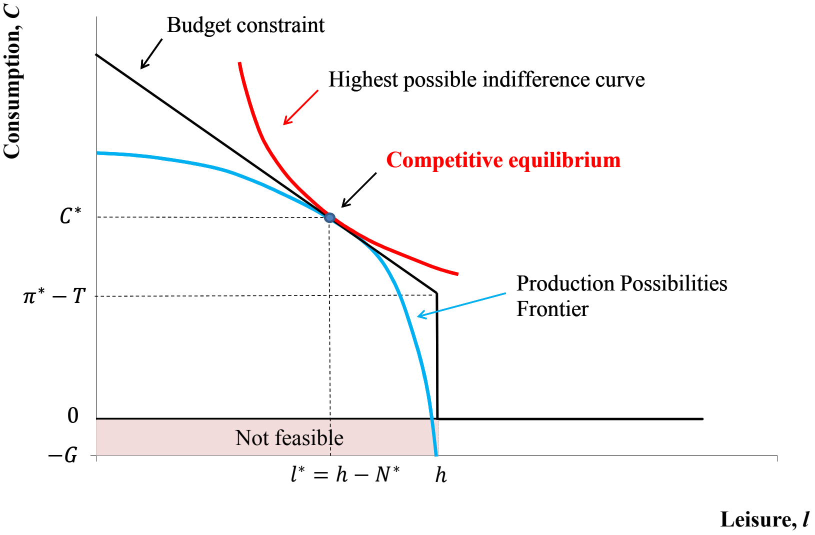


Production as function of leisure

$$Y = z \cdot F(\bar{K}, h - l)$$




The Production Possibilities Frontier
$C = z \cdot F(\overline{K}, h - l) - G$
Consumption, C
$Y^{\max} - G$
Slope $= -MRT = -MP_N$
C_1
0
$-G$
Not feasible
$l_1 = h - N_1$
h
Leisure, l



Consumption, C
Budget constraint
Highest possible indifference curve
Competitive equilibrium
C^*
$\pi^* - T$
Production Possibilities Frontier
0
$-G$
Not feasible
$l^* = h - N^*$
h
Leisure, l

- Optimality condition

  Tangency of the highest possible indifference curve to the budget constraint and to the production possibilities frontier

$$MRS_{l,C} \;=\; MRT_{l,C} \;=\; MP_N \;=\; w$$

  - $MRS_{l,C} \;=\; MRT_{l,C}$

    $\rightarrow$ maximization of utility given the technological constraint
  - $MRS_{l,C} \;=\; w$

    $\rightarrow$ maximization of utility given the budget constraint
  - $MP_N \;=\; w$

    $\rightarrow$ maximization of profit given the technological constraint

Pareto-optimality

- An allocation is Pareto-optimal if it is not possible to reallocate the resources so that someone is better off without making someone else worse off

- First Theorem of Welfare Economics

  Under certain conditions, a competitive equilibrium is Pareto-optimal

  - Absence of externalities
  - Absence of monopoly power
  - Absence of distorting taxes

  $\rightarrow$ Competitive markets guarantee an efficient allocation of resources

- Second Theorem of Welfare Economics

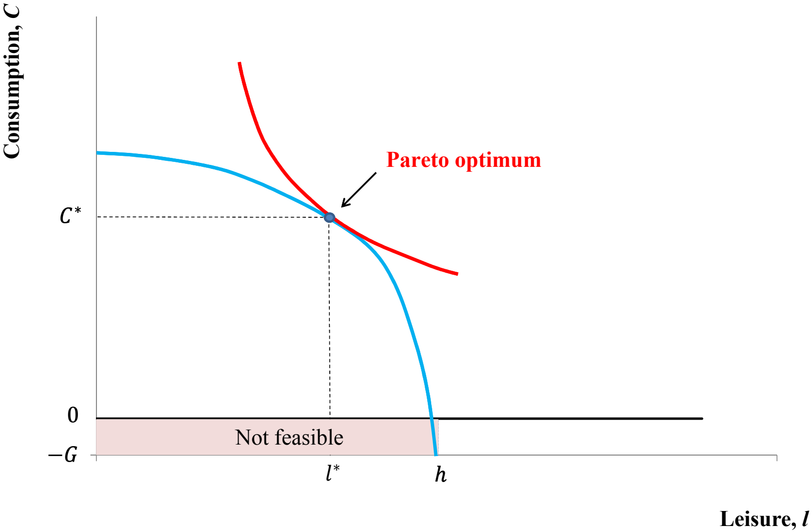
  Under certain conditions, any Pareto-optimum can be achieved in a competitive market with a redistribution of resources

Social planner

- A fictitious social planner chooses the levels of C and l
  - That maximize the utility of the representative agent
  - That belong to the Production Possibilities Frontier
- The optimality condition for the social optimum is:

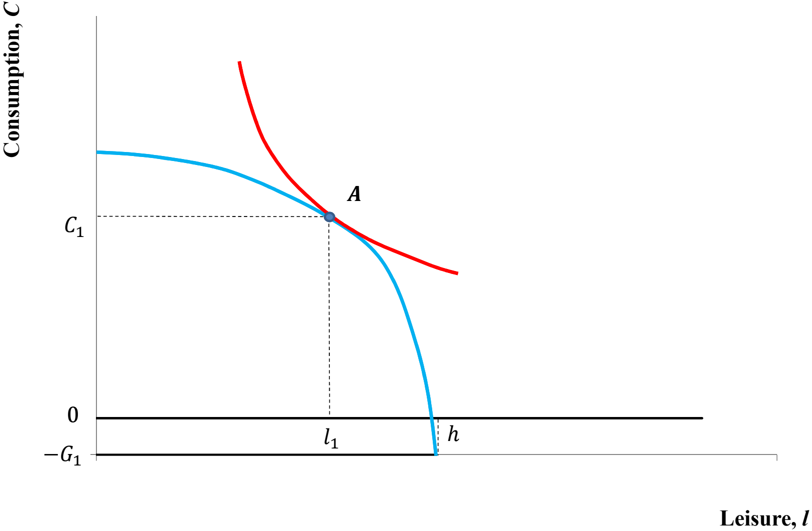
$$MRS_{l,C} = MRT_{l,C} = MP_N$$

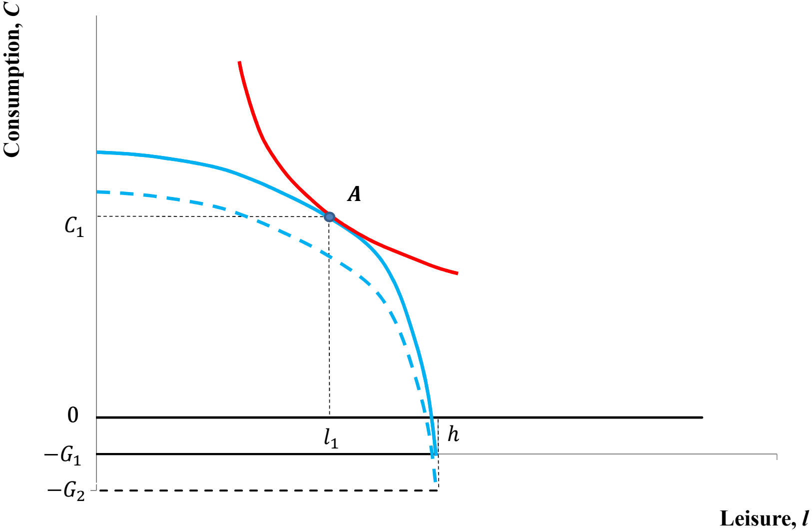
- The solution of the social planner (which is Pareto-optimal) coincides with the competitive equilibrium
- The theorems of welfare economics hold

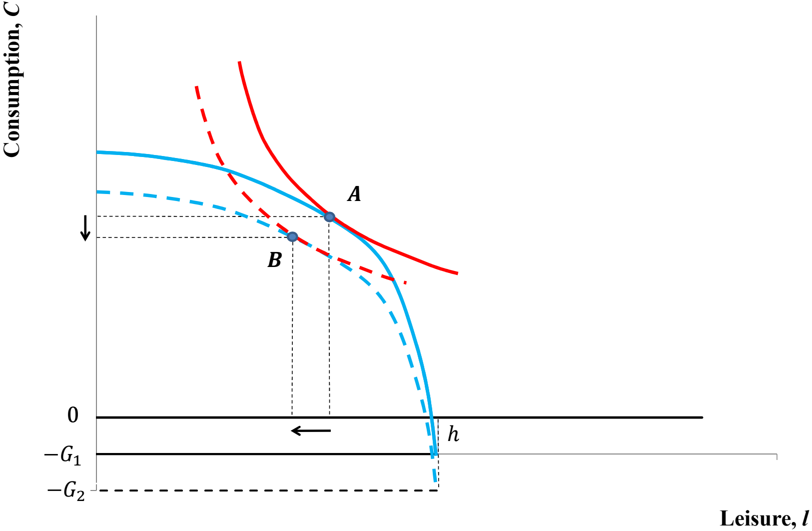

Consumption, C
Pareto optimum
C^*
0
$-G$
Not feasible
l^*
h
Leisure, l

Increase in government expenditures G

- G increases $\rightarrow$ The PPF $C = z \cdot F(K, h - l) - G$ shifts downward
- T increases $\rightarrow$ The consumer budget constraint $C = (w \cdot h + \pi - T) - w \cdot l$ shifts to the left
- Consequences:
  - Consumption C and leisure l decrease (negative income effect)
  - Employment N increases
  - Real GDP Y increases
  - Real wage w decreases
- Model predictions:
  - Consumption is countercyclical (in contrast with data)
  - Employment is procyclical (consistent with data)
  - Real wage is countercyclical (in contrast with data)
- Changes in government spending are not a good candidate as a cause of business cycles


Consumption, C
A
C_1
0
l_1
h
$-G_1$
Leisure, l


Consumption, C
A
C_1
0
l_1
h
$-G_1$
$-G_2$
Leisure, l


Consumption, C
A
B
0
h
−G1
−G2
Leisure, l

Numerical evaluation

$$\begin{aligned} Y &= z \cdot \overline{K}^{\alpha} \cdot (N^d)^{1-\alpha} \\ U &= C^{\beta} \cdot \ l^{1-\beta} \end{aligned}$$

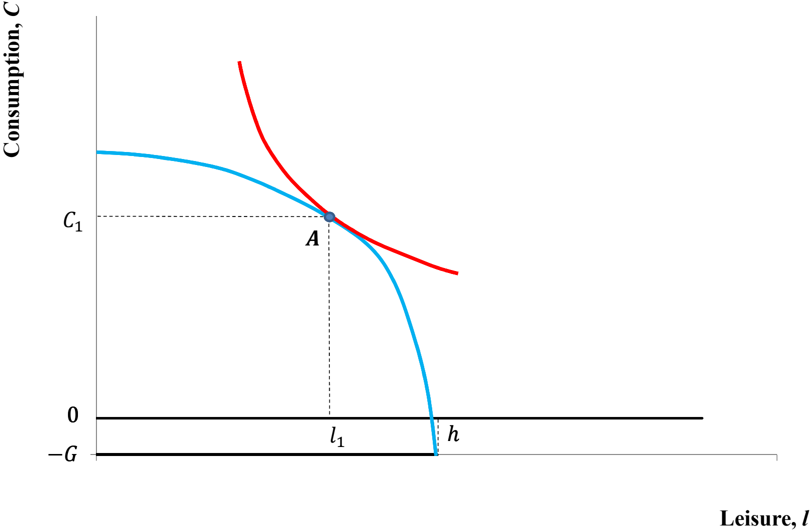
with $z = 10$, $\overline{K} = 100$, $\alpha = 1/3$, $\beta = 1/2$, $T = G = 100$, $h = 100$

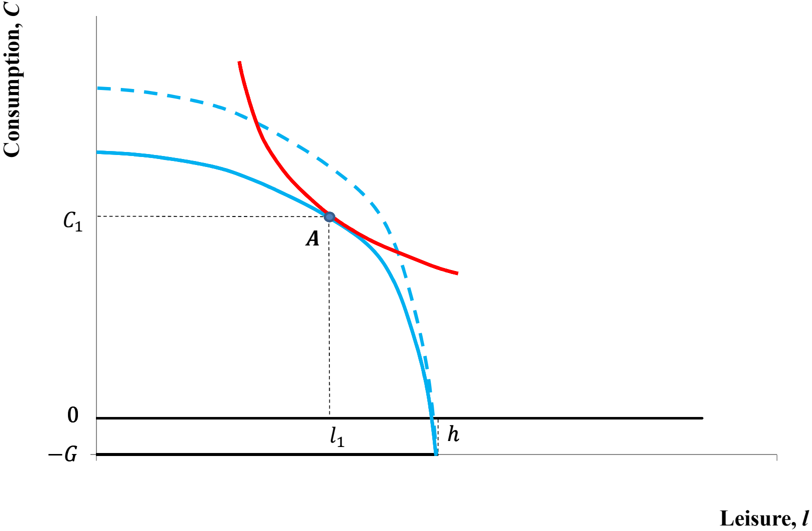
Shock: $G = 200$

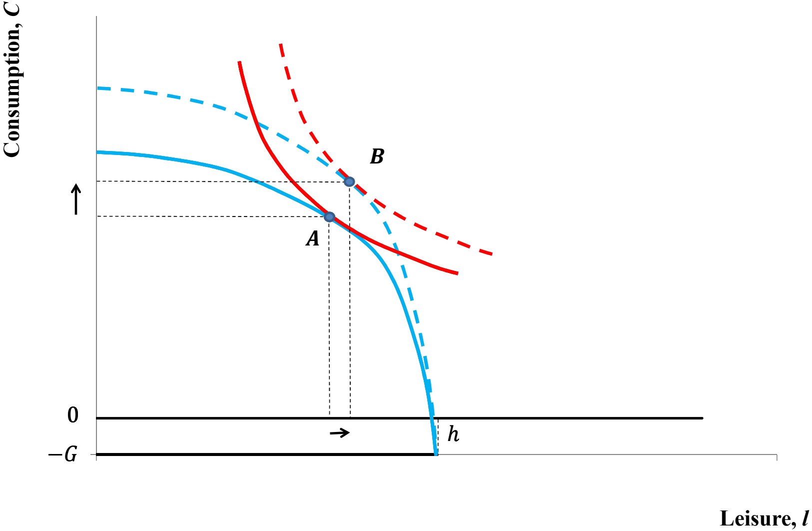
		Initial	Final	Change
Leisure	l	55.4	50.5	-8.9%
Consumption	C	483.6	425.7	-12.0%
Utility	U	163.7	146.6	-10.4%
Employment	N	44.6	49.5	11.0%
Real GDP	Y	583.6	625.7	7.2%
Profits	π	194.5	208.6	7.2%
Real wage	w	8.7	8.4	-3.4%
Lump-sum tax	T	100.0	200.0	100.0%

Increase in the total factor productivity z

- z increases $\rightarrow$ The PPF $C = z \cdot F(K, h - l) - G$ shifts to the right $\rightarrow$ Profits increase
- MP_N increases $\rightarrow$ Real wage increases $\rightarrow$ The consumer budget constraint $C = (w \cdot h + \pi - T) - w \cdot l$ shifts to the right
- Consequences:
  - Consumption C increases
  - Ambiguous effect on leisure l
  - Ambiguous effect on employment N
  - Real GDP Y increases
- Model predictions:
  - Consumption is procyclical (consistent with data)
  - Real wage is procyclical (consistent with in data)
  - Employment is procyclical or countercyclical (in contrast with data)
- Shocks in TFP could be a primary cause of business cycles


Consumption, C
C_1
A
0
$-G$
l_1
h
Leisure, l


Consumption, C
C_1
A
0
l_1
h
$-G$
Leisure, l


Consumption, C
B
A
0
−G
h
Leisure, l

Numerical evaluation

$$
\begin{aligned}
Y &= z \cdot \overline{K}^{\alpha} \cdot (N^d)^{1-\alpha} \\
U &= C^{\beta} \cdot l^{1-\beta}
\end{aligned}
$$

with $z = 10$, $\overline{K} = 100$, $\alpha = 1/3$, $\beta = 1/2$, $T = G = 100$, $h = 100$

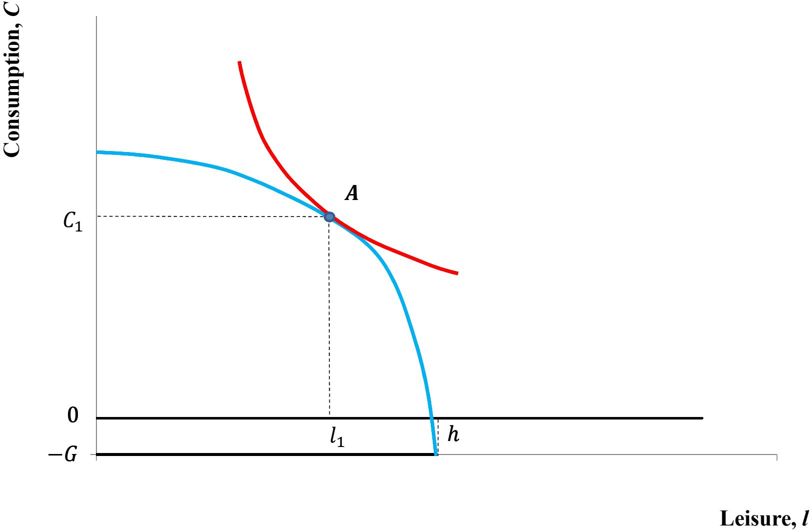
Shock: $z = 12 \quad (+20\%)$

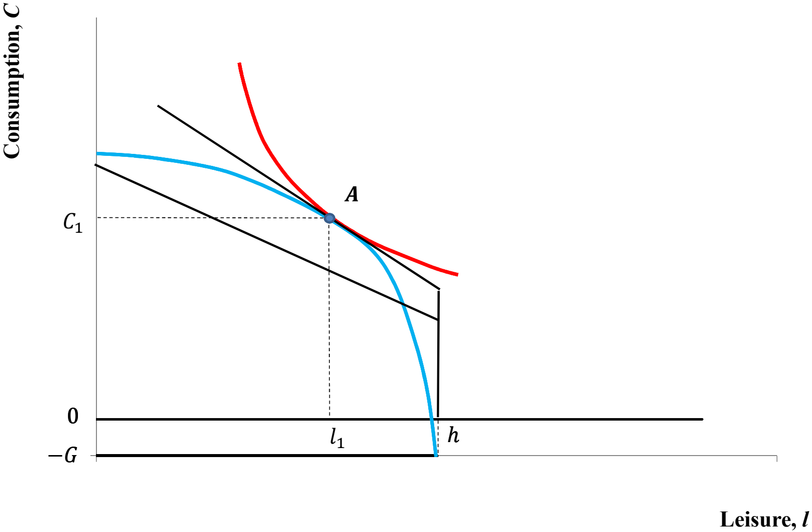
		Initial	Final	Change
Leisure	l	55.4	56.2	1.4%
Consumption	C	483.6	592.1	22.4%
Utility	U	163.7	182.4	11.4%
Employment	N	44.6	43.8	-1.8%
Real GDP	Y	583.6	692.1	18.6%
Profits	π	194.5	230.7	18.6%
Real wage	w	8.7	10.5	20.7%

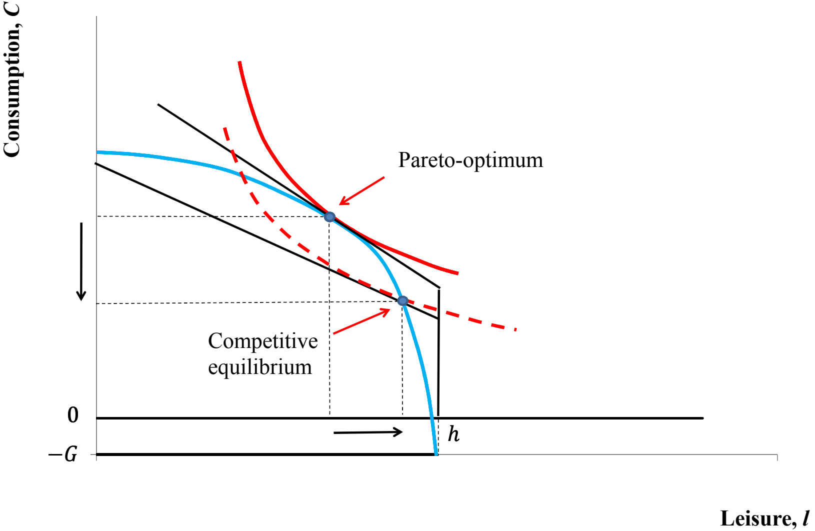
Distorting taxation (Tax on labor incomes)

- The PPF $C = z \cdot F(K, h - l) - G$ remains unchanged
- The consumer budget constraint $C = (w \cdot h + \pi - T) - w \cdot l$ becomes $C = [w \cdot (1 - \tau) \cdot h + \pi] - w \cdot (1 - \tau) \cdot l$
- The tax rate τ must be determined in order to satisfy the government budget constraint: $\tau \cdot w \cdot N = G$
- The new budget constraint is not tangent to the PPF

  $\rightarrow$ The competitive equilibrium is not Pareto-efficient
- Model predictions:
  - Leisure l increases (Distorting income tax $\rightarrow$ Disincentive to work)
  - Consumption C decreases
  - Employment N decreases
  - Real GDP Y decreases


Consumption, C
A
C_1
0
l_1
h
$-G$
Leisure, l


Consumption, C
A
C_1
0
$-G$
l_1
h
Leisure, l


Consumption, C
Pareto-optimum
Competitive
equilibrium
0
$-G$
h
Leisure, l

Numerical evaluation

$$\begin{aligned} Y &= z \cdot \overline{K}^{\alpha} \cdot (N^d)^{1-\alpha} \\ U &= C^{\beta} \cdot l^{1-\beta} \end{aligned}$$

with $z = 10$, $\overline{K} = 100$, $\alpha = 1/3$, $\beta = 1/2$, $T = G = 100$, $h = 100$

Shock: $T = 0$ and τ such that $\tau \cdot w \cdot N = G$

		Initial	Final	Change
Leisure	l	55.4	63.0	13.8%
Consumption	C	483.6	415.1	-14.2%
Utility	U	163.7	161.8	-1.2%
Employment	N	44.6	37.0	-17.1%
Real GDP	Y	583.6	515.1	-11.7%
Profits	π	194.5	171.7	-11.7%
Real wage	w	8.7	9.3	6.4%
Lump-sum tax	T	100.0		
Tax rate	τ		29.1%	