

ECON 303 : Exercise 2, Winter 2018

- Suppose that the production function of the representative firm is a function of both labor and capital and has the form $Y = zK^\alpha N^{1-\alpha}$, where the variables have the same interpretations as what we have had in class. Also, as always, the representative firm is assumed to be only interested in its profit.
 - Suppose that $\alpha = \frac{1}{3}$, $z = 30$, $K = 1$, and $w = 10$. Write down and solve the maximization problem of the representative firm. What will be the optimal level of hours hired, production, and profits by the firms?
 - Represent in a graph the connection between the marginal benefit and the marginal cost of labor within the production function.
 - Find the labor demand curve of the representative firm.
- Consider a Cobb-Douglas production function with three inputs: in addition to capital (K) and labor (L) we introduce a new variable, human capital shown by H, which stands for the quality of labor. You can interpret H as the knowledge and skill that people obtain from education (e.g., number of college degrees among the workers). For simplicity, we normalized the TFP to one:

$$Y = K^{\frac{1}{3}} H^{\frac{1}{3}} L^{\frac{1}{3}}$$

- Derive an expression for the marginal product of labor. How does an increase in the amount of human capital affect the marginal product of labor?
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- What is the income share paid to labor? What is the income share paid to human capital? In the National Income Accounts of this economy, what share of total income do you think workers would appear to receive? (Hint: Consider where the return to human capital shows up).
- An unskilled worker earns the marginal product of labor, whereas a skilled worker earns the marginal product of labor plus the marginal product of human capital. Using the answers to (a) and (b), find the ratio of the skilled wage to the unskilled wage. How does an increase in the amount of human capital affect this ratio? Explain.
- Some people advocate government funding of college scholarships as a way of creating a more egalitarian society. Others argue that scholarships help only those who are able to go to college. Do your answers to the above questions shed light on this debate?

- By looking at the growth accounting of many countries over the past century we can deduce the two following facts: (1) The economy's capital/labor ratio rises, and (2) The share of labor and capital in national income are roughly constant. How can this be? What do these two empirical facts suggest about the form of the production function? (Hints: (1) The share of labor is $w \cdot N/Y$ and the share of capital is $R \cdot K/Y$ (w is the wage rate and R is the rental rate of capital), (2) Assume factor markets are competitive, so factors are paid their marginal products).

- Consider a representative household with the utility function $U(C, l) = \theta(\ln C + \ln l)$ and the budget constraint $C = wN^s + \pi - T$. Here, like always, w is the real wages for work, N^s stands for amount of hours worked, π the profits transfers delivered by the representative firm, and T the amount of lump-sum taxes the government imposes. The representative household has a limited amount of hours a day, $N^s + l = h$.

Also suppose output, Y , is produced by competitive firms with technology $Y = zN^d$. The government in this economy taxes the representative household in order to finance a fix G in the following way $T = G$.

- Define the competitive equilibrium of this economy. Be careful to specify the problems that must be solved individually and the variables that each problem take as given

For the rest of the problem, assume that $h = 1, K = 1, z = 1$, and $G = \frac{1}{10}$.

b. Solve the utility maximization problem of the representative household. This is, in terms of what the representative household takes as given, define the optimal values of the decisions of the representative household.

c. Solve the profit maximization problem of the representative firm. This is, in terms of what the representative firm takes as given, define the optimal values of the decisions of the representative firm.

d. What would the balanced budget of the government will be. How many markets must be cleared in this economy? Write down the law that ensures us that we do not need to check the clearance of all the markets. How does it apply for this economy.

e. Draw clearly the production function and specify what kind of link does it have with production possibility frontier.

f. Draw clearly what is the link between the production possibility frontier, the utility curve and the real wages.

g. What would happen if government spending increases? Draw clearly how your graph will move. In which direction consumption, leisure, hours worked, and production will move?

h. Is there a maximum amount of government spending allowed in this economy? What interpretation can you give to this?

i) Determine the optimal value of λ that is the Lagrange multiplier on the household's budget constraint.

k) Write down the planner's problem.

l) What are the first-order conditions of the planner's problem?

m) Find the solution to the planner's problem.

n) Decentralize the planner's problem (that is, find the equilibrium prices so that the social planner problem and the competitive equilibrium problem have the same solution).

o) What does this tell you about the economy. (Recall the first and second Welfare Theorems).

5. Following the Great Depression of 1929 that led to more than 25% an unemployment rate in the the U.S. and other western economies, John Maynard Keynes argued that the best way to stimulate the econmy and lower the huge unemployment is the government intervention into the economy and substantial increases in its spending. He in fact famously said in a recession it is a good idea to pay people to dig holes and fill them up again! In this question you are asked to evaluate the logic behind his advice.

(a) Suppose the representative household has the following preferences

$$U(C, l, G) = \sqrt{C + G} + \sqrt{l}$$

where G stands for the government spending and it pays for them by imposing lump-sum taxes, T . Now the household's budget constraint is

$$C + w.l = w - T = w - G$$

For simplicity, suppose that the production function is only a function of labor, that is,

$$Y = N^d$$

a. Write down the Lagrangian problem, derive the first-order conditions and the labor supply curve. Does the labor supply is a function of gov. spending? Explain? What happens to output, employment, and consumption when G increases?

(b) Now suppose the utility function is not anymore a function of government spending and the government purchases useless things (e.g., holes in the ground). More precisely assume that

$$U(C, l) = \sqrt{C} + \sqrt{l}$$

Does labor supply now depend on the government expenditures? Explain. What happen for output and employment once G increases? Can we say that Keynes was right? Explain.