Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

MAT 1341 – The Diagnostic Test (v.1) **Instructor:**

Last name: ______________________

First name: ______________________

Student number:

Please, read the following instructions carefully:

- You have 80 minutes to complete this test. **Do not detach** the pages of this examination. Read each question carefully. Where it is possible to check your work, do so.
- Answer all questions by choosing (crossing) the respective box. You can use the backs of the pages and the last page for computations.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, laptops, pagers or any text storage or communication device is not permitted.

THIS SPACE IS RESERVED FOR THE MARKER:

Question	1	2	3	4	5	6	7	8	9	10	Total
Mark											
Out of	1	1	1	1	1	1	1	1	1	1	10

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1. Find an equation for the plane which contains two lines with parametric equations

$$x = -1 - t,\ y = -1 + t,\ z = 1 + 3t \quad \text{and} \quad x = 3 + 2s,\ y = -3 - s,\ z = 7 + 3s.$$

`cross (X) the correct answer`:

- [A] $6x + 9y - z = -16$
- [B] $6x + 9y + z = -14$
- [C] $-2x + 6y + z = -3$
- [D] $-2x + 11y + 9z = 0$
- [E] $-6x + 3y + z = 4$
- [F] $-11x + 7y + 2z = 6$

Solution: The normal vector for the plane will be $\pm d_1 \times d_2$, where $d_1 = (-1, 1, 3)$ and $d_2 = (2, -1, 3)$ are direction vectors for the lines above. A computation shows

$$d_1 \times d_2 = \begin{vmatrix} i & j & k \\ -1 & 1 & 3 \\ 2 & -1 & 3 \end{vmatrix} = (6, 9, -1).$$

So the normal vector for the plane is $n = (6, 9, -1)$. Since the plane contains $(-1, -1, 1)$, we get $n \cdot (-1, -1, 1) = -16$. Hence, an equation for the plane is $6x + 9y - z = -16$ and the correct answer is [A].

2. Find an equation of the plane passing through the points $P = (0, -3, 0)$ and $Q = (-1, 1, 2)$, and which is parallel to the line L with parametric equation $x = t$, $y = -t$, $z = t$.

`cross (X) the correct answer`:

- [A] $4x + y = -3$
- [B] $2x + z = -0$
- [C] $2x + y - z = -3$
- [D] $-2x + y - 3z = -3$
- [E] $y - 2z = -3$
- [F] $6x + y + z = -3$

Solution. Normal vectors to the plane are perpendicular to the difference $u = Q - P = (-1, 4, 2)$ and to the direction vector $v = (1, -1, 1)$ of L. Hence, normal vectors of the plane are in proportion to $u \times v = (6, 3, -3) = 3(2, 1, -1)$. So an equation of the plane has the form $2x + y - z = d$. Since $(0, -3, 0)$ is on the plane, $d = -3$ and the correct answer is [C].

3. Find an equation of the plane which passes through the point $(1, 1, 1)$ and which is perpendicular to the line whose parametric equations are

$$x = 1 - 4t,\ y = -6 + 2t,\ z = -3 + 3t.$$

`cross (X) the correct answer`:

- [A] $-4x + 2y + 3z = -25$
- [B] $-4x + 2y + 3z = -10$
- [C] $-4x + 2y + 3z = 1$
- [D] $x + 6y - 3z = 2$
- [E] $4x + 2y + 3z = 9$
- [F] $-4x + 2y + 3z = 10$

Solution: A normal vector for the plane will be a direction vector for this line which is $(-4, 2, 3)$. Since the plane contains $(1, 1, 1)$, we get $n \cdot (1, 1, 1) = (-4, 2, 3) \cdot (1, 1, 1) = 1$. Hence, an equation for the plane is $-4x + 2y + 3z = 1$ and the correct answer is [C].

4. Find parametric equations for the line containing the points $(-2, 2, 3)$ and $(4, -2, 0)$.

`cross (X) the correct answer`:

- [A] $x = -1 - 6t,\ y = 3 + 4t,\ z = 6 + t$
- [B] $x = -2 - 6t,\ y = 2 + 4t,\ z = 3 + 3t$
- [C] $x = -2 + 4t,\ y = 2 - 2t,\ z = 3$
- [D] $x = -1 - 6t,\ y = 1 - t,\ z = 4 + 3t$
- [E] $x = 4 + 6t,\ y = -2 + 4t,\ z = 1 + 3t$
- [F] Such a line does not exists

Solution: A direction vector for such a line is the difference $P - Q = (-6, 4, 3)$. There is only one line above with this direction vector, namely [E]. This line does pass through both P and Q, so the correct answer is [B].

5. Find a scalar equation for the plane with vector parametric equation

$$v = (2, 0, -2) + s(-1, 1, 2) + t(-4, 2, -1), \quad s, t \in \mathbb{R}.$$

`cross (X) the correct answer`:

- [A] $-9x + 4y + 6z = -30$
- [B] $-5x + 9y + 2z = -14$
- [C] $-2x + 9y + 5z = -14$
- [D] $2x + 9y + 5z = -6$
- [E] $5x + 9y + 2z = 6$
- [F] $5x + 9y - 2z = 14$

Solution: A normal vector for such a plane will be parallel to

$$d_1 \times d_2 = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ -4 & 2 & -1 \end{vmatrix} = (-5, -9, 2).$$

There is only one plane in the list above with this normal vector, namely [F]. One can check that this plane does contain the point $(2, 0, -2)$, so the correct answer is [F].

6. Find all vectors in $\mathbb{R}^3$ which are perpendicular to both $u = (1, -1, 5)$ and $v = (1, 2, 2)$.

`cross (X) the correct answer`:

- $\boxed{\text{A}}$ $\{(0, 0, 0)\}$
- $\boxed{\text{B}}$ $\{(-12, 3, 3)\}$
- $\boxed{\text{C}}$ $\{(-8, 2, 2)\}$
- $\boxed{\text{D}}$ $\{(-8, t+1, t+1) \mid t \in \mathbb{R}\}$
- $\boxed{\text{E}}$ $\{(-4t, t, t) \mid t \in \mathbb{R}\}$
- $\boxed{\text{F}}$ $\{(0, -t, t) \mid t \in \mathbb{R}\}$

Solution: Such vectors will be parallel to

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 5 \\ 1 & 2 & 2 \end{vmatrix} = (-12, 3, 3) = 3(-4, 1, 1).$$

So $\boxed{\text{E}}$ is the correct answer.

7. A triangle has vertices $A = (1,1,1)$, $B = (3,2,1)$ and $C = (2,1,3)$. Find the cosine of the interior angle at A.

`cross (X) the correct answer`:

$\boxed{\text{A}}$ 0

$\boxed{\text{B}}$ 1

$\boxed{\text{C}}$ 1/5

$\boxed{\text{D}}$ 2/5

$\boxed{\text{E}}$ 3/5

$\boxed{\text{F}}$ 4/5

Solution: We take $u = B - A = (2,1,0)$ and $v = C - A = (1,0,2)$. Then

$$\cos\theta = \frac{u \cdot v}{||u|| \cdot ||v||} = \frac{2}{\sqrt{5}\sqrt{5}} = 2/5.$$

So the correct answer is $\boxed{\text{D}}$.

8. Let L be the line through the point $(1,2,0)$ parallel to the vector $u = (-1,1,2)$. Find the point of intersection of L with the plane P given by the equation $x + y + 2z = 23$.

`cross (X) the correct answer:`

- [A] $(4,11,4)$
- [B] $(1,2,10)$
- [C] $(4,7,6)$
- [D] $(1,11,4)$
- [E] $(-4,10,7)$
- [F] $(-4,7,10)$

Solution: The line will have parametric equation $x = 1 - s$, $y = 2 + s$, $z = 2s$. Since in addition $x + y + 2z = 23$, we find

$$(1 - s) + (2 + s) + 2(2s) = 23,$$

which gives $s = 5$. Hence, the point is $(-4,7,10)$, so the correct answer is [F]

9. Let $u = (3, 3, 6)$ and $v = (-1, 2, 1)$.

Find the length of the projection of the vector u along v.

`cross (X) the correct answer`:

A 0

B $\frac{\sqrt{6}}{2}$

C $\frac{3\sqrt{6}}{2}$

D $\frac{3\sqrt{2}}{2}$

E $\frac{2\sqrt{6}}{3}$

F $\frac{2\sqrt{2}}{3}$

Solution: Since $proj_v u = \frac{u \cdot v}{||v||^2}$ we obtain

$$||proj_v u|| = \frac{|u \cdot v|}{||v||^2}||v|| = \frac{|u \cdot v|}{||v||} = \frac{9}{\sqrt{6}} = \frac{3\sqrt{6}}{2}.$$

So the correct answer is C.

10. Find the area of the triangle whose vertices are the points $P = (-1, 3, 2)$, $Q = (1, 1, 0)$ and $R = (2, 1, -1)$.

`cross (X) the correct answer:`

- $\boxed{\text{A}}$ 0
- $\boxed{\text{B}}$ $\sqrt{2}$
- $\boxed{\text{C}}$ 2
- $\boxed{\text{D}}$ $2\sqrt{2}$
- $\boxed{\text{E}}$ 4
- $\boxed{\text{F}}$ $4\sqrt{2}$

Solution: The area is $\frac{1}{2}||u \times v||$. Since $u = Q - P = (2, -2, -2)$ and $v = R - P = (3, -2, -3)$, we find

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 3 & -2 & -3 \end{vmatrix} = 2(1, 0, 1).$$

So $\frac{1}{2}||u \times v|| = \frac{1}{2}(2\sqrt{2}) = \sqrt{2}$, so the correct answer is $\boxed{\text{B}}$.

The last page (use it for computations)

$$\sin(\tfrac{\pi}{6}) = \cos(\tfrac{\pi}{3}) = \tfrac{1}{2}, \quad \sin(\tfrac{\pi}{3}) = \cos(\tfrac{\pi}{6}) = \tfrac{\sqrt{3}}{2}$$
$$\sin(\tfrac{\pi}{4}) = \cos(\tfrac{\pi}{4}) = \tfrac{\sqrt{2}}{2}, \quad \sin(0) = \cos(\tfrac{\pi}{2}) = 0, \quad \sin(\tfrac{\pi}{2}) = \cos(0) = 1.$$