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VOTRE LIEN AVEC CE QUI COMPTE — CONNECTS YOU TO WHAT MATTERS

Business Analytics - ADM 2302 M, N, P and Q

Midterm Exam - Winter 2018

SOLUTIONS

Duration: 2 hours

Professors: Jonathan Li and Rim Jaber

Last Name: ________________________ **First Name:** _____________________

Student #: ________________________ **Section:** _____________________

Instructions:

1- Write in your last and first name, your Student ID number, and your section in the spaces above, and sign the Statement of Academic Integrity below.
2- Verify that your exam copy has 8 pages (including this title page). If yours does not, please inform the professor now.
3- Answer all questions on this examination copy. **Use the opposite (blank) side, if necessary**. Only answers in this exam booklet will be marked. Show all work.
4- This is a closed-book exam: however, one double-sided cheat sheet letter (8.5" x 11") paper, and a calculator are allowed for arithmetic use only. NO restrictions on the content of the sheet, however **it must be handed over with the exam copy at the end of the exam period, or else your copy will not be marked.**
5- Read each question very carefully: only provide what is asked and do not hesitate to ask for clarifications when needed.
6- **NO COMMUNICATION DEVICES MAY BE WITHIN SIGHT.**
7- Good luck and keep smiling.

DO NOT WRITE ON THE TABLE BELOW

Questions	1	2	3	4	Total
Notes					
Points	26	25	22	27	100

Statement of Academic Integrity
The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

Statement to be signed by the student:
I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed:____

Note: an examination copy or booklet without that signed statement will not be graded and will receive a final exam grade of zero.

Question 1: Graphical Method and miscellaneous topics (26 points)

Consider the following linear programming problem:

Maximize Z = \$3x + \$4y (total revenue)

Subject to :

(1) $4x + 8y \leq 40$ → coordinates: (0,5), (10,0)

(2) $1.6x + y \geq 8$ → coordinats : (0,8), (5,0)

$x, y \geq 0$

a) Graph the constraint lines and mark them clearly with the numbers (1) and (2) to indicate which line corresponds to which constraint. Darken the feasible region.(8 points)

b) Determine the values for x and y that will maximize the total revenue. Fill the blanks below. Provide all necessary details to justify your answers. (6 points)

(4 points)

Optimal value of x = ____10_____

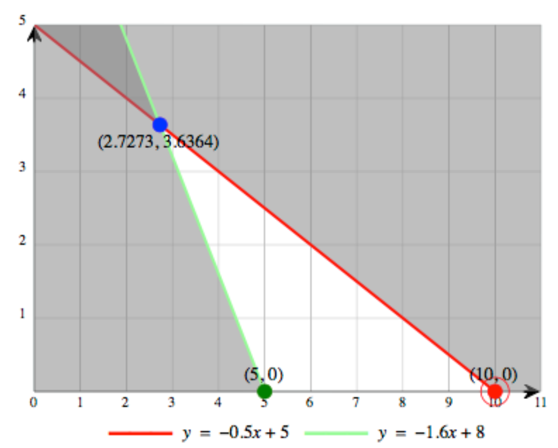
Optimal value of y = _____0_____

Optimal value of objective function Z = _30 (2 pts)

c) Based on the above optimal values, what is the amount of slack associated with the second constraint? (2 points)

Slack of the second constraint = 10*1.6+0 – 8 =16-8=8

Solution: a) recommend to plot isoprofit line by hand: coordinates: (0,3) and (4,0)



corner points:

X = 2.727 and Y = 3.636 → Z = 22.68

X = 5 and Y = 0 → Z = 15

2 points for plotting each constraint;

3 points to darken the feasible region,

1 point for labeling the graph

b) X = 10 and Y = 0 (4 points), Z = 30 (2 points)

d) Circle the proper answer(s) to question (1) to (6). There can be more than one answers.
(10 points= 2 points each)

1. If a linear programming problem has redundant constraints, then removing the redundant constraints will affect the optimal solution.
   A) True
   B) False

2. If the feasible region for a linear programming problem is unbounded, then the solution to the corresponding linear programming problem is _________ unbounded.
   A) Always
   B) Sometimes
   C) never

3. Consider the following constraints and choose the correct answer:
   Constraint T: $2X_1^2 + 3X_2 \leq 100$ Constraint S: $2X_1.X_2 + 3X_2 \leq 1000$
   A) constraint T can be used in a linear program
   B) constraint S can be used in a linear program
   C) neither can be used in a linear program
   D) both may be used in a linear program
   E) both may be used in an integer linear program, but not in a linear program

4. In a transportation problem, when would the supply constraints become non-binding?
   A) when total supply is more than total demand
   B) when total supply is less than total demand
   C) when total supply is equal to total demand
   D) it never happens
   E) it can happen in all cases A), B), and C)

Consider the transportation problem and its optimal solution in the tables below. (Note: In the first table, each entry corresponding to a pair of Source-Destination refers to the cost of transportation; each entry in the column of "Supply" refers to the capacity associated with the corresponding Source (at the same row); each entry in the row of "Demand" refers to the required demand for the corresponding destination (at the same column). In the second table, each entry corresponding to a pair of Source-Destination refers to the optimal amount to ship.

COSTS	Dest. 1	Dest. 2	Dest. 3	Dest. 4	Supply
Source 1	12	18	9	11	105
Source 2	19	7	30	15	145
Source 3	8	10	14	16	50
Demand	80	60	70	90	300 \ 300

Shipments	Dest. 1	Dest. 2	Dest. 3	Dest. 4	Row Total
Source 1	30	0	70	5	105
Source 2	0	60	0	85	145
Source 3	50	0	0	0	50
Col. Total	80	60	70	90	300 \ 300

5. What is the value of the objective function?
   A) 0
   B) $169
   C) 300 units
   D) $2,100
   E) $3,140

Question 2: Linear Programming Formulation (25 points)

An English wine merchant introduces two types of wine, A and B, from vineyards that are far away and after the process, puts it in bottles. Thus, he produces two brands of product: the Fein Wein and Party Plonk.

Wine A costs 0.80 dollars per liter and wine B costs 0.20 dollars per liter. The Fein Wein brand consists of 60% wine A and 40% wine B while the Party Plonk brand has 20% wine A and 80% wine B. The merchant shop sells the Fein Wein brand at 2 dollars per liter and the Party Plonk brand at 1.20 dollars per liter. The processing, bottling and distribution cost 0.5 dollars per liter for both brands.

The merchant has agreed to buy at least 24,000 liters of wine A this year. Also, there are available up to 120,000 liters of wine B that can be purchased. It is estimated that sales of Fein Wein brand during the year will reach 50,000 liters, however the demand for the Party Plonk brand is uncertain. The merchant has only 60,000 dollars available to buy wine A and wine B.

Determine how many liters of wine A and B should be used in the two brands in order for the merchant to maximize his profit.

Formulate algebraically the linear programming model for this problem. Define the decision variables, objective function, and constraints. DO NOT SOLVE the model.

Ans.
(2 PTS)
A1 : liters of wine A for the brand Fein Wein
A2 : liters of wine A for the brand Party Plonk
B1 : liters of wine B for the brand Fein Wein
B2 : liters of wine B for the brand Party Plonk

(4PTS = 2 PTS for profit + 2 PTS for cost)
Maximize (2-0.5)*(A1+B1)+(1.2-0.5)*(A2+B2)-0.8*(A1+A2)-0.2*(B1+B2)

Subject to:

A1+A2>=24,000	(Total Wine A)	(2 PTS)
B1+B2 <= 120,000	(Total Wine B)	(2 PTS)
0.8*(A1+A2)+0.2*(B1+B2)<=60,000	(Total budget)	(3 PTS)
A1+B1 >= 50,000	(Meeting the demand of Fein Wein)	(2 PTS)
A1 = 60%*(A1+B1)	(60% wine A for Fien Wein)	(2PTS)
B1 = 40%*(A1+B1)	(40% wine B for Fien Wein)	(2PTS)
A2 = 20%*(A2+B2)	(20% wine A for Party Plonk)	(2PTS)
B2 = 80%*(A2+B2)	(80% wine B for Party Plonk)	(2PTS)
A1,A2,B1,B2 >=0.		(2PTS)

Question 3: Linear Programming Formulation (22 points)

The J. Mehta Company's production manager is planning a series of one-month production periods for stainless steel sinks. The forecasted demand for the next three months is as follows:

Month	Demand for Stainless Steel Sinks
1	120
2	240
3	160

The Mehta firm can normally produce 100 stainless steel sinks in a month. This is done during regular production hours at a cost of $100 per sink. If demand in any one month cannot be satisfied by regular production, the production manager has three other choices:

(1) He/she can produce up to 50 more sinks per month in overtime but at a cost of $130 per sink;
(2) He/she can purchase a limited number of sinks from a friendly competitor for resale. The maximum number of subcontracting over the three-month period is 200 sinks (NOT 200 sinks each month), at a cost of $150 each;
(3) Or, he/she can satisfy the demand from his/her on-hand inventory (i.e. sinks in stock at the end of the month). The inventory holding cost is $10 per sink per month.

NO Inventory on hand at the beginning of month 1.

Formulate algebraically the linear programming model for this production problem to minimize cost. DO NOT SOLVE the model.
Write the objective function and constraints using the following decision variables:

- xi: # of sinks to produce in regular time in month i,
- yi: # of sinks to produce in overtime in month i,
- si: # of sinks supplied by subcontractors in month i,
- Ii: # of sinks in stock at the end of month i,

Where i = 1, 2 and 3

Minimize 100(x1+x2+x3) + 130(y1+y2+y3) + 150(s1+s2+s3) + 10(I1+I2+I3)
(4 points)

Subject to

Demand Constraints

x1+y1+s1-I1 = 120 (month1) (3 points)
I1+x2+y2+s2 -I2 = 240 (month 2) (2 points)
I2+x3+y3+s3-I3= 160 (month 3) (2 points)

x1 <= 100, x2 <= 100, and x3 <= 100 (capacity regular production) (3 points)
y1<= 50, y2 <= 50, and y3<= 50 (capacity overtime production) (3 points)
s1+s2+s3 <= 200 (maximum subcontracting) (3 points)
xi, yi, si, Ii >= 0 for all i = 1,2, and 3 (non-negativity) (2 points)

Question 4: Sensitivity Analysis (27 points)

Canadian Market Research (CMR) is a marketing research firm that handles consumer surveys. One of its clients is a national press service that periodically conducts political polls on issues of widespread interest. In a survey for the press service, CMR determines that it must fulfill several requirements in order to draw statistically valid conclusions on the sensitive issue of Canadian immigration laws:

1. Survey at least 2,300 Canadian households in total.
2. Survey at least 1000 households with heads who are 30 years of age or younger.
3. Survey at least 600 households with heads who are between 31 and 50 years of age.
4. Ensure that at least 15% of those surveyed live in the city that is a major destination for immigrants (such as Toronto)
5. Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a destination city.

CMR decides that all surveys should be conducted in person. It Estimates that the costs of reaching people in each age and region category are as follows:

Cost per person surveyed ($)

Region	Age <= 30	Age 31-50	Age >= 51
Destination City	$7.5	$6.80	$5.50
Not in destination City	$6.90	$7.25	$6.10

CMR's objective is to meet the five sampling requirements at the least possible costs.

The following correct output for this problem is provided below as Excel Solver output. You will need this output to answer the following questions **(a) through (f) below each part of which is to be considered <u>independently</u> of all others.**

	D_1	D_2	D_3	N_1	N_2	N_3			
	<= 30 and destination	31-50 and destination	>= 51 and destination	<= 30 and not destination	31-50 and not destination	>= 51 and not destination			
Number of Households	0.00	600.00	140.00	1000.00	0.00	560.00			
Interview Cost	$7.50	$6.80	$5.50	$6.90	$7.25	$6.10		<- Objective	
Constraints:									
Total Households	1	1	1	1	1	1	2300.00	>=	2300
<= 30 Households	1			1			1000.00	>=	1000
31-50 Households		1			1		600.00	>=	600
Destination Cities	0.85	0.85	0.85	-0.15	-0.15	-0.15	395.00	>=	0
Limit on >= 51 in Destination			0.8			-0.2	0.00	<=	0
							LHS	**Sign**	**RHS**

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number of Households <= 30 and destination	0.00	0.60	7.5	1E+30	0.6
C5	Number of Households 31-50 and destination	600.00	0.00	6.8	0.45	0.82
D5	Number of Households >= 51 and destination	140.00	0.00	5.5	0.6	29.9
E5	Number of Households <= 30 and not destination	1000.00	0.00	6.9	0.6	0.92
F5	Number of Households 31-50 and not destination	0.00	0.45	7.25	**W**	**X**
G5	Number of Households >= 51 and not destination	560.00	0.00	6.1	1.025	0.6

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H8	Total Households	2300.00	5.98	2300	1E+30	700
H9	<= 30 Households	1000.00	0.92	1000	700	1000
H10	31-50 Households	600.00	0.82	600	700	493.75
H11	Destination Cities	395.00	0.00	0	395	1E+30
H12	Limit on >= 51 in Destination	0.00	-0.60	0	560	140

(a) What is the optimal solution to CMR's marketing research problem? How much would such households survey cost? (4 points).

CMR's need to survey 600 households between age 31 to 50 and living in a destination city,140 age greater >= to 51 and living in a destination city, 1000 households between with the age <= 30 and not living in a destination city, and 560 households age >= 51 and not living in destination city. → no need to be managerial statement ***(2 points)***

The total cost of the survey is $15,166 ***(2 points)***

(b) If the cost of surveying one person 51 years of age or older and do not live in a destination city (e.g. N_3) is decreased to $5.9, will the optimal solution change? Will the cost change? If yes, by how much. Justify. (6 points)

An decrease of $0.2 is within the allowable decrease of 0.6 ***(2 points)***
thus the optimal solution will not change *(2 points)*

*However, the cost will go DOWN by 0.2*560 = $112 to $15,054.* ***(2 points)***

(c) What is the impact on the optimal solution and the value of the objective function, if CMR wants to increase the sample size to 2500 (i.e. survey at least 2500 Canadian households in total)? Justify. (6 points)

This represent an increase of 200 (=2500-2300), which within the allowable increase of infinity, thus the value of the shadow price (5.98) will remain valid ***(2 points)***
The optimal solution would change to an unknown value (need to solve) ***(2 points)***

Each additional person who needs to be included in the survey will increase cost by $5.98. Hence, the total cost will increase to
*$16,362(=15, 166 + 5.98*200= 15,166 + 1,196)* ***(2 points)***

(d) What is the impact on the value of the objective function, if we can reduce the minimum required number of respondents' age of 30 or younger to 800, provided we raise the number of respondents 31-50 years of age to 900? Justify. (5 points)

This is a simultaneous change in the RHS of the second and third constraint.

100% Rule ***(3 points)***
*[(200/1000) +(300/700)] * 100 = 20% +%42.86 = 62.82% < 100 thus the value of the shadow price will not change.*

The revised cost is = 15,166 + (0.92)(-200) + (0.82)(300) = $15,228 ***(2 points)***

(e) Two numbers have been removed from the objective function sensitivity table by your professor (the letters W and X appear instead of the numbers). What are the correct values of W and X? Justify. (4 points)
If no justification → ok

W = Infinity → because at a cost of $7.25 we are NOT surveying people between the age of 31 and 50 as it is not cost effective thus this would remain true if the cost of the survey increase up to infinity. ***(2 points)***

X = 0.45 → the value of the reduced cost is 0.45 thus for every person surveyed in this category you will cost will go up by 0.45….thus in order be cost effective the cost of surveying has to be reduced at least by 0.45. ***(2 points)***

(f) Is the solution to this problem a unique optimal solution or there is a multiple optimal solution? Justify. (2 points)

It is a unique solution because NO zero in the allowable increase or decrease column