Reference material for exam and tutorial tests - PHYS1004 Winter 2015

$N_A = 6.022 \times 10^{23}$ mol^{-1}
$G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}
$g = 9.81$ m s^{-2}
$m_e = 9.109 \times 10^{-31}$ kg
$m_p = 1.673 \times 10^{-27}$ kg
$e = 1.602 \times 10^{-19}$ C
$k = 1/(4\pi\epsilon_0) = 8.988 \times 10^9$ N m^{2}C^{-2}
$c = 2.998 \times 10^8$ m/s
Permittivity, free space,
$\epsilon_0 = 8.854 \times 10^{-12}$ C^{2}N^{-1}m^{-2}
Permeability, free space,
$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}= (H/m)
1 J = 1 N m   1 eV = 1.602×10^{-19} J
1 C = 1 A s   1 V = 1 J/C
1 Ω = 1 V/A   1 F = 1 C/V
1 Wb = 1 T m^2 1 H = 1 T m^2 /A
1 Hz = 1 s^{-1} 1 T = 1 N/(A m) [10^4 Gauss]
$(1+x)^n = 1 + nx + n(n-1)x^2/2! +$
$n(n-1)(n-2)x^3/3! + ...$if $|x| < 1$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
Uniform linear acceleration: $v_f^2 = v_i^2 + 2a\Delta s$
$s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \quad v_f = v_i + a\Delta t$
Uniform circular acceleration: $|\vec{a}| = \frac{v^2}{r}$
Spring-block:$F = -k\Delta s \quad U_{sp} = \frac{1}{2}k(\Delta s)^2$
$W > 0 \Rightarrow$ energy transferred to an object by force
$K = \frac{1}{2}mv^2, \quad \Delta K = K_f - K_i \quad U_g = mgh$
Ignoring dissipative energy losses:
$\Delta E_{\text{sys}} = \Delta K + \Delta U = W_{\text{ext}}$ or $E_f = E_i + W_{\text{ext}}$
Conservation E_{mech} in an isolated system ($W_{\text{ext}} = 0$):
$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$
Work-KE theorem: $\Delta K = W_{\text{net}}$ [J]
Work $W_{force} = \int_i^f \vec{F} \cdot d\vec{s}$ [J]
Power $P = dW/dt = \vec{F} \cdot \vec{v}$ [W]
$\Delta U = U_f - U_i = -W_{force}$ [J]
If external agent does work against force:
$\Delta U = W_{ext}$
A force is conservative if the work to move a mass or charge between two points is path independent.
Coulomb's Law:
$F_{\text{1on2}} = F_{\text{2on1}} = \frac{k|q_1||q_2|}{r^2}$ [N]

Shell Theorems: A shell of uniform charge: 1) attracts or repels an external charge as if all of the shell's charge were at its centre; and 2) exerts no net electrostatic force on a charge in its interior.
$\vec{E} = \vec{F}/q_0$ [N C^{-1} or V/m]
Electric Dipole
2 charges $+q, -q$ separated by s; dipole moment $|\vec{p}| = qs$ directed from $-q$ to $+q$
$\vec{\tau} = \vec{p} \times \vec{E} \quad U = -\vec{p} \cdot \vec{E}$
Electric Fields:
point charge: $|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ and $\vec{E} = |\vec{E}|\hat{r}$
non-conducting ∞ sheet: $|\vec{E}| = \sigma/(2\epsilon_0)$
conducting ∞ sheet: $|\vec{E}| = \sigma/(\epsilon_0)$
Gauss's Law:
$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$
Electric potential, $V = U/q_0$ [V] where U is the electrostatic potential energy
$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$
$\vec{E}$ from V: $E_s = -\frac{\partial V}{\partial s}$
V of point charge: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
System of 2 charges: $U_{12} = V_1 q_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$
for several charges: $U = U_{12} + U_{13} + U_{23} + ...$
Capacitance: $C = Q/\Delta V_C$ [F]
parallel-plate capacitor: $C = \frac{\epsilon_0 A}{d}$
in series: $1/C_{eq} = \sum_i 1/C_i$
in parallel: $C_{eq} = \sum_i C_i$
$U_C = \frac{1}{2}C(\Delta V_C)^2$
$u_E = \frac{1}{2}\epsilon_0 E^2$
with dielectrics: $\epsilon_0 \rightarrow \epsilon = \kappa\epsilon_0$,
dielectric constant: $\kappa = \epsilon/\epsilon_0$
Current: $I = dQ/dt$ [A]
Current density: $\vec{J} : |\vec{J}| = I/A$ in direction of $\vec{E}$.
v_d is drift speed: n_e is conduction e$^-$/m^3
$J = n_e e v_d$
Resistance: $R = \Delta V_R / I$ [Ω]
Ohms law $\Rightarrow R$ independent of ΔV_R
$R = \rho L/A$ units of resistivity ρ are [Ωm].
Conductivity $\sigma = 1/\rho$ [Ω^{-1}m^{-1}]
$J = \sigma E \quad J = n_e e v_d$
R in parallel: $\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$

R in series: $R_{eq} = \sum_i R_i$

Power, $P = iV$ (general case).

If Ohm's law holds: $P = i^2 R = V^2/R$

emf: $\mathcal{E} = \frac{dW}{dQ}$ [V]

Kirchhoff's Rules:

Loop: $\sum_i \Delta V_i = 0$

Junction $\sum_i I_i = 0$

RC circuit:

Discharging: $Q(t) = Q_0 e^{-t/\tau}$ $\tau = RC$ [s]

Charging: $Q(t) = Q_{max}(1 - e^{-t/\tau})$ $Q_{max} = C\mathcal{E}$

Magnetic fields: Force from $\vec{B}$

moving charge: $\vec{F}_{\text{on q}} = q\vec{v} \times \vec{B}$

wire current: $\vec{F}_{\text{wire}} = I\vec{\ell} \times \vec{B}$

2 parallel wires: $F_{||\text{ wires}} = \frac{\mu_0 \ell I_1 I_2}{2\pi d}$

(|| attract, anti|| repel)

Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \mu_0 \epsilon_0 \frac{d\Phi_e}{dt}$

long straight wire: $\vec{B} = \frac{\mu_0}{2\pi} \frac{I}{d}$(tangent to circle,rhr)

$B_{\text{solenoid}} = \mu_0 n I$ where $n = N/\ell$

Biot-Savart Law, current element $Id\vec{s}$:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{r}}{r^2}$$

Magnetic dipoles:

loop's magnetic dipole moment: $\vec{\mu} = NIA$ (rhr I)

mag field on axis: $\vec{B}_{\text{loop}} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{z^3}$

Torque on current loop: $\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential energy of magnetic dipole: $U = -\vec{\mu} \cdot \vec{B}$

Magnetic Flux through loop:

$\Phi_m = N \int_{\text{loop}} \vec{B} \cdot d\vec{A}$ [Wb]

Faraday's Law: $\mathcal{E} = -\frac{d\Phi_m}{dt}$, or $|\mathcal{E}| = \left|\frac{d\Phi_m}{dt}\right|$

with direction of induced current such that induced $\vec{B}$ will oppose the change in Φ_m.

Induced electric field: $\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$

Inductance: L [H] =[Wb/A]

solenoid: $L = \frac{N\Phi_m}{I} = \mu_0 n^2 \ell A$

$\mathcal{E}_{\text{coil}} = L\left|\frac{dI}{dt}\right|$ direction from Lenz's Law

$\Delta V_L = -L\frac{dI}{dt}$

$U_{\text{inductor}} = \frac{1}{2}LI^2$ $u_B = \frac{1}{2\mu_0}B^2$

LR circuit: $I = I_0 e^{-t/\tau}$ $\tau = \frac{L}{R}$

oscillatory motion: $x(t) = A\cos(\omega t + \phi_0)$

angular frequency ω [rad/s]

frequency $f = \omega/2\pi$ (Hz)

period $T = 1/f$ [s]

Spring-block system: $U = \frac{1}{2}kx^2$

x displacement, k spring constant $\omega = \sqrt{\frac{k}{m}}$

LC Circuit: $\omega = \sqrt{\frac{1}{LC}}$

$Q(t) = Q_0 \cos(\omega t + \phi_0)$

AC circuits:

capacitive reactance: $X_C = 1/(\omega C)$ [Ω]

inductive reactance: $X_L = \omega L$ [Ω]

$I_{rms} = I_{max}/\sqrt{2}$ $V_{rms} = V_{max}/\sqrt{2}$

$\mathcal{E}_{rms} = \mathcal{E}_{max}/\sqrt{2}$ $P_{ave} = I_{rms}^2 R$

Travelling waves:

$v = \lambda f$ $k = 2\pi/\lambda$ $\omega = vk$

$D(x,t) = A\sin(kx - \omega t + \phi_0)$

Electromagnetic waves:

$E = E_{max}\sin(kx - \omega t), B = B_{max}\sin(kx - \omega t)$

$c = 1/\sqrt{\epsilon_0 \mu_0}$ $E = cB$ $\vec{E} \perp \vec{B}$

Poynting vector: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ [W/m^2]

$I = S_{ave} = E_{rms}^2/(c\mu_0)$

index of refraction: $n = c/v$

Circumference of a circle: $2\pi R$

Area of a circle: πR^2

Surface area of sphere: $4\pi R^2$

Volume of sphere: $\frac{4}{3}\pi R^3$

Error Propagation Equations

$$\sigma_z = \left[\left(\frac{\partial z}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial z}{\partial y}\sigma_y\right)^2 + ...\right]^{\frac{1}{2}}$$

for $z = ax + by - cu + ...$

$$\sigma_z = \left[(a\sigma_x)^2 + (b\sigma_y)^2 + (c\sigma_u)^2 + ...\right]^{\frac{1}{2}}$$

for $z = Ax^n y^m u^{-p}$

$$\sigma_z = z\left[(n\sigma_x/x)^2 + (m\sigma_y/y)^2 + (p\sigma_u/u)^2 + ...\right]^{\frac{1}{2}}$$

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