Math Whisperer

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Improper Integral

Evaluate the following IMPROPER Integral

$$\int_0^4 \frac{1}{x-3}\,dx$$

$$= \int_0^{3^-} \frac{1}{x-3}\,dx + \int_{3^+}^4 \frac{1}{x-3}$$

$$= \lim_{b\to 3^-} \int_0^b \frac{1}{x-3}\,dx + \lim_{c\to 3^+} \int_c^4 \frac{1}{x-3}\,dx$$

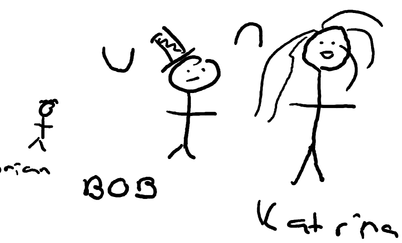
$$= \lim_{b\to 3^-} \ln|x-3|\Big]_0^b + \lim_{c\to 3^+} \ln|x-3|\Big]_c^4$$

$$= \lim_{b\to 3^-} \left[\ln|b-3| - \ln|0-3|\right] + \lim_{c\to 3^+} \left[\ln|4-3| - \ln|c-3|\right]$$

$$\lim_{b\to 3^-} \ln|3^- - 3| = -\infty \quad \text{Divergent}$$

Comparison Test

Does the following integral C or D



$$\int_1^\infty \frac{3\sin^2 x}{x^2+4}\,dx$$

$$-1 \le \sin x \le 1$$

$$0 \le \sin^2 x \le 1$$

$$0 \le 3\sin^2 x \le 3$$

$$x^2+4 \ge x^2$$

$$\frac{1}{x^2+4} \le \frac{1}{x^2}$$

$$\frac{3\sin^2 x}{x^2+4} \le \frac{3}{x^2}$$

Bob Kat

C

p-test

$$\int_1^\infty \frac{1}{x^p}\,dp \quad p>1 \text{ C} \quad p\le 1 \text{ D}$$

$$\int_0^1 \frac{1}{x^p}\,dp \quad p\ge 1 \text{ D} \quad p<1 \text{ C}$$

$\therefore$ converges

Solve: $\frac{y'}{x^2} = 1+y^2$; $y(0)=1$

Solve: $\frac{y'}{x^2} = 1 + y^2$; $y(0) = 1$

$\frac{dy}{dx} \cdot \frac{1}{x^2} = 1 + y^2$

$\int \frac{1}{1+y^2}\, dy = \int x^2 dx$

$\arctan x = \frac{x^3}{3} + C$

$x = 0, y = 1$

$\arctan(1) = \frac{0^3}{3} + C$

$\frac{\pi}{4} = C$

$\arctan y = \frac{x^3}{3} + \frac{\pi}{4}$

$y = \tan\left(\frac{x^3}{3} + \frac{\pi}{4}\right)$

Use Euler's Method and 3 steps to evaluate $y(6)$ for $y' = 3x + 2y$ when $y(0) = 2$

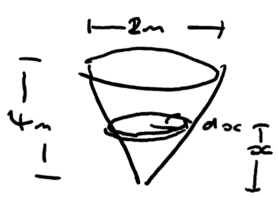
n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	4	10
1	2	10	26	62
2	4	62	136	334
3	6	334		

$\Delta x = \frac{b-a}{n}$

$\Delta x = 2$

$x(6) = 334$

How much work is required to pump all of the Tide Pod liquid $\rho = 1200$ kg/m³ out of your body to a distance 2m above your head.



$W = Fd$

$F = mg$

$m = \rho V$

$V = A\,dx$

Similair triangles

$\frac{4}{2} = \frac{x}{2r}$

$r = \frac{x}{4}$

$A = \pi r^2$

$A = \pi x^2$

$$\overline{16^2}$$

$$V = \frac{\pi x^2}{16^2}\,dx$$

$$m = \frac{1200}{16}\pi x^2\,dx$$

$$F = \frac{1200}{16}\cdot 9.81\,\pi x^2$$

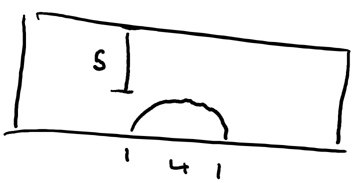
$$= 735\,\pi x^2\,dx \qquad d = 6-x$$

$$W = 735\,\pi x^2(6-x)\,dx$$

$$W_T = \int_0^4 735\,\pi x^2(6-x)\,dx$$

$$W_T = 47040\pi\,J$$

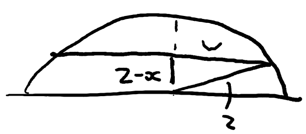
Evaluate the hydrostatic force on the submersed object. Set up but do not evaluate.



$F = PA \qquad h = 5+x$

$P = \rho g h \qquad P = \rho g h$

$= 9800(5+x)$



Pyth theorem

$(2-x^2)+w^2 = 2^2$

$w = \sqrt{4x-x^2}$

$F = PA$

$F = 19600(5+x)\sqrt{4x^2+x^2}$

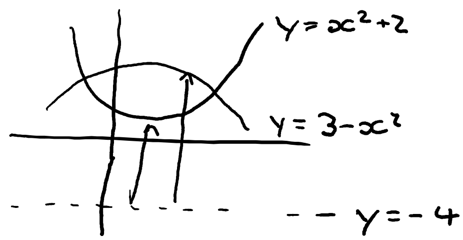
$$F_T = \int_0^2 19600(5+x)\sqrt{4x+x^2}\,dx$$

Find the volume of the solid found when the region bounded by $y = x^2+2$, $y = 3-x^2$ is rotated around $y=-4$

Region: $y = f(x)$

Rotating Line $y = f(x)$ $\Big]$ match $\therefore$ washers

$r_{in} = (x^2+2)-(-4) = x^2+6$



$r_{in} = (x^2+2)-(-4) = x^2+6$

$r_{out} = (3-x^2)-(-4) = 7-x^2$

$A_{in} = \pi r_{in}^2 = \pi(x^2+6)^2 = \pi(x^4+12x^2+36)$

$A_{out} = \pi r_{out}^2 = \pi(7-x^2)^2 = \pi(49-14x^2+x^4)$

$A_{answer} = \pi(13-26x^2)$

$x^2+2 = 3-x^2$

$2x^2-1$

$V = \pi(13-26x^2)\,dx$

$$V = \pi\int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \pi(13-26x^2)\,dx$$

$V \approx 38.5$

Find the volume of the solid formed when the region bounded by $y=x^3+2$, $x=0$, $x=3$, $y=0$ is rotated around

$r = x+5$

$h = x^3+2$

$SA = 2\pi r h$

$SA = 2\pi(x+5)(x^3+2)$

$SA = 2\pi(x^4+5x^3+2x+10)$

$$V = \int_0^3 2\pi(x^4+5x^3+2x+10)\,dx$$

$= \frac{3777}{10}\pi$