

NAME

S.N.

Section: A B C D E F G H

I have read the text on academic integrity at the top of the question sheet and I pledge not to have committed or attempted to commit academic fraud in this examination. Signed:

Q1 a) [ 2 ]

b) [ 3 ]

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c) [ 4 ]

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d) [ 5 ]

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e) [ 2 ]

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Q2 a) [ 4 ]

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b) [ 5 ]

Q3 a) [ 2 ]

b) [ 2 ]

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c) [ 4 ]

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Q4 a) [ 2 ]

b) [ 2 ]

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c) [ 3 ]

d) [ 3 ]

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Q5 a) [ 2 ]

b) [ 2 ]

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c) [ 3 ]

d) [ 3 ]

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Q6 a) [ 2 ]

b) [ 2 ]

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c) [ 2 ]

d) [ 2 ]

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e) [ 2 ]

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Q1 a) [ 2 ]

1 z value for 90% (0.4 on table) is 1.28  
 1  $1.28 = (x-47.8)/2.8 \Rightarrow x = 51.384 \text{ cm}$   
 \*\*\* (0.5 no units)

TO PROFS: I've put asterisks on units.  
 Normally I'd deduct, but it appears that unit issues not made clear in some classes.  
 I've kept 1/2 mark deduction on \$ Q4a

b) [ 3 ]

To be normal (unfortunate wording!),  
 forearm length between 45.0 and 52.2 cm  
 $P(45 < x < 52.2) =$   
 1  $P[(45-47.8)/2.8 \leq z \leq (52.2-47.8)/2.8]$   
 1  $= P(-1 \leq Z \leq 1.57) = 0.3413 + .4418 = 0.7831$   
 $P(\text{Abnormal}) = 1 - P(\text{Normal}) = 0.2169$   
 (probability man has abnormal forearm len.)  
 $P(\text{Two Abnormals}) = P(\text{Abnormal}) * P(\text{Abnormal})$   
 1  $= 0.2169 * 0.2169 = 0.047$

c) [ 4 ]

1 z-value for 90. percentile is  $z = 1.28$   
 1 Sample size (144) is large enough to use CLT. Sample means are approximately Gaussian.

$y_{\text{bar}}$  : observed forearm length of 144 students  
 2  $1.28 = (y_{\text{bar}} - 47.8)/(2.8/\sqrt{144}) \Rightarrow y = 48.1 \text{ cm}$   
 -0.5 no units \*\*\*\*

d) [ 5 ]

2 sample mean = 48.02 cm sample stdev = 2.49 cm n = 9  
 population mean = 48 cm population stdev = ? we need to use t distribution.

1  $t = (48.02 - 48)/(2.49/\sqrt{9}) = 0.024 \text{ df} = 9 - 1 = 8$   
 1  $P(t \geq 0.024) = \text{more than } 25\%$

1 This is a large probability so we would accept sample is drawn from the stated population.

e) [ 2 ]

1 In this case we can use the GAUSSIAN, because we get an EXACT distribution.

$z = (48.02 - 48)/(2.00/\sqrt{9}) = 0.03 \quad P(Z \geq 0.03) = 0.488$

There is no change in our conclusion. This is a large probability so we accept that the sample is drawn from stated population.

Q2 a) [ 4 ]

1 mark	$N, S, n, x = 14 \quad 4 \quad 8 \quad 0$	$N, S, n, x = 14 \quad 4 \quad 8 \quad 1$
	$C(S, x) = 1$	$C(S, x) = 4$
	$C((N-S), (n-x)) = 45$	$C((N-S), (n-x)) = 120$
	$C(N, n) = 3003$	$C(N, n) = 3003$
1 mark	$P(x=0) = 0.014985$	1 mark $P(x=1) = 0.15984$
1 mark	$P(x < 2) = 0.17482$	

b) [ 5 ]

1 mark Binomial large n  
 1 mark  $n, p, x = 1000 \quad 0.32 \quad 400$   
 $np = 320 > 5$   
 $nq = 680 > 5$   
 1 mark Can use Normal approximation  
 Mean = 320  
 Variance = 217.6  
 1 mark  $z = 1.999828$  With Correction for continuity else 2.033723  
 1 mark  $P(x \geq 350) = 0.022759$  else 0.02099  
 -0.5 no correction for continuity (10% difference in probability!)

(Minitab binomial gives 0.0233842)

Q3 a)[ 2 ]

2marks  $47/(47+3) = 0.94$   
=P(T+ | meningitis)

b)[ 2 ]

2marks  $47/(47+13) = 0.783333$   
= P(meningitis | T+)

c) [ 4 ]

1 mark Bayes (or showing 2-way table)

	Meningitis	WELL
Test +ve	0.94	0.028889
Test -ve	0.06	0.971111

1 mark Prior 0.2 0.8

1 mark Condit 0.94 0.028889

Joint 0.188 0.023111

1 mark Posterior 0.890526 0.109474

Q4 a)[ 2 ]

$E(aX + bY) = aE(X) + bE(Y)$   
 $= 100*10 + 200*15 = \$4,000$

1 point for the formula,

1 point for the correct answer

-0.5 no \$ somewhere in answer

Let's keep this one - it's money!!!

b)[ 2 ]

$Cov(X,Y) = COR(X,Y)*SD(X)*SD(Y) = 0.6*5*6 = 18$

Units are  $\$^2$

1 point for the formula,

1 point for the correct answer

\*\*\*-0.5 for no units

c)[ 3 ]

$V(aX+bY)_{indep} = a^2 V(X) + b^2 V(Y)$   
 $= 200^2 * 25 + 300^2 * 36 = 5,410,000 \text{ } \$^2$

-1 point for not squaring constants

-1 for using s.d. instead of the variance

\*\*\*units

d)[ 3 ]

$V(aX+bY) = a^2 V(X) + b^2 V(Y) + 2ab COV(X,Y)$

$SD(aX + bY) = \sqrt{100^2 * 25 + 200^2 * 36 + 2 * 100 * 200 * 0.6 * 5 * 6} =$

$= 1,552.42 \text{ } \$$

1 for "formula", 2 for OK answer

Deductions (down to 0) of 1 mark for each

no "2", no "ab", V() vs SD(); \*\*\*½ for units

Q5 a)[ 2 ]

Poisson model - constant RATE/time of hits

with mean NUMBER of successes for period

= time interval \* rate

$P(k \text{ hits} | \mu \text{ expected}) = \mu^k \exp(-k)/k!$

1 mark Poisson, 1 mark for a reason

b)[ 2 ]

For 30 mins = ½ hr expect  $1/2 * 1/2 = 1/4$  hits

$P(0 | .25) = \exp(-0.25) = 0.778801$

1 mark setup, 1 answer (-.5 not evaluated)

c)[ 3 ]

1 hour expect ½ hit.

$P(>2 | 0.5) = 1 - P(0) - P(1)$

$= 1 - \exp(-0.5) * (1 + .5)$

$= 1 - 0.606531 * (1.5) = 1 - 0.909796$

$= 0.090204$

1 mark mean, 1 mark setup, 1 mark answer

d)[ 3 ]

The condition is irrelevant - the prob of a hit is constant per unit time.

$P(2 | 1/2 \text{ expected}) = .5^2 \exp(-.5)/2!$

$= .25 * .606531 / 2 = .07582$

1 for justification, 1 for setup, 1 for #

Q6 a)[ 2 ]

Use  $\mu = \text{mean} = 0.5 * (.46 + .49) = .475\text{g}$

Range = 6 \* sigmas so use

$\sigma = (.49 - .46) / 6 = 0.005\text{g}$

-0.5 no units \*\*\*

b)[ 2 ]

NNN has 2 "low" sachets, 8 "high"

SS has 2 "low" and 6 "high" <-- slightly better quantitative performance

SS has tighter distribution but long tails

1 mark counts some way, 1 mark for "shape" comment

c)[ 2 ]

NNN appears to have a Gaussian dist of salt

(1 mark) but SS is highly non-gaussian

-- many outliers, though symmetric.

d)[ 2 ]

Need to use the bottom set - it uses

the "specification" sigma (see  $S_{\bar{}}$ )

rather than the data-derived values,

which are too generous in variability.

1 mark each for mentioning spec/historical value AND data-generated value.

e)[ 2 ] Because there are several failures in keeping the VARIATION low enough (S chart)

for the B (bottom) graphs, the process is UNCONTROLLABLE - need to revise the whole

PROCESS. 1 mark UNCONTROLLABLE or equivalent (not OUT OF CONTROL unless applied to

VARIABILITY), 1 mark change whole process.