



## GNG 1105 C ENGINEERING MECHANICS

Mid-term Exam  
November 2<sup>nd</sup>, 2017  
Time: 75 minutes

Page 1 of 1

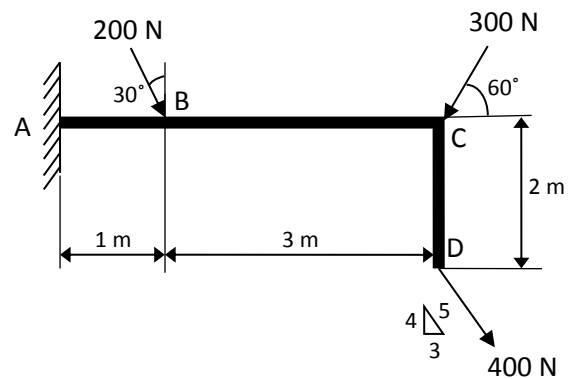
Prof. A. Skaff

**Closed book examination. Only non-programmable calculators are allowed.**

### Problem 1 (15 marks)

Rod ACD is acted upon by 200N force at B, 300N at C and 400N at D. This rod is being held in equilibrium by the fixed support at A.

- Draw the Free-Body-Diagram of rod ABCD.
- Reduce the 3 forces into a force-couple system at point C.
- Find the reactions at point A.

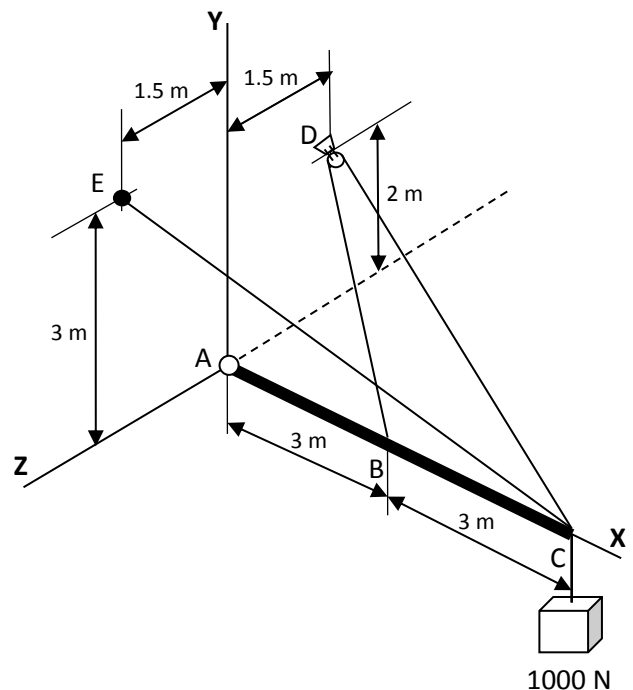


### Problem 2 (15 marks)

Boom AC is supported by a ball and socket joint at A and by cables EC and BDC which goes over pulley D (cable BDC is a continuous cable going around the pulley).

This boom is carrying a 1000N load at its end C and it is in equilibrium.

- Draw the Free-Body-Diagram of boom AC.
- Write the tension in cables BD, CD and CE in vector form.
- Calculate the tension in cables BD, CD and CE.

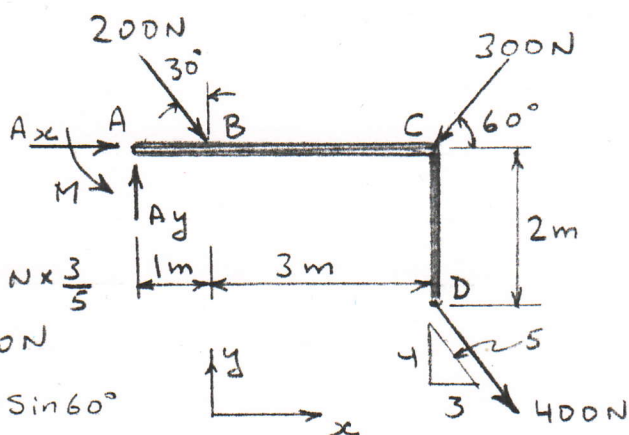


GNG 1105 A & C  
Mid-Term Examination  
SOLUTIONS

Nov. 1, 2017

1. a) FBD

See diagram to the right.



b)  $R_x = \sum F_x = 200 \text{ N} \sin 30^\circ$

$$-300 \text{ N} \cos 60^\circ + 400 \text{ N} \times \frac{3}{5}$$

$$= 100 \text{ N} - 150 \text{ N} + 240 \text{ N} = 190 \text{ N}$$

$$R_y = \sum F_y = -200 \text{ N} \cos 30^\circ - 300 \text{ N} \sin 60^\circ$$

$$-400 \text{ N} \times \frac{4}{5}$$

$$= -173.21 \text{ N} - 259.81 \text{ N} - 320 \text{ N} = -753.02 \text{ N}$$

$$\therefore R = \sqrt{(190)^2 + (-753.02)^2} = 776.62 \text{ N}$$

$$\angle \theta = \tan^{-1} \frac{-753.02}{190} = 75.84^\circ$$

$$\therefore R = \underline{\underline{776.62 \text{ N}}}$$

$$\nabla \theta = 75.84^\circ \text{ ANS.}$$

$$\downarrow R = 776.62 \text{ N}$$

+)  $\sum M_c = 200 \text{ N} \cos 30^\circ \times 3 \text{ m} + 400 \text{ N} \times \frac{3}{5} \times 2 \text{ m}$

$$= 519.62 + 480 = 999.62 \text{ N}\cdot\text{m}$$

$\therefore$  The Force-Couple system at C is:

$$R = 776.62 \text{ N} \nabla 75.84^\circ \text{ and } M_c = 999.62 \text{ N}\cdot\text{m} \text{ ANS.}$$

c)  $\rightarrow \sum F_x = 0$ ;  $A_x + 200 \sin 30^\circ - 300 \cos 60^\circ + 400 \times \frac{3}{5} = 0$

$$\therefore A_x = -100 + 150 - 240 = -190 \text{ N} = 190 \text{ N} \leftarrow$$

$$+\uparrow \sum F_y = 0$$
;  $A_y - 200 \cos 30^\circ - 300 \sin 60^\circ - 400 \times \frac{4}{5} = 0$

$$\therefore A_y = 173.21 + 259.81 + 320 = 753.02 \text{ N} \uparrow$$

Hence,  $A = \sqrt{(-190)^2 + (753.02)^2} = 776.62 \text{ N}$

$$\angle \theta = \tan^{-1} \frac{753.02}{-190} = 75.84^\circ$$

$$\nearrow A = 776.62 \text{ N}$$

$$75.84^\circ \text{ ANS.}$$

1. Cont'd

You could have used the results in part (b) but give them opposite signs.

Now,  $\Sigma M_A = 0$

$$M - 200 \cos 30^\circ \times 1\text{m} - 300 \sin 60^\circ \times 4\text{m} + 400 \times \frac{3}{5} \times 2\text{m} - 400 \times \frac{4}{5} \times 4 = 0$$

$$M = 173.21 + 1039.23 - 480 + 1280 = \underline{\underline{2012.44 \text{ N.m}}} \uparrow \text{ Ans.}$$

2.

a) FBD : See diagram

$$b) \vec{BD} = -3m\vec{i} + 2m\vec{j} - 1.5m\vec{k}$$

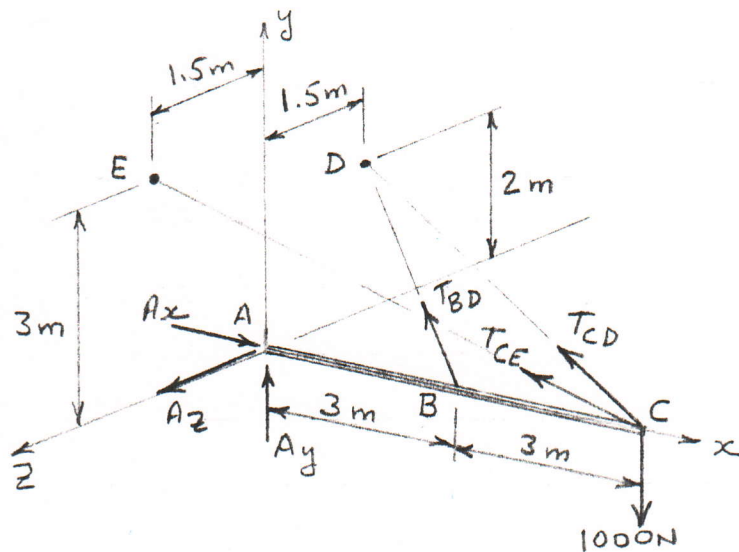
$$BD = \sqrt{(-3)^2 + (2)^2 + (-1.5)^2} \\ = 3.91m.$$

$$\vec{CD} = -6m\vec{i} + 2m\vec{j} - 1.5m\vec{k}$$

$$CD = \sqrt{(-6)^2 + (2)^2 + (-1.5)^2} \\ = 6.50m.$$

$$\vec{CE} = -6m\vec{i} + 3m\vec{j} + 1.5m\vec{k}$$

$$CE = \sqrt{(-6)^2 + (3)^2 + (1.5)^2} = 6.87m.$$



$$\vec{T}_{BD} = T_{BD} \vec{\lambda}_{BD} = T_{BD} \frac{\vec{BD}}{BD} = \frac{T_{BD}}{3.91} (-3\vec{i} + 2\vec{j} - 1.5\vec{k})$$

$$\vec{T}_{CD} = T_{CD} \vec{\lambda}_{CD} = T_{CD} \frac{\vec{CD}}{CD} = \frac{T_{CD}}{6.50} (-6\vec{i} + 2\vec{j} - 1.5\vec{k})$$

$$\vec{T}_{CE} = T_{CE} \vec{\lambda}_{CE} = T_{CE} \frac{\vec{CE}}{CE} = \frac{T_{CE}}{6.87} (-6\vec{i} + 3\vec{j} + 1.5\vec{k})$$

$$c) \quad \sum M_A = \vec{r}_{B/A} \vec{T}_{BD} + \vec{r}_{C/A} \vec{T}_{CD} + \vec{r}_{C/A} \vec{T}_{CE} + \vec{r}_{C/A} (-1000N) = 0$$

where,

$$\vec{r}_{B/A} = (3.0m)\vec{i}, \quad \vec{r}_{C/A} = (6.0m)\vec{i}$$

$$\therefore \sum \vec{M}_A = (3.0m)\vec{i} \times \frac{T_{BD}}{3.91} (-3\vec{i} + 2\vec{j} - 1.5\vec{k})$$

$$+ (6.0m)\vec{i} \times \frac{T_{CD}}{6.5} (-6\vec{i} + 2\vec{j} - 1.5\vec{k})$$

$$+ (6.0m)\vec{i} \times \frac{T_{CE}}{6.87} (-6\vec{i} + 3\vec{j} + 1.5\vec{k}) + (6.0m)\vec{i} \times (-1000N)\vec{j} = 0$$

2. Cont'd.

$$\begin{aligned} \therefore \Sigma M_A &= \frac{6.0}{3.91} T_{BD} \bar{k} + \frac{4.5}{3.91} T_{BD} \bar{j} \\ &+ \frac{12.0}{6.5} T_{CD} \bar{k} + \frac{9.0}{6.5} T_{CD} \bar{j} \\ &+ \frac{18.0}{6.87} T_{CE} \bar{k} - \frac{9.0}{6.87} T_{CE} \bar{j} - 6000 \text{ N.m } \bar{k} = 0 \end{aligned}$$

Equate the coefficients of  $\bar{j}$  and  $\bar{k}$  to zero.

$$\textcircled{j} : \frac{4.5}{3.91} T_{BD} + \frac{9.0}{6.5} T_{CD} - \frac{9.0}{6.87} T_{CE} = 0$$

Since  $T_{BD} = T_{CD}$  (continuous cable) then,

$$\begin{aligned} \frac{4.5}{3.91} T_{BD} + \frac{9.0}{6.5} T_{BD} - \frac{9.0}{6.87} T_{CE} &= 0 \\ \frac{29.25 + 35.19}{25.415} T_{BD} - \frac{9.0}{6.87} T_{CE} &= 0 \end{aligned}$$

$$\text{i.e. } 2.54 T_{BD} - 1.31 T_{CE} = 0 \quad \text{----- } \textcircled{1}$$

$$\textcircled{k} : \frac{6.0}{3.91} T_{BD} + \frac{12.0}{6.5} T_{CD} + \frac{18.0}{6.87} T_{CE} - 6000 = 0$$

$$\frac{39.0 + 46.91}{25.415} T_{BD} + \frac{18.0}{6.87} T_{CE} = 6000 \quad (T_{BD} = T_{CD})$$

$$3.38 T_{BD} + 2.62 T_{CE} = 6000 \quad \text{----- } \textcircled{2}$$

Solving eqs  $\textcircled{1}$  &  $\textcircled{2}$ , multiply  $\textcircled{1}$  by 2 and add to  $\textcircled{2}$ :

$$8.46 T_{BD} = 6000$$

$$\therefore T_{BD} = T_{CD} = \frac{6000}{8.46} = \underline{\underline{709.22 \text{ N.}}} \quad \text{ANS.}$$

Insert in eq.  $\textcircled{2}$ :

$$3.38 \times 709.22 + 2.62 T_{CE} = 6000$$

$$2.62 T_{CE} = 3602.84 \text{ N.}$$

$$\therefore T_{CE} = \frac{3602.84}{2.62} = \underline{\underline{1375.13 \text{ N.}}} \quad \text{ANS.}$$

2. Cont'd.

Determinant form:

$$\begin{aligned} \Sigma M_A = & \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3.0 & 0 & 0 \\ -\frac{3}{3.91} & \frac{2}{3.91} & -\frac{1.5}{3.91} \end{vmatrix} T_{BD} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 6.0 & 0 & 0 \\ -\frac{6}{6.5} & \frac{2}{6.5} & -\frac{1.5}{6.5} \end{vmatrix} T_{CD} \\ & + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 6.0 & 0 & 0 \\ -\frac{6}{6.87} & \frac{3}{6.87} & \frac{1.5}{6.87} \end{vmatrix} T_{CE} + 6.0\bar{i} \times (-1000\text{N})\bar{k} = 0 \end{aligned}$$

Bonus)

$$\Sigma F_x = 0,$$

$$A_x - (T_{BD})_x - (T_{CD})_x - (T_{CE})_x = 0$$

$$A_x - 709.22 \times \frac{3}{3.91} - 709.22 \times \frac{6}{6.50} - 1375.13 \times \frac{6}{6.87} = 0$$

$$A_x - 544.16 - 654.66 - 1200.99 = 0$$

$$\therefore A_x = \underline{\underline{2399.81\text{N}}} \quad \text{ANS.}$$

$$\Sigma F_y = 0,$$

$$A_y + (T_{BD})_y + (T_{CD})_y + (T_{CE})_y - 1000\text{N} = 0$$

$$A_y + 709.22 \times \frac{2}{3.91} + 709.22 \times \frac{2}{6.50} + 1375.13 \times \frac{3}{6.87} - 1000\text{N} = 0$$

$$A_y + 362.77 + 218.22 + 600.49 - 1000 = 0$$

$$\therefore A_y = \underline{\underline{-181.48\text{N}}} \quad \text{ANS.}$$

$$\Sigma F_z = 0,$$

$$A_z - (T_{BD})_z - (T_{CD})_z + (T_{CE})_z = 0$$

$$A_z - 709.22 \times \frac{1.5}{3.91} - 709.22 \times \frac{1.5}{6.50} + 1375.13 \times \frac{1.5}{6.87} = 0$$

$$A_z - 272.08 - 163.67 + 300.25 = 0$$

$$\therefore A_z = \underline{\underline{135.5\text{N}}} \quad \text{ANS}$$

$$\text{Hence, } A = \underline{\underline{(2399.81\text{N})\bar{i} - (181.48\text{N})\bar{j} + (135.5\text{N})\bar{k}}} \quad \text{ANS.}$$