Lecture

Friday, September 21, 2018 11:22 AM

$f''_{xx} = \frac{\partial^2 f}{\partial x^2} = (f'_x)'_x \; \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$

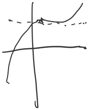
$f''_{xy} = f''_{yx}$

$(f'_x)_y = (f'_y)'_x$ | $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$

Taylor's approximation

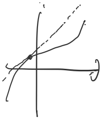
One variable: approximate a function of with a polynomial function.

near x_0 $x - x_0$ is small



$f(x) \cong f(x_0)$ order.

order (Linear)



$f(x) \cong f(x_0) + \underbrace{f'(x_0)(x-x_0)}_{\text{Linear term}}$

even Lette? consider quadratic function

$f(x) \cong f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2$

2nd order (quadratic approximation)

$q(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)$

$q(x_0) = f(x_0)$ | $q''(x_0) = f''(x_0)$.

$q'(x_0) = f'(x_0)$

$f: \underset{x,y}{\mathbb{R}^2} \to \mathbb{R}$ approximate of f

for (x, y) close to (x_0, y_0)

$f(x,y) \simeq f(x_0, y_0)$ — 0 order

(consistent)

1 order (linear)

$$f(x,y) = \underbrace{f(x_0,y_0) + f'(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0)}_{L(x,y)}$$

$l(x_0,y_0) = f(x_0,y_0)$, $l'_x(x_0,y_0) = f'_x(x_0,y_0)$

$l'_y = f'(y)$

2ed order (quadratic) approximation.

$q(x,y)$ — a quadratic function

$q(x_0,y_0) = f(x_0,y_0)$.

$q'(x_0,y_0) = f'_x(x_0,y_0)$; $q'_y(x_0,y_0) = f'_y(x_0,y_0)$

$$\underbrace{\frac{\partial^2}{\partial x^2} \quad \frac{\partial^2}{\partial y^2}, \quad \frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}}_{3.}$$

partial derivatives.

$g''_{xx}(x_0,y_0) = f''_{xx}(x_0,y_0)$

$g''_{yy}(x_0,y_0) = f''_{yy}(x_0,y_0)$.

$g''_{xy}(x_0,y_0) =$
$f''_{xy}(x_0,y_0)$.

$f(x,y) \approx g(x,y)$ $\quad g(x,y) = f(x_0,y_0)$.

$$g(x,y) = f(x_0,y_0) + f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0).$$

$$\left.\begin{aligned} &+ \frac{f'_{xx}(x_0,y_0)}{2}(x-x_0)^2 \\ &+ \frac{f''_{yy}(x_0,y_0)}{2}(y-y_0)^2 \end{aligned}\right\} \text{pure terms.}$$

no division by 2 $\quad + f''_{xy}(x_0,y_0)(x-x_0)(y-y_0)$.

Ex. $f(x,y) = \sin(x+2y)$ $\quad (x_0,y_0)=(0,0)$

$f'_x(x,y) = \cos(x+2y)$.

$f'_y(x,y) = 2\cos(x+2y)$

$$f''_{xx}(x,y) = -\sin(x+2y)$$

$$f''_{yy}(x,y) = -4\sin(x+2y)$$

$$f''_{xy}(x,y) = (\cos(x+2y))'_y$$

$$= -2\sin(x+2y)$$

$$f(x_0,y_0) = f(0,0) = \sin(0) = 0$$

$$f'_x(x_0,y_0) = \cos(x_0+2y_0) = 1$$

$$f'_y(x_0,y_0) = 2\cos(x_0+y_0) = 2$$

$$f''_{xx}(x_0,y_0) = -\sin(x_0+y_0) = 0$$

$$f''_{yy}(x_0,y_0) = -4\sin(x_0 y_0) = -4$$

$$g(x) = 0 + 1x\cancel{x} + 2\cdot y = x+2y$$

Find the second order approximate value of $\frac{(3.98-1)^2}{(5.97-3)^2}$

$$f(x,y) = \frac{(x-1)^2}{(y-3)^2} \qquad x_0 = 4 \quad y_0 = 6$$

$$f'_x(x,y) = \frac{2(x-1)}{(y-3)^2}$$

$$f'_y(x,y) = \frac{-2(x-1)^2}{(y-3)^3}$$

$$f''_{xx}(x,y) = \frac{2}{(y-3)^2} \qquad f''_{yy}(x,y) = \frac{6(x-1)^2}{(y-3)^4}$$

$$f''_{xy}(x,y) = \left(\frac{2(x-1)}{(y-3)^2}\right)'_y = \frac{-4(x-1)}{(y-3)^2}$$

$$f(x_0,y_0) = \frac{(x-1)^2}{(6-3)^2} = 1$$

$$f'_x(x_0,y_0) = \frac{2(4-1)}{(6-3)} = \frac{2}{3}$$

$$f'_y(x_0,y_0) = \frac{-2(4-1)^2}{(6-3)^3} = -\frac{2}{3}$$

$$f''_{xx}(x_0,y_0) = \frac{2}{(6-3)^2} = \frac{2}{9}$$

$$f''_{yy}(x_0,y_0) = \frac{6(4-1)^2}{(6-3)^4} = \frac{6\cdot 3^2}{3^4} = \frac{2}{3}$$

f''

$f_{xy}(x_0,y_0) = \frac{-4(4-1)}{(6-3)3} = \frac{-4}{9}$

$x = 3.98$ $\qquad \Delta x = x - x_0 = 3.98 - 4$

$= -0.02$

$y = 6.97$ $\qquad \Delta y = y - y_0 = -0.03$

$$f(x,y) \cong 1 + \frac{2}{3}\Delta x - \frac{2}{3}\Delta y + \frac{1}{9}(\Delta x)^2 + \frac{1}{3}(\Delta y)^2$$

$$+ -\frac{4}{9}(\Delta x)(\Delta y)$$

$$f(x,y) \cong 1 + \frac{2}{3}(-0.02) - \frac{2}{3}(-0.03) \ldots$$

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