Second Midterm Practice Sheet
MAT1322C

Applications of differential equations

1. A turkey with a temperature of $-15°$ is put in a $245°$ oven. We denote by $h(t)$ the temperature of the turkey t minutes after being placed in the oven. After 20 minutes the temperature of the turkey is 85°C.

   Give a problem with initial value satisfied by $h(t)$, then solve the problem to find $h(t)$. The problem may depend on some parameter that you will need to determine using the data given.

   At what time does the temperature of the turkey reaches 150°?

2. A vat contains 500 gallons of beer at 4% of alcohol. Beer with 6% of alcohol is pumped into the vat a rate of 5 gallons per minute, and the mixture is pumped out of the vat at the same rate. We denote by $Q(t)$ the quantity of alcohol (in gallons) in the vat at time t (in minutes).

   Give a problem with initial value satisfied by $Q(t)$, then solve the problem to find $Q(t)$.

   What is the quantity of alcohol in the vat after 30 minutes?

Value of the sum of a series

1. Compute the sum of the series $\sum\limits_{n=0}^{\infty} \frac{2^{3n}-(-1)^n 5^{n+1}}{3^{2n}}$.

2. Show that the following series is convergent $S = \sum\limits_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$.

   Give an upper bound on the absolute value of the error if we use the 100-th partial sum s_{100} to approximate its sum.

3. Show that the following series is convergent $S = \sum\limits_{n=0}^{\infty} (-1)^n \frac{\sqrt{n+1}}{3n+1}$.

   Give an upper bound on the absolute value of the error if we use the first 10 terms of the series to approximate its sum.

Tests for convergence:

(a) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$,

(b) $\sum_{n=2}^{\infty} \frac{1}{n(ln(n))^2}$,

(c) $\sum_{k=1}^{\infty} \frac{1}{k^2+k^3}$,

(d) $\sum_{i=1}^{\infty} \frac{i}{\sqrt{i^5+1}}$,

(e) $\sum_{s=1}^{\infty} \frac{s^2-5s}{s^3+s+1}$,

(f) $\sum_{i=1}^{\infty} \frac{ln(i)}{i}$,

(g) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n!}$,

(h) $\sum_{n=1}^{\infty} (-1)^{n+3} \frac{n^2}{n^3+4}$,

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$,

(j) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

(k) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

(l) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$

Do the following series **converge absolutely**?

(a) $\sum_{n=1}^{\infty}(-1)^{n+3}\frac{n^2}{n^3+4}$

(b) $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{n^3+4}$

(c) $\sum_{n=1}^{\infty}(-1)^{n}\frac{\cos^2 n+1}{n^{7/8}}$

(d) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+2}$

(e) $\sum_{n=0}^{\infty}(-1)^{n-1}\frac{\sqrt{n+2}}{\sqrt{n^2+7}}$

Find radius and Interval of Convergence:

1. $\sum_{n=1}^{\infty}\frac{(2x-1)^n}{5^n\sqrt{n}}$
2. $\sum_{n=1}^{\infty}\frac{x^n}{n\cdot 3^n}$
3. $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n^2+1}$
4. $\sum_{n=1}^{\infty}\frac{n\cdot(x+1)^n}{4^n}$

Power series and functions

Consider the function $f(x)=\int_0^x \frac{1}{1+2t^4}dt$.

1. Determine the representation as a power series centered at 0 of $f(x)$.
2. What is the interval of convergence?
3. Use your result to express $\int_0^{0.1}\frac{1}{1+2t^4}dt = f(0.1)$ as a series.
4. Approximate $f(0.1)$ with two correct decimals?

Do you remember these facts?

1. What is the series $\sum_{n=1}^{\infty}\frac{1}{n}$ called? Does it converge?
2. What is the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ called? Does it converge?
3. What is the series $\sum_{n=0}^{\infty}q^n$ called? Does it converge? If so, what is its sum?
4. What is absolute convergence? Give an example of a series that converges but does not converge absolutely.
5. If a series converges absolutely, does it converge?
6. If a series converges, does it converge absolutely?
7. Give the formula for the Taylor Series of a function $f(x)$ centered at $x=a$.
8. Give the formula for the MacLaurin Series of a function $f(x)$.

Good luck with studying!