Department of Civil Engineering and Applied Mechanics

CIVE 460: Matrix Structural Analysis

Fall 2017

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Midterm Exam

Date: November 6th, 4:05 PM (In Class)

Name:________________________ **ID**:______________________

Problem 1 (75 Points)

You are given a set of results from running the MATLAB source code for Beam analysis. Some of the results are omitted (you should complete the missing results wherever applicable). Note that results use a unit force Kilo-Newton and a unit length meter. Determine the following:

a) Draw the Analytical Model. Also, draw an idealization of the structure. (**10 Points**)

b) Find the Structure Stiffness Matrix [***S***] and Total Joint Load Vector [***P***-***P$_f$***]. (**25 Points**)

c) The resulting Joint Displacement Vector of the structure is [-0.0267 -0.0057 0.0045]^T. Find the Structure Reaction Vector. (**15 Points**)

d) Suppose that we removed the last member of the Beam along with its member-load(s) and end-joint load(s), if any. Determine the following:

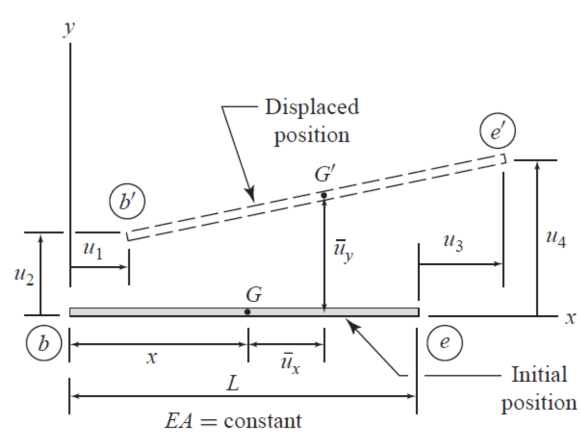
   I. Draw the Analytical Model. Also, draw an idealization of the structure. (**5 Points**)
   II. Find the new Structure Stiffness Matrix [***S***] and Total Joint Load Vector [***P***-***P$_f$***]. (**5 Points**)
   III. Find the new Structure Reaction Vector. (**5 Points**)
   IV. Draw the Shear Diagram of the structure. (**10 Points**)

Problem 2 (25 Points)

For the truss element shown here, determine the following:

a) Derive the Displacement Functions, $\bar{u}_x$ and $\bar{u}_y$, which are in the form of complete polynomials. (**10 Points**)

b) Specify the relation between the Shape Functions and the Displacement Functions. Also, draw the Shape Functions along their domain. (**15 Points**)



Solution 1

a)

The analytical model should be drawn

b)

```
The structure stiffness matrix is:
S =
   1.0e+05 *
    0.1966   -0.1493         0
   -0.1493    1.8667         0
         0         0    0.8960

The structure equivalent joint load vector, {Pe}=-{Pf} is:
P =
 -189.8148
   -6.9444
   49.5000

The total load vector, {P}-{Pf}, is:
P =
 -439.8148
 -656.9444
  399.5000

The joint displacement vector is:
d =
   -0.0267
   -0.0057
    0.0045
```

c)

```
The support reaction vector is:
R =
  293.4131
  683.4625
  621.4369
 -681.6687
  -74.8500
>>
```

d-i)

The analytical model should be drawn similar to that in (a) but with the last member eliminated, this will only affect the numbering of the members, free and restrained coordinates.

d-ii to d-iv)

Because the last member (member 3) is rigidly connected to the adjacent member (member 2), the removal effect on the reaction vector, the stiffness matrix [**S**], the joint load vector [**P-P$_f$**] and the joint displacement vector [**d**] can be easily isolated. There is no interaction between the structure stiffness elements in terms of the last member's contribution to it. Hence:

```
The structure stiffness
S =
   1.0e+05 *
     0.1966   -0.1493
   -0.1493    1.8667
```

```
The total load vector, {P}-{Pf}, is:
P =
 -439.8148
 -656.9444
```

```
The joint displacement vector is:
d =
   -0.0267
   -0.0057
```

The support reaction for all the reaction are the same except for the last one, where the contributions of the member loads on member 3 should be deducted from the reaction vector in the first part. Finally, the shear diagram can be easily drawn starting from the reaction at the first node and subsequently adding the member and joint loads as we go through the beam, where a shear-jump is highlighted at each of the subsequent joints as well.

Solution 2

The derivation of the displacement functions and the resulting shape functions are shown in the lecture notes.

$$\bar{u}_x = \left(1 - \frac{x}{L}\right)u_1 + \left(\frac{x}{L}\right)u_3 \qquad \bar{u}_y = \left(1 - \frac{x}{L}\right)u_2 + \left(\frac{x}{L}\right)u_4$$

$$\bar{u}_x = N_1u_1 + N_3u_3 \qquad \bar{u}_y = N_2u_2 + N_4u_4$$