CARLETON UNIVERSITY

FINAL EXAMINATION
DECEMBER 2008

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, 69.1004 A, B, C, D.
Course Instructor(s): Dr. A.B. Mingarelli (A, B), Dr. M.R.R. Sadeghi (C), Dr. A. Masuda (D).

AUTHORIZED MEMORANDA

NON-PROGRAMMABLE CALCULATOR PERMITTED.

This exam may be released to the Library and may be taken away by the student.

1. Please count your pages now. **This examination has pages numbered from 1 - 15**, excluding this page. The pages numbered 8-15 are blank and are to be used for rough work only. If you feel there is a page missing please report this to your Proctor.
2. Next, please verify that you are in possession of a Scantron FORM
3. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
4. **The examination is out of a total of 100** and consists of 28 multiple choice of varying numerical values displayed next to the question number. **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question). Additional pages are added for your convenience and for rough work and are not to be submitted for grading purposes.

Return only the duly completed Scantron form, not the examination nor your work.

Grading Instructions

- Please note that there are **28 questions** and most have **different values**. The maximum grade on this examination is 100% (which gets reduced to 50% of your final grade for the course by rounding up). Thus, for example, 53% = 27/50.
- In addition, the grade distribution (out of 100) is as follows:

1. 3 marks	11. 3 marks	21. 4 marks
2. 4 marks	12. 4 marks	22. 4 marks
3. 3 marks	13. 4 marks	23. 3 marks
4. 4 marks	14. 4 marks	24. 4 marks
5. 3 marks	15. 5 marks	25. 3 marks
6. 3 marks	16. 4 marks	26. 4 marks
7. 3 marks	17. 5 marks	27. 4 marks
8. 3 marks	18. 3 marks	28. 4 marks
9. 2 marks	19. 4 marks	
10.3 marks	20. 3 marks	

CORRECT ANSWERS HAVE BEEN CIRCLED
-G. H.

Multiple-Choice Questions
Please choose only one answer and insert in PENCIL in your Scantron sheet.

1. **[3 marks]** Evaluate $\lim_{x\to 0} \dfrac{\sin 8x}{16x}$.

   (a) -1
   (b) $1/2$
   (c) $3/2$
   (d) 0

2. **[4 marks]** Let $f(x) = \text{Arctan}\sqrt{x^2+1}$. Evaluate $f'(1)$. (Note that Arctan x and $tan^{-1}x$ represent the same function).

   (a) $f'(1) = \dfrac{1}{\sqrt{3}}$
   (b) $f'(1) = \dfrac{1}{2}$
   (c) $f'(1) = \dfrac{1}{3\sqrt{2}}$
   (d) $f'(1) = 1$

3. **[3 marks]** Let $f(x) = 2|x+1|+1$. Calculate

$$L = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}.$$

   (a) $L = 0$
   (b) $L = 1$
   (c) $L = -1$
   (d) This limit does not exist

4. **[4 marks]** Find the derivative of the function f defined by $f(x) = x^{3x}$.

   (a) $3x^{3x}(1+\ln x)$
   (b) $3x^{3x-1}$
   (c) $3x^{3x}$
   (d) $x^{3x}(1+2\ln x)$

5. **[3 marks]** A differentiable function f with a differentiable inverse, F, has the property that $f'(1) = 1/2$ and $f(1) = 1/2$. What is the value of the derivative of the inverse of f at $x = 1/2$? That is, calculate $F'(1/2)$.

Don't we need $F(x)$?

(a) 1

(b) 2

(c) 0

(d) 1/2

6. **[3 marks]** Let $f(x) = 5^{\sqrt{x+1}}$. Evaluate $f'(8)$. In other words, find the derivative of f at $x = 8$.

(a) $\dfrac{125 \ln 5}{6}$

(b) $\dfrac{9 \ln 3}{2}$

(c) 125

(d) $\dfrac{9 \ln 5}{2}$

7. **[3 marks]** Find the derivative of the function f defined by $f(x) = \dfrac{x}{x^2+1}$.

(a) $f'(x) = \dfrac{1}{2x}$

(b) $f'(x) = \dfrac{1}{(x^2+1)^2}$

(c) $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$

(d) $f'(x) = \dfrac{1}{x(x^2+1)}$

8. **[3 marks]** Evaluate the limit: $L = \lim_{x \to \infty} \sqrt{x}\,(\sqrt{x+1} - \sqrt{x})$.

(a) $L = 1/3$

(b) $L = 1/2$

(c) $L = 1$

(d) $L = 0$

9. **[2 marks]** Let y be given implicitly as a differentiable function of x by $ye^{xy-1} = 1$. Calculate the value of the derivative $\dfrac{dy}{dx}$ at the point (x, y) where $x = 1$, $y = 1$:

(a) 1,

(b) $-1/3$,

(c) 0,

(d) $-1/2$

10. **[3 marks]** Evaluate

$$I = \lim_{x \to 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{2t}\, dt$$

(a) $I = 0$

(b) $I = \frac{1}{4}$

(c) This limit does not exist

(d) $I = \frac{1}{2}$

11. **[3 marks]** Evaluate $L = \lim_{x \to 0^+} \frac{\log_2 \sqrt{x}}{\ln x}$ using any method.

(a) $L = \frac{1}{2 \ln 2}$

(b) $L = 1$

(c) $L = 2$

(d) $L = \frac{1}{\log_2 2}$

12. **[4 marks]** The function f defined by $f(x) = 2x^3 - 6x^2 - 12x + 1$ is concave up at each point of which of the following intervals?

(a) $-12 < x < 0$

(b) $0 < x < 1$

(c) $1 < x < \infty$

(d) $-\infty < x < 1$

13. **[4 marks]** Determine ALL the horizontal asymptotes of the function f defined by

$$f(x) = \frac{x|x|}{x^2 + 1}.$$

(a) $y = 0$ is the only asymptote

(b) $y = -1$, $y = 1$

(c) $y = 1/2$, $y = 1/3$

(d) $y = 1$, $y = -1/2$

14. **[4 marks]** For what value of a does the function defined by

$$f(x) = x^4 + ax^3 - 6$$

have a point of inflection at $x = 1$?

(a) $a = -2$
(b) $a = 1$
(c) $a = -4$
(d) $a = 0$

15. **[5 marks]** Find all the critical points of the function f defined by $f(x) = x^2\, e^{-x}$.

(a) $x = 0$ only
(b) $x = 0$ and $x = 1$ only
(c) $x = e$ and $x = -1$ only
(d) $x = 0$ and $x = 2$ only

16. **[4 marks]** Determine the largest interval I on which the function f, whose domain is the set of all real numbers, and which is defined by $f(x) = e^{-x^2}$ is increasing on I.

(a) $0 < x < \infty$
(b) This function is never increasing.
(c) $-\infty < x < \sqrt{2}/2$
(d) $-\infty < x < 0$

17. **[5 marks]** Find the most general antiderivative of the function f defined by

$$f(x) = \frac{1 - 2\sin 2x}{x + \cos 2x}.$$

(a) $\ln|1 + 2\cos 2x| + C$, where C is a constant
(b) $\ln|1 - \sin 2x| + C$, where C is a constant
(c) $\ln|x + \cos 2x| + C$, where C is a constant
(d) $\ln|x - \sin 2x| + C$, where C is a constant

18. **[3 marks]** Evaluate $f(x) = \int_0^{\pi/8} \tan 2x \; dx$.

(a) $\frac{1}{2} \ln 2 - 1$

(b) $\frac{1}{2} \ln 8$

(c) $\frac{1}{4} \ln 2$

(d) $\frac{1}{2} \ln |\sec^2 2x|$

19. **[4 marks]** Evaluate $\int_0^1 x \, e^x \, dx$

(a) $1 - \frac{2}{e}$

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{e-1}{2}$

20. **[3 marks]** The improper integral $\int_0^\infty x \, 3^{-x^2} \, dx$ has the value

(a) $-\frac{1}{3 \ln 3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{\ln 3}$

(d) $\frac{1}{2 \ln 3}$

21. **[4 marks]** Evaluate $\int x^2 \ln x \; dx$.

(a) $\frac{1}{3} x^3 (\ln x - \frac{1}{3}) + C$

(b) $\frac{1}{3} x^3 \ln x - \frac{x}{9} + C$

(c) $3(\ln x)^2 + \frac{1}{9} + C$

(d) $2x^2 (\ln x) + \frac{x}{9} + C$

22. **[4 marks]** Evaluate and simplify the indefinite integral: $\int \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$.

(a) $\frac{3^{\sqrt{x}}}{\ln x} + C$

(b) $\frac{3^{\sqrt{x}+1}}{4} + C$

(c) $\frac{2\,3^{\sqrt{x}}}{\ln 3} + C$

(d) $\frac{4\,3^{\sqrt{x}}}{\ln 2} + C$

23. **[3 marks]** Evaluate

$$\int \frac{3x+1}{x(x-1)}\,dx$$

using the method of partial fractions.

(a) $2\ln|x-1| + 3\ln|x| + C$

(b) $\ln|x-1| - 2\ln|x| + C$

(c) $4\ln|x-1| - \ln|x| + C$

(d) $2\ln|x-1| + 4\ln|x| + C$

24. **[4 marks]** Evaluate the indefinite trigonometric integral

$$\int \sec^6 x\, \tan^3 x\, dx.$$

(a) $\frac{\sec^3 x\, \tan^3 x}{3} + C$

(b) $\frac{\sec^8 x}{8} - \frac{\sec^6 x}{6} + C$

(c) $\frac{\sec^8 x}{7} + \frac{\sec^6 x}{5} + C$

(d) $\frac{\sec^3 x}{5} - \frac{\tan^4 x}{4} + C$

25. **[3 marks]** Evaluate the improper integral $I = \int_0^1 x \ln x \, dx$

(a) $I = -\frac{1}{4}$

(b) $I = 0$

(c) $I = \frac{1}{3}$

(d) $I = -\frac{5}{12}$

26. **[4 marks]** Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \sin x$ between $x = 0$ and $x = \pi$ about the $y-$axis.

(a) $2\pi \int_0^\pi \sin^2 x \, dx$

(b) $2\pi \int_0^\pi x^2 \sin^2 x \, dx$

(c) $2\pi \int_0^\pi x \sin x \, dx$

(d) $2\pi \int_0^\pi \sin x \, dx$

27. **[4 marks]** Find the area A of the region bounded by the curves defined by $y = x^2$, $y = x^3$ and the lines $x = 0$ and $x = 1$.

(a) $A = \frac{3}{2}$

(b) $A = \frac{3}{4}$

(c) $A = \frac{1}{12}$

(d) $A = \frac{1}{3}$

28. **[4 marks]** Evaluate $I = \int_0^1 \sqrt{1 - x^2} \, dx$. (Hint: There is an easy way to do this.)

(a) $I = \pi$

(b) $I = \frac{\pi}{4}$

(c) $I = \frac{\pi}{2}$

(d) $I = \frac{\pi}{3}$

END
Total: [100 marks]