Math 1362 - Fall 2018
Assignment 1
Theo Espagnol ; Student #: 0300052140

The proposition and axiom numbers in this assignment are taken from the textbook.

Q1. Prop 1.11(i)

Suppose $a, b, c, d \in \mathbb{Z}$. Prove that $(a+b)(c+d) = (ac+bc)+(ad+bd)$

Suppose $a, b, c, d \in \mathbb{Z}$, let $n = (a+b)$

then, $(a+b)(c+d) = n(c+d)$ — replacement (iv)

$n(c+d) = nc + nd$ — Ax 1.1 (iii)

$nc + nd = (a+b)c + (a+b)d$ — replacement (iv)

$(a+b)c + (a+b)d = (ca+cb)+(da+db)$ — prop 1.6

$(ca+cb)+(da+db) = (ac+bc)+(ad+bd)$ — Ax 1.1 (iv).

∎

Q2. Prop 1.24(ii)

Suppose $a \in \mathbb{Z}$. Prove that $-a = (-1)a$

We know that $0 \cdot a = 0$ — prop 1.14

We also know that $1 + (-1) = 0$ — Ax 1.4

$\therefore (1+(-1)) \cdot a = 0$ — replacement (iv)

$\Rightarrow 1 \cdot a + (-1) \cdot a = 0$ — prop 1.6

$\Rightarrow a + (-1) \cdot a = 0$ — Ax 1.3

$\Rightarrow (-a) + a + (-1) \cdot a = (-a) + 0$ — replacement (iv)

$\Rightarrow 0 + (-1) \cdot a = (-a) + 0$ — Ax 1.4

$\Rightarrow (-1)a = (-a)$ — Ax 1.2

$\Rightarrow -a = (-1)a$ — symmetry (ii)

∎

Q3. Prop. 1.28 (ii)

Suppose $a, b, c, d \in \mathbb{Z}$. Prove that $(a-b)-(c-d)=(a+d)-(b+c)$

Suppose $a, b, c, d \in \mathbb{Z}$. Let $(c-d)=n$. Then

$(a-b)-(c-d)=(a-b)-n$	replacement (iv)
$(a-b)-n=(a-b)+(-1)n$	Prop 1.24 (ii)
$(a-b)+(-1)n=(a-b)+(-1)(c-d)$	replacement (iv)
$(a-b)+(-1)(c-d)=(a-b)+((-1)c+(-1)(-d))$	Ax 1.1 (iii)
$(a-b)+((-1)c+(-1)(-d))=(a-b)+(-c+(-1)(-d))$	Prop 1.24 (ii)
$(a-b)+(-c)+(-1)(-d)=(a-b)+(-c)+(1\cdot d)$	Prop 1.20
$(a-b)+(-c)+(1\cdot d)=(a-b)+(-c)+d$	Ax 1.3
$(a-b)+(-c)+d=(a+d)+(-b)+(-c)$	Ax 1.1 (ii)
$(a+d)+(-b)+(-c)=(a+d)+((-b)+(-c))$	Ax 1.1 (ii)
$(a+d)+((-b)+(-c))=(a+d)+((-1)(b)+(-1)(c))$	Prop 1.24 (ii)
$(a+d)+((-1)(b)+(-1)(c))=(a+d)-(b+c)$	Symmetry of Ax 1.1 (iii)

Proof of Prop 1.24(ii)

Q4. Prop 2.4 (i)

Suppose $a, b, c \in \mathbb{Z}$ and $a<b$. Prove that $a+c<b+c$

We know that $a<b \therefore b-a \in \mathbb{N}$	Def. 2.5
$\Rightarrow (b-a)+0 \in \mathbb{N}$	Ax 1.2
$\Rightarrow (b-a)+(c+(-c)) \in \mathbb{N}$	Ax 1.4
$\Rightarrow (b+c)+((-a)+(-c)) \in \mathbb{N}$	Ax 1.1 (ii)
$\Rightarrow (b+c)-(a+c) \in \mathbb{N}$	Prop 1.24 (i)
$\Rightarrow a+c<b+c \in \mathbb{N}$	Ax 2.1 (i)

Q5. Prop 2.13

Suppose $a \in \mathbb{N}$ and $b \in \mathbb{Z}$ satisfy $ab \in \mathbb{N}$ Prove that $b \in \mathbb{N}$

Suppose $b=0$	Suppose $-b \in \mathbb{N}$	Suppose $b \in \mathbb{N}$
Then $ab \in \mathbb{N}$ Ax 2.1(ii)	Then $a(-b) \in \mathbb{N}$ Ax 2.1(ii)	By Ax 2.1(iv)
$\Rightarrow a \cdot 0 \in \mathbb{N}$ replacemet (iv)	$\Rightarrow -(ab) \in \mathbb{N}$ Ax 1.19 (notes)	$b \in \mathbb{N} \because$
$\Rightarrow 0 \in \mathbb{N}$ Prop 1.14	$\Rightarrow (ab)+(-(ab)) \in \mathbb{N}$ Ax 2.1(i)	$b \neq 0$ and
Contradiction to Ax 2.1(iii)	$\Rightarrow 0 \in \mathbb{N}$ Prop 1.8	$-b \notin \mathbb{N}$
$\therefore b \neq 0$	Contradiction to Ax 2.1(iii)	∎
	$\therefore -b \notin \mathbb{N}$	