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Comp 3804 - Test 2B

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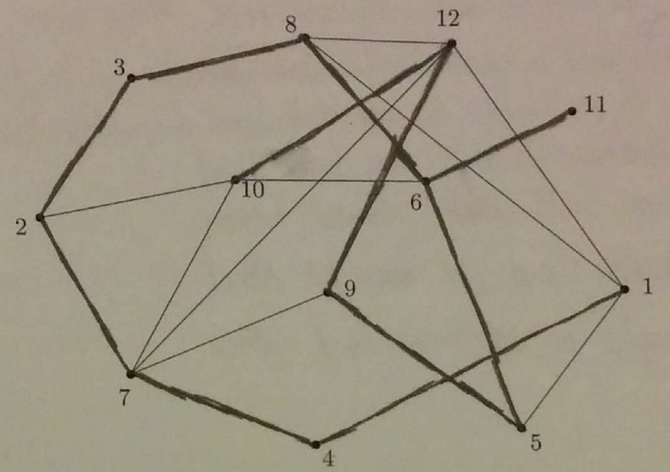
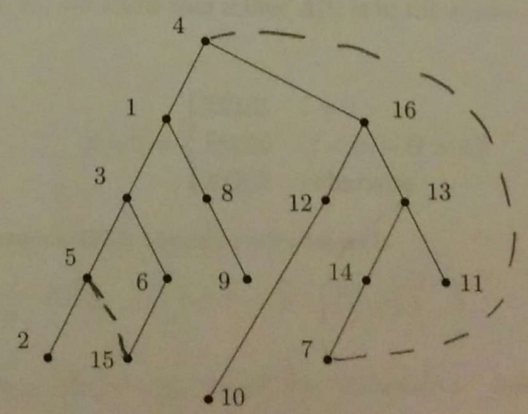
26 points total
Instructions: This is a closed book exam. No calculators are allowed. All questions should be answered in the space provided. You may do rough work on the back of the pages. You have 80 minutes.

1. (5 pts) Run DFS on the graph below, starting at vertex 1. Darken the solid edges of the DFS tree. When you have a choice on which vertex to visit next, always select the vertex with lowest index.



2. Suppose the tree given in the figure below is a tree resulting from running DFS on a graph.
   (a) (3 pts) List the order of the vertices in a preorder traversal of the tree from the root:
   4, 1, 3, 5, 2, 6, 15, 8, 9, 16, 12, 10, 13, 14, 7, 11
   (b) (1 pts) Draw an edge from vertex 7 that can be a valid *back edge*.
   (c) (1 pts) Draw an edge from vertex 15 that *cannot* be a valid back edge.



3. (4 pts) Prove that every tree on $n \geq 2$ vertices is bipartite. Recall that a graph $G = (V, E)$ is bipartite provided that V can be partitioned into two disjoint sets V_1 and V_2, such that every edge has one endpoint in V_1 and the other in V_2. (Hint: you may use the fact that every tree $n \geq 2$ vertices has at least 2 vertices of degree 1 without proof.)

Base: $n=2$

Ind Hypo: Assume that every tree on $2 \leq n \leq k$ vertices is bipartite.

Ind Step: Tree with $k+1$ vertices, since it has at least 2 vertices of degree 1 we can remove one which leaves a tree with k vertices. By induction, we know that tree is bipartite, if we put the vertex back and set it to the opposite color of the parent vertex then the new edge is in both V_1 and V_2 and therefore the tree with $k+1$ vertices is bipartite.

4. (6 pts) Let $A[1 \ldots n]$ be an array of n positive integers. Suppose you are given an integer T and the goal is to determine if there is a subset of elements in A whose sum is T. For example, if $A = 4, 2, 9, 3, 6$ and $T = 5$ then the answer is TRUE since $A[2] + A[4] = 5$. If $T = 27$ then the answer is FALSE.

Define the following function $S[i, t]$ = TRUE if there is a subset of elements in $A[i \ldots n]$ whose sum is exactly t and $S[i, t]$ = FALSE otherwise. The goal of this problem is to determine whether $S[1, T]$ is TRUE or FALSE.

Complete the recursive definition of $S[i, t]$ by specifying what should be defined in XXXX. (Hint: Since the goal is to compute $S[1, T]$, we know that either $A[1]$ is in the solution or not. Use this property to define the recursion.)

$$S[i,t] = \begin{cases} \text{TRUE} & t = 0 \\ \text{FALSE} & (t < 0) \vee (i > n) \\ \text{XXXX} & \text{Otherwise} \end{cases}$$

What is the recursive statement XXXX (justify your answer):

return $(S[i+1, t]$ OR $S[i+1, t - A[i]])$

Case 1: A[i] is not in solution, then we want to increment i and run again with t

Case 2: A[i] is in solution, then we want to increment i and run again WITH the difference between A[i] and t.

This will end if the difference between A[i] and t is zero and return true. It will also end if A[i] − t < 0 or once we've checked the last element in A and return false.

5. (6 pts) Let $A[1\dots n]$ be an array of n positive integers. Let $B[1\dots n]$ be the n elements of A in sorted order. For example, if $A = 5,3,4,1,2$ then $B = 1,2,3,4,5$. Prove that the longest increasing subsequence (LCS) in A is the same as the longest common subsequence (LCS) between A and B. That is $\text{LIS}(A) = \text{LCS}(A,B)$.

(a) Let $c_1,\dots,c_k$ be an increasing subsequence of A. Show that $c_1,\dots,c_k$ is a common subsequence of A and B.

Since $c_1,\dots,c_k$ is an increasing subsequence of A then $A[i_{c_1}] < A[i_{c_2}] < \dots < A[i_{c_k}]$ and $i_{c_1} < i_{c_2} < \dots < i_{c_k}$ where i_x is the index of c_x in A.

Since B is the elements of A in sorted order then the order of the subsequence $c_1,\dots,c_k$ has not changed as B is in sorted (increasing) order. and $c_1 \leq c_2 \leq \dots \leq c_k$.

As such $c_1,\dots,c_k$ is subsequence of B and therefor a common subsequence of A and B.

(b) Let $c_1,\dots,c_k$ be a common subsequence of A and B. Show that $c_1,\dots,c_k$ is an increasing subsequence of A.

Since $c_1,\dots,c_k$ is a subsequence of B and B is the sorted (in increasing order) elements of A then $c_1 \leq c_2 \leq \dots \leq c_k$.

Since $c_1,\dots,c_k$ is also a subsequence in A and we know that $c_1 \leq c_2 \leq \dots \leq c_k$ then seeing as it is sorted it is an increasing subsequence.