University of British Columbia
Econ 451
Midterm
Fall 2017

Name:
Student Number:
Total marks: 35

Instructor: Alfred Kong
Instructions: This exam is 80 minutes long. Answer all the questions in part A and B. Good luck.

Part A (3 marks each, 15 marks in total)
Answer all of the following questions. Your grade will depend almost entirely on the quality of your explanation.

1. There are only 3 consumers in a country (and they have a government). They agreed to vote for the amount of public expenditures spent on national defence (a public good), G, which is enjoyed by all 3 consumers. The 3 consumers also agreed that the government should always balance the budget. To finance the public good, a uniform income tax, t, will apply on each consumer's income. The GDP of the country is 60.

The preference of each consumer is the same and is given by the following utility function:
$u_i(t, G) = [1 - t]y_i + G^{0.5}$

a) If the income distribution of the country is: y_1 = \$5, y_2 = \$5 and y_3 = \$50, how much G will be provided by the government? What is the ratio of G to GDP in this case?
b) What if the income distribution is: y_1 = \$20, y_2 = \$20 and y_3 = \$20, What is the ratio of G to GDP in this situation instead? What can you conclude about the relationship between the G to GDP ratio and income inquality in this country?

a) For each consumer, the level of G they prefer is given by:
$y_i/60 = 0.5G^{-0.5}$

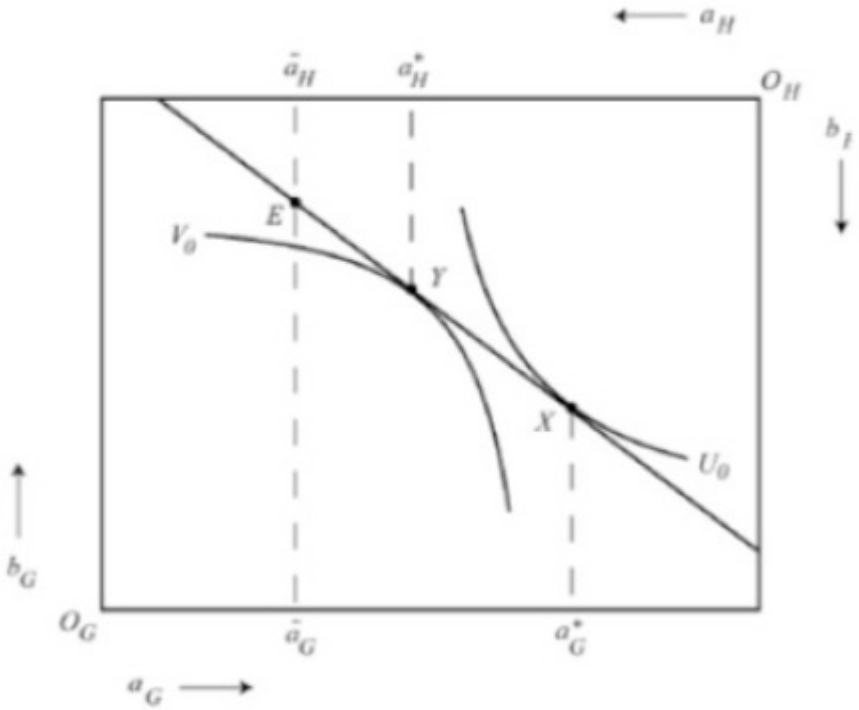
For consumer 1 and 2, they both prefer G = 36.
For consumer 3, he/she prefers G = 0.36

G = 36 gets 2 votes so the government will provide G = 36. And the G to GDP ratio = 36/60 = 60%
b)
All consumers prefer the same level of G = 2.25.
And the G to GDP ratio = 2.25/60 = 3.75%

The more inequal (or equal) the income distribution is, the bigger (or smaller) the size of G relative to GDP.

2. In an Edgeworth box containing an endowment point and a budget line, with the total amount of ale available on the x-axis, draw a stricly convex indifference curve for Gary and a stricly

convex indifference curve for Harry, both tangent to the budget line, such that Gary wants to buy ale, Harry wants to sell ale, and Gary wants to buy more ale than Harry wants to sell at that tangent point.



In the graph, E is the endowment point, X is Gary's best attainable consumption bundle given the budget line (the price ratio of the two goods), and Y is Harry's best bundle.

The amount of ale that Gary wants to consume is a_G^*, which is more than his endowment, $\tilde{a}_G$ so Gary wants to buy $a_G^* - \tilde{a}_G$ amount of ale from Harry.

The amount of ale that Harry wants to consume is a_H^*, which is less than his endowment, $\tilde{a}_H$ so Harry wants to sell $\tilde{a}_H - a_H^*$ amount of ale to Gary. Notice that $a_G^* - \tilde{a}_G > \tilde{a}_H - a_H^*$ so this is not the market equilibrium.

3. Explain whether the following 3 statements are true or false based on the required readings in the course.

a) "The Wagner's Law applies in many OECD countries."
True. The study shows the income elasticity of demand for public goods is greater than 1 for many OECD coutnries, and that was the reason why the public sector grew faster than GDP in those countries.

b) "A country with a higher income inequality should put a higher income tax rate on top income earners."
False. It depends on the country's social preference such as the belief on whether luck affects income. For example, US has a higher pretax income inequality than Europe but US has a lower income tax rate.

c) “Dr. Gregory Mankiw prefers using a permit trading system over Pigouvian tax to control pollution.”
False. In his view, the main advantage of using pigouvian tax for pollution control rather than cap and trade is the tax revenue received from the tax can be used to reduce other distortionary taxes (such as income tax) in the society. The carbon tax would improve efficiency both in the pollution market and the market in which the distortionary tax has been reduced.

4. According to the development model, describe the expenditures at each stage of the development process in terms of Pareto efficiency and equity.

The early stage of development is the period of industrialization during which the population moves from the countryside to the urban areas. To meet the needs of the urban population there is a requirement for infrastructural expenditure. The typically rapid growth experienced at this stage of development results in a significant increase in expenditure and the need for infrastructure determines the nature of expenditure. This is primarily expenditure for reasons of efficiency.

In the middle stages of development, the infrastructural expenditure of the public sector becomes increasingly complementary with expenditure by the private sector. For example, factory construction will be supported by the building of connecting roads. As urbanization proceeds, population density increases which produces a range of externalities such as pollution and crime. An increasing proportion of public expenditure is then diverted towards the control of these externalities. This expenditure is again directed to the reduction of market failure so is for reasons of efficiency.

In the developed phase of the economy expenditure is driven by the desire to react to issues of equity. This results in transfer payments, such as social security, health and education, becoming the main items of expenditure.

5. Consider an economy composed of two people, Alfred and Wendy.
a) Suppose there are two possible allocations, and neither Alfred nor Wendy likes these allocations equally well. Give an example in which only one of the allocations is Pareto efficient, and then give another example in wich both allocations are Pareto efficient.
b) Now imagine that there are five possible allocations. Again there are no two allocations that Alfred likes equally well, and there are no two allocations that Wendy likes equally well. give another example in wich all 5 allocations are Pareto efficient.

a) Both allocations may or may not be Pareto Optimal.

Suppose that the allocations are identified by the nubmers 1 and 2, and that Alfred’s and Wendy’s preferences are described by the Table below:

	Preference
Alfred	**1 > 2**
Wendy	**1 > 2**

In this case, since both persons prefer 1 than 2. Allocation 1 is the only Pareto efficient allocation.

However, if the preferences are like:

	Preference
Alfred	**1 > 2**
Wendy	**2 > 1**

Allocation 1 is Pareto optimal because Alfred would be worse off under the only alternative allocation, and allocation 2 is also Pareto optimal because Wendy would be worse off under the only alternative allocation. So in both allocation1 and 2, we cannot make Alfred (or Wendy) happier without making the other person worse off.

b) All of the allocations can be Pareto optimal. Let Alfred's and Wendy's preferences over allocations 1 to 5 be given by the table below:

	Preference
Alfred	**1 > 2 > 3 > 4 > 5**
Wendy	**5 > 4 > 3 > 2 > 1**

Consider Allocation 1. Alfred is very happy for having 1 but 1 is Wendy's least prefered allocation. Swtiching from 1 to 2 would make Wendy better off but Alfred worse off. So allocation 1 is Pareto efficient. The same argument can be applied to allocation 2 to 5 as well.

Part B (10 marks each, 20 marks in total)
Answer all of the following questions. Your answers to the following questions should take advantage of the relevant economic tools. Where calculations are involved, show your work.

1. A factory's total costs c are:
$c = 4q^2$

where q is the level of output. It can sell any number of units of output at a price of 64. However, production inflicts damage on its neighbours.

The total damage D inflicted depends on the factory's output:
$D = 4q + q^2$

Assume that the factory has the property rights.
a) In the absence of an agreement with its neighbours, what would its level of output be? (2 points)
b) Suppose that the neighbours negotiate with the factory. To what level of output would the negotiations lead? (2 points)
c) What is the minimum total payment that the neighbours must make to the factory to achieve this change in output? (Hint: a diagram which contains private MC and social MC would help you get the payment numbers.) (2 points)
d) What is the maximum total payment? (Hint: a diagram which contains private MC and social MC would help you get the payment numbers.) (2 points)
e) If the government wishes to control the externality by imposing a per unit tax on the factory's output, how much should the tax be? How much revenue does the government collect? (2 points)

a) Without an agreement, the factory will maximize its profit by equalizing MR and MC:
PMB = PMC
64 = 8q
q = 8
Thus the output will be 8 without government interventions or agreements between the polluter and its neighbors.

b) The negotiation will lead to an efficient output level defined by:
SMB = SMC
SMB = PMC + Marginal External Damage
64 = 8q + (4 + 2q)
q* = 6 which is the efficient level of production (and pollution it will cause).

c) The smallest possible bribe just compensates the factory for its lost profits when it reduces output from 8 to 6. Since the vertical distance between p and PMC is the factory's marginal profits, the reduction in total profits is the area between p and PMC curves between output levels 6 and 8.

Minimum bribe = (1/2)(64-48)(8-6) = 16

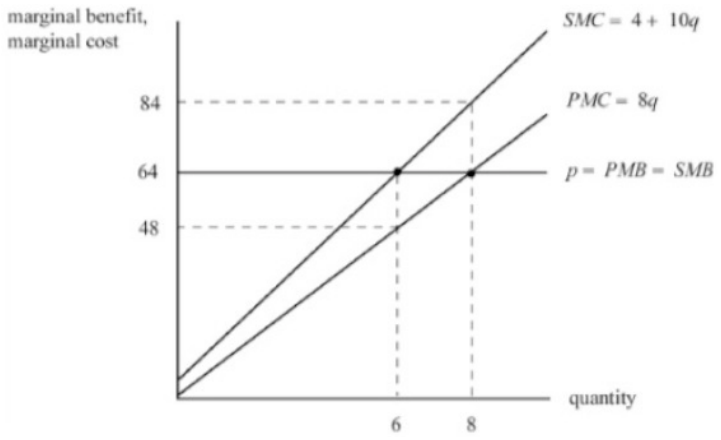
d) The largest possible bribe is the reduction in total damages accociated with the reduction of output from 8 to 6. Since the vertical distance between SMC and PMC is the marginal damages, the reduction in total damages is the area between SMC and PMC curves between output levels 6 and 8. It is the sum of the areas of two triangles:

Maximum bribe = (1/2)(64-48)(8-6) + (1/2)(84-64)(8-6) = 36

e) The ideal tax forces the factory to recognize the damages that its activities imposes upon the neighbours, and is therefore equal to the marginal damage (MD) evaluated at the optimal level of output:

MD at optimal output level = 4+2q = 4+2(6) = 16 = per unit tax on output

The government revenue = 16*6 = 96



2. Aldo and Betty are living in the same apartment. Aldo likes to watch sports programs (X_A) and listen to rock music (Y_A), and Betty likes to watch dramas (X_B) and listen to country music (Y_B). Betty is indifferent to what Aldo is watching or listening, but Aldo is annoyed by the music that Betty chooses. The preferences of the two people can be represented by the utility functions below:
Aldo: $U(X_A,Y_A,Y_B) = 2X_AY_A - Y_B^2$
Betty: $U(X_B,Y_B,X_A) = X_BY_B$

Where X_A is the number of hours of sports programs chosen by Aldo, X_B is the number of hours of dramas chosen by Betty, Y_A is the number of hours of rock music chosen by Aldo, and Y_B is the number of hours of country music chosen by Betty.

Assume they both have $10 income and turning on the TV or radio costs $1 per hour in electricity ($P_X = P_Y = 1$).

The social welfare function for this question is:
social welfare function = U(Aldo) + U(Betty)

a) What would be the allocation (X_A, Y_B) if each person ignored the other person, and made consumption decisions based only on their own well-being? What will be the level of social welfare in the apartment? (2 points)
b) Is the allocation in a) pareto efficient? Why? (2 points)
c) There is an arbitrator who wants to maximize the social welfare in the department. Assuming the arbitrator uses the social welfare function as given above, what should be the socially optimal allocation? No income reallocation is done here. (2 points)
d) Betty is not willing to reduce her consumption of country music. The arbitrator (the government) would like to put a per unit tax, t, on each hour of country music chosen by Betty to

force her to choose the socially optimal allocation. How much should be the per unit tax, t, on country music? (2 points)
e) In addition to the per unit tax imposed on country music, what else would the arbitrator need to do to reach the socially optimal allocation you have found in c)? (2 points)

a)

$X_A = 5$
$Y_A = 5$
$X_B = 5$
$Y_B = 5$
social welfare = $2(5)(5) - 5^2 + (5)(5) = 50$

b) The allocation is pareto efficient since

$$\frac{MU_X^{Betty}}{MU_Y^{Betty}} = \frac{P_X}{P_Y} \qquad \frac{MU_X^{Alfred}}{MU_Y^{Alfred}} = \frac{P_X}{P_Y}$$

Or you can say no one can be better off without making someone worse off.

c)

$$L = 2X_AY_A - Y_B^2 + X_BY_B + \lambda(10 - X_A - Y_A) + \gamma(10 - X_B - Y_B)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial X_A} &= 2Y_A - \lambda = 0 \\ \frac{\partial L}{\partial Y_A} &= 2X_A - \lambda = 0 \end{aligned}\right\} \quad X_A = Y_A \quad \cdots (1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial X_B} &= Y_B - \gamma = 0 \\ \frac{\partial L}{\partial Y_B} &= -2Y_B + X_B - \gamma = 0 \end{aligned}\right\} \quad X_B = 3Y_B \quad \cdots (2)$$

$$\frac{\partial L}{\partial \lambda} = 10 - X_A - Y_A = 0 \quad \cdots (3)$$

$$\frac{\partial L}{\partial \gamma} = 10 - X_B - Y_B = 0 \quad \cdots (4)$$

Sub (1) into (3),

$X_A^* = 5 \quad Y_A^* = 5$

Sub (2) into (4),

$X_B^* = 7.5 \quad Y_B^* = 2.5$

social welfare = $2(5)(5) - (2.5)^2 + (7.5)(2.5) = 62.5$

d)

$$\mathcal{L} = X_B Y_B + \gamma(10 - X_B - (1+t)Y_B)$$

$$\frac{\partial \mathcal{L}}{\partial X_B} = Y_B - \gamma = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y_B} = X_B - (1+t)\gamma = 0$$

$$\Rightarrow \frac{Y_B}{X_B} = \frac{1}{(1+t)}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = 10 - X_B - (1+t)Y_B = 0$$

$$10 - (1+t)Y_B - (1+t)Y_B = 0$$

$$5 = (1+t)Y_B$$

$$Y_B^* = 2.5 \Rightarrow t = 1$$

The per unit tax on country music should be $1

e) Betty's consumption after tax would be
$X_B = 5$
$Y_B = 2.5$

The arbitrator can give 2.5 units of X to Betty by using the $2.5 tax collected $2.5 from Betty. So Betty consumption could then be:
$X_B = 7.5$
$Y_B = 2.5$
like part c).