1. (a) $(4\sqrt{2}-3)^2 = (4\sqrt{2}-3)(4\sqrt{2}-3)$

$$= (4\sqrt{2})(4\sqrt{2}) + 4\sqrt{2}(-3) - 3(4\sqrt{2}) + 9$$

$$= 16 \cdot 2 - 12\sqrt{2} - 12\sqrt{2} + 9$$

$$= 32 - 24\sqrt{2} + 9$$

$$= 41 - 24\sqrt{2}$$

Alternatively, could also have used the formula

$$(a-b)^2 = a^2 - 2ab + b^2, \text{ with } a = 4\sqrt{2} \text{ and } b = 3$$

$$(4\sqrt{2}-3)^2 = (4\sqrt{2})^2 - 2(4\sqrt{2})(3) + 3^2$$

$$= 16 \cdot 2 - 24\sqrt{2} + 9$$

$$= 32 - 24\sqrt{2} + 9$$

$$= 41 - 24\sqrt{2}$$

NOTE: $(a-b)^2 \neq a^2 - b^2$

Imagine that it <u>was</u> equal. Let's see what this means when $a = 3$ and $b = 1$:

$$\left.\begin{aligned} (3-1)^2 &= 2^2 = 4 \\ 3^2 - 1^2 &= 9 - 1 = 8 \end{aligned}\right\} \text{Obviously, it's not true that } 4 = 8 \text{ !}$$

To avoid this mistake, remember that the exponent 2 in $(a-b)^2$ tells you to <u>multiply</u> $(a-b)$ by itself:

$$(a-b)^2 = (a-b)(a-b)$$

By expanding $(a-b)(a-b)$, we get:

$$(a-b)^2 = (a-b)(a-b)$$

$$= a^2 + a(-b) - b(a) - b(-b)$$

$$= a^2 - ab - ab + b^2$$

$$= a^2 - \underline{2ab} + b^2$$

↑ don't forget this!!!

1. (b) $5\sqrt{12} - 4\sqrt{75} + 2\sqrt{48} = 5\sqrt{3\cdot 4} - 4\sqrt{3\cdot 25} + 2\sqrt{3\cdot 16}$

$= 5\sqrt{3}\sqrt{4} - 4\sqrt{3}\sqrt{25} + 2\sqrt{3}\sqrt{16}$

$= 5\sqrt{3}\cdot 2 - 4\sqrt{3}\cdot 5 + 2\sqrt{3}\cdot 4$

$= (2\cdot 5)\sqrt{3} - (4\cdot 5)\sqrt{3} + (2\cdot 4)\sqrt{3}$

$= 10\sqrt{3} - 20\sqrt{3} + 8\sqrt{3}$

$= -2\sqrt{3}$

what if you didn't get right away that $12 = 4\cdot 3$, $75 = 25\cdot 3$, and $48 = 16\cdot 3$? You can still simplify, you just need to break things down more! Example I often saw:

$5\sqrt{12} - 4\sqrt{75} + 2\sqrt{48} = 5\sqrt{2\cdot 6} - 4\sqrt{5\cdot 15} + 2\sqrt{4\cdot 12}$

$= 5\sqrt{2\cdot 2\cdot 3} - 4\sqrt{5\cdot 5\cdot 3} + 2\sqrt{4\cdot 4\cdot 3}$

$= 5\sqrt{2\cdot 2}\sqrt{3} - 4\sqrt{5\cdot 5}\sqrt{3} + 2\sqrt{4\cdot 4}\cdot\sqrt{3}$

$= 5\cdot 2\sqrt{3} - 4\cdot 5\sqrt{3} + 2\cdot 4\sqrt{3}$

$= 10\sqrt{3} - 20\sqrt{3} + 8\sqrt{3}$

$= -2\sqrt{3}$

2. (a) $\dfrac{20}{2\sqrt{3}+\sqrt{7}} = \dfrac{20}{2\sqrt{3}+\sqrt{7}} \cdot \dfrac{2\sqrt{3}-\sqrt{7}}{2\sqrt{3}-\sqrt{7}}$

$$= \frac{20(2\sqrt{3}-\sqrt{7})}{(2\sqrt{3}+\sqrt{7})(2\sqrt{3}-\sqrt{7})}$$

$$= \frac{40\sqrt{3}-20\sqrt{7}}{(2\sqrt{3})(2\sqrt{3})+2\sqrt{3}(-\sqrt{7})+\sqrt{7}(2\sqrt{3})+\sqrt{7}(-\sqrt{7})}$$

$$= \frac{40\sqrt{3}\cancel{40}-20\sqrt{7}}{4\cdot 3\cancel{-2\sqrt{3}\sqrt{7}}\cancel{+2\sqrt{3}\sqrt{7}}-7}$$

$$= \frac{40\sqrt{3}-20\sqrt{7}}{12-7}$$

$$= \frac{40\sqrt{3}-20\sqrt{7}}{5} = 8\sqrt{3}-4\sqrt{7}$$

(b) $\dfrac{3+\sqrt{2}}{3-\sqrt{2}} = \dfrac{3+\sqrt{2}}{3-\sqrt{2}} \cdot \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$

$$= \frac{(3+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

$$= \frac{3^2+2(3)(\sqrt{2})+(\sqrt{2})^2}{3^2-(\sqrt{2})^2}$$ $\leftarrow (a+b)^2 = a^2+2ab+b^2$

$\leftarrow (a+b)(a-b) = a^2-b^2$

$$= \frac{9+6\sqrt{2}+2}{9-2} = \frac{11+6\sqrt{2}}{7}$$

Many of you didn't know by what you should multiply the rational expression to ~~simplify~~ rationalize the denominator.

First, understand that "rationalizing the denominator" means to rewrite the rational expression in such a way that the denominator no longer contains any $\sqrt{\ }$.

How can you do that without actually changing the value of the expression?

Look at something simple:

$$\frac{2}{3} = \frac{2}{3} \cdot \frac{6}{6} = \frac{2 \cdot 6}{3 \cdot 6} = \frac{12}{18}$$

So $\frac{2}{3}$ and $\frac{12}{18}$ are the same number, they're just written differently. That's because doing "$\cdot \frac{6}{6}$" is like doing "$\cdot 1$" (times 1)

Another example: $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$$= \frac{2\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3}$$

So $\frac{2}{\sqrt{3}}$ and $\frac{2\sqrt{3}}{3}$ represent the same number. Why? All we did was multiply by $\frac{\sqrt{3}}{\sqrt{3}}$, which is like multiplying by 1 (since $\frac{\sqrt{3}}{\sqrt{3}} = 1$). So we didn't change the <u>value</u> of $\frac{2}{\sqrt{3}}$, just the way it's written.

What if we have a rational expression where the denominator isn't just one term?

Ex: $\frac{5+\sqrt{2}}{2\sqrt{3}-\sqrt{5}}$

How do we get rid of <u>two</u> $\sqrt{\ }$?

We use the fact that if I multiply $a-b$ by $a+b$, I get this:

$$(a-b)(a+b) = a^2 + \cancel{ab} - \cancel{ba} - b^2 = a^2 - b^2$$

Why is this helpful? Because if both my a and my b contain $\sqrt{\ }$, then a^2 and b^2 will no longer contain $\sqrt{\ }$. $\longrightarrow$

Ex: $\dfrac{5+\sqrt{2}}{\underbrace{2\sqrt{3}-\sqrt{5}}_{a-b,}} = \dfrac{5+\sqrt{2}}{2\sqrt{3}-\sqrt{5}} \cdot \dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$

where $a = 2\sqrt{3}$, $b = \sqrt{5}$

So multiply by $a+b$.

$$= \frac{(5+\sqrt{2})(2\sqrt{3}+\sqrt{5})}{(2\sqrt{3}-\sqrt{5})(2\sqrt{3}+\sqrt{5})}$$

$$= \frac{10\sqrt{3}+5\sqrt{5}+2\sqrt{3}\sqrt{2}+\sqrt{2}\sqrt{5}}{(2\sqrt{3})(2\sqrt{3}) - (\sqrt{5})(\sqrt{5})}$$

$$= \frac{10\sqrt{3}+5\sqrt{5}+2\sqrt{6}+\sqrt{10}}{4\cdot 3 - 5}$$

$$= \frac{10\sqrt{3}+5\sqrt{5}+2\sqrt{6}+\sqrt{10}}{7}$$

If my denominator is something like $a+b$, I'll multiply by $a-b$.

Ex: $\dfrac{\sqrt{3}}{10+\sqrt{11}} = \dfrac{\sqrt{3}}{10+\sqrt{11}} \cdot \dfrac{10-\sqrt{11}}{10-\sqrt{11}}$

$$= \frac{\sqrt{3}(10-\sqrt{11})}{(10+\sqrt{11})(10-\sqrt{11})}$$

$$= \frac{10\sqrt{3}-\sqrt{33}}{100-11}$$

$$= \frac{10\sqrt{3}-\sqrt{33}}{89}$$

3. (a) $(3x^4 + x^2 + x) + 2x(6x^4 - 2x^3 + 5x^2)$

$= 3x^4 + x^2 + x + 2x(6x^4 - 2x^3 + 5x^2)$

$= 3x^4 + x^2 + x + (2x)(6x^4) + (2x)(-2x^3) + (2x)(5x^2)$

$= 3x^4 + x^2 + x + (2\cdot 6)(x\cdot x^4) + (2\cdot(-2))(x\cdot x^3) + (2\cdot 5)(x\cdot x^2)$

$= 3x^4 + x^2 + x + 12x^5 - 4x^4 + 10x^3$

$= 12x^5 - x^4 + 10x^3 + x^2 + x$

3. (b) $\dfrac{4}{x^3 - 4x} + \dfrac{1}{x-2}$

$= \dfrac{4}{x(x^2-4)} + \dfrac{1}{x-2}$

$= \dfrac{4}{\boxed{x(x+2)(x-2)}} + \dfrac{1}{x-2}$ use this as common denominator

$= \dfrac{4}{x(x+2)(x-2)} + \dfrac{1}{x-2} \cdot \dfrac{x(x+2)}{x(x+2)}$

$= \dfrac{4}{x(x+2)(x-2)} + \dfrac{x(x+2)}{x(x+2)(x-2)}$

$= \dfrac{4 + x(x+2)}{x(x+2)(x-2)} = \dfrac{4 + x^2 + 2x}{x(x+2)(x-2)} = \dfrac{x^2+2x+4}{x^3-4x}$

Always factor out the common factor, if any!

4, (a) $3x^2+3x-18 = 3(\underbrace{x^2+x-6}_{\text{Factor this}})$

To factor x^2+x-6, need two integers m, n for which $m \cdot n = -6$ and $m+n = 1$.

$m=-2$, $n=3$ works since $(-2)(3)=-6$ and $-2+3=1$.

So: $x^2+x-6 = (x-2)(x+3)$, and:

$3x^2+3x-18 = 3(x^2+x-6) = 3(x-2)(x+3)$

Alternate solution:

$3x^2+3x-18$

$3\cdot(-18) = -54$

Find two integers m, n whose product is -54 and whose sum is 3:

$$\left.\begin{aligned} m\cdot n &= -54 \\ m+n &= 3 \end{aligned}\right\} m=9,\ n=-6 \text{ works}$$

Then: $3x^2+\underbrace{3x}-18 = \underline{3x^2+9x}\ \underline{-6x-18}$

rewrite as $9x-6x$

$= 3x(x+3)-6(x+3)$

$= (3x-6)(x+3)$

$= 3(x-2)(x+3)$

Factor out all the common factors

4. (b) $2x^5 - 32x = 2x(x^4 - 16)$

Notice that $x^4 - 16$ is a difference of squares:

$$\underbrace{x^4}_{(x^2)^2} \overset{\text{difference}}{-} \underbrace{16}_{4^2} \quad \text{squares!}$$

So: $2x^5 - 32x = 2x(x^4 - 16)$

$$= 2x(x^2+4)(\underbrace{x^2 \overset{\text{difference}}{-} 4}_{\text{squares}})$$

$$= 2x(x^2+4)(x+2)(x-2)$$

5. Note: $x+3 = x-(-3)$

~~and $3x^5+2x^4-20x+24 = 3x^5+2x^4+0x^3+0x^2$~~

and $3x^5+2x^4-20x+24 = \underline{3}x^5+\underline{2}x^4+\underline{0}x^3+\underline{0}x^2-\underline{20}x+\underline{24}$

So:

$$\begin{array}{r|rrrrrr} -3 & 3 & 2 & 0 & 0 & -20 & 24 \\ & \downarrow & -9 & 21 & -63 & 189 & -507 \\ \hline & 3 & -7 & 21 & -63 & 169 & \boxed{-483} \end{array}$$

$\uparrow$ remainder

$$\frac{3x^5+2x^4-20x+24}{x+3} = 3x^4-7x^3+21x^2-63x+169x - \frac{483}{x+3}$$

Since the quotient has a non-zero remainder, $x+3$ is not a factor of $3x^5+2x^4-20x+24$.

6. (a) $2x^2 - 2x - 1 = 0$

$a = 2, \quad b = -2, \quad c = -1$

$b^2 - 4ac = (-2)^2 - 4(2)(-1)$

$= 4 + 8 = 12 > 0$ so there are 2 solutions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{12}}{2(2)}$$

$$= \frac{2 \pm 2\sqrt{3}}{2(2)} = \frac{\cancel{2}(1 \pm \sqrt{3})}{\cancel{2}(2)}$$

$$= \frac{1 \pm \sqrt{3}}{2}$$

So $x = \frac{1+\sqrt{3}}{2}$ or $x = \frac{1-\sqrt{3}}{2}$

The solution set is $\left\{ \frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2} \right\}$

6(b) $9x(x-2) = 2x(3x-7)$

$\Rightarrow 9x^2 - 18x = 6x^2 - 14x$ Expand

$\Rightarrow 3x^2 - 4x = 0$ Do $-6x^2 + 14x$ on both sides.

$\Rightarrow x(3x-4) = 0$ Factor out the common factor X.

$\Rightarrow x = 0$ or $3x - 4 = 0$

$\Rightarrow x = 0$ or $x = \frac{4}{3}$

The solution set is $\{0, \frac{4}{3}\}$.

6(c) $\sqrt{2x+5} - 1 = x - 6$

$\Rightarrow \sqrt{2x+5} = x - 5$ Add 1 to both sides

$\Rightarrow (\sqrt{2x+5})^2 = (x-5)^2$

$\Rightarrow 2x+5 = (x-5)^2$ since $(\sqrt{\square})^2 = \square$

$\Rightarrow 2x+5 = x^2 - 10x + 25$ since $(a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow 0 = x^2 - 12x + 20$

$\Rightarrow x^2 - 12x + 20 = 0$ $(a=b \Rightarrow b=a)$

$\Rightarrow (x-10)(x-2) = 0$ $\left.m \cdot n = +20,\ m+n = -12\right\} \begin{matrix} m=-10 \\ n=-2 \end{matrix}$

$\Rightarrow x = 10$ or $x = 2$ The solution set is $\{2, 10\}$.

7. (a) $-5 \le \frac{3x-4}{2} < 7$

$\Rightarrow \quad -10 \le 3x-4 < 14$

$\Rightarrow \quad -10+4 \le 3x < 14+4$

$\Rightarrow \quad -6 \le 3x < 18$

$\Rightarrow \quad \frac{-6}{3} \le x < \frac{18}{3}$

$\Rightarrow \quad -2 \le x < 6$

Interval notation: The solution set is $[-2, 6)$

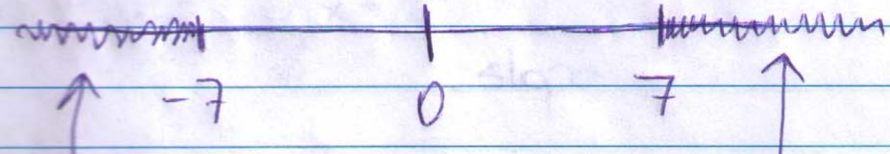
Set notation: The solution set is $\{x \mid -2 \le x < 6\}$

7. (b) $2 \le |x+4| - 5$

$\Rightarrow \quad 2+5 \le |x+4| - 5 + 5$

$\Rightarrow \quad 7 \le |x+4|$

Visually, we want all x for which $x+4$ is at least 7 units away from zero:



what x will make $x+4$ go

what x will make $x+4$ go

So we have to solve 2 inequalities:

Either $\quad x+4 \le -7 \quad$ or $\quad x+4 \ge 7$:

$\Leftrightarrow \quad x \le -7-4 \quad$ or $\quad x \ge 7-4$

$\Leftrightarrow \quad x \le -11 \quad$ or $\quad x \ge 3$

Interval notation: the solution set is $(-\infty, -11] \cup [3, \infty)$

Set notation: the solution set is $\{x \mid x \le -11 \text{ or } x \ge 3\}$

8. We want the equation of a line parallel to ~~the~~ $2x+5y-1=0$. This means they have the same slope. Find the slope of $2x+5y-1=0$:

$$2x+5y-1=0$$

$$\Leftrightarrow \quad 5y = -2x+1$$

$$\Leftrightarrow \quad y = -\frac{2}{5}x + \frac{1}{5}$$

slope

So our line has the equation $y = -\frac{2}{5}x + b$.

To find b, use the fact that this line goes through the point $(-10, 1)$. This means that when $x=-10$, $y=1$:

$$1 = -\frac{2}{5}(-10) + b$$

$$\Rightarrow \quad 1 = \frac{20}{5} + b$$

$$\Rightarrow \quad 1 = 4 + b$$

$$\Rightarrow \quad -3 = b$$

So $\boxed{y = -\frac{2}{5}x - 3}$

8. (b) A circle has the equation

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h,k) is the center and r the radius.

So a circle with center at $(-3, 4)$ and radius 6 has the equation

$$(x-(-3))^2 + (y-4)^2 = 6^2$$

$$\Rightarrow \boxed{(x+3)^2 + (y-4)^2 = 36}$$

9. (a) $f(x) = \sqrt{12-2x}$

Restriction: need $12-2x \geq 0$

$$\Rightarrow 12 \geq 2x$$

$$\Rightarrow 6 \geq x$$

The domain of f is $(-\infty, 6]$.

9. $g(x) = \dfrac{3x-1}{x^2-2x-24}$

Restriction: $x^2-2x-24 \neq 0$

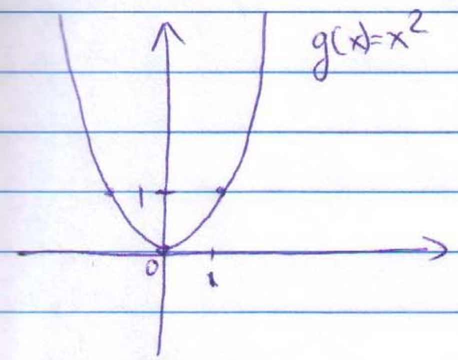
$x^2-2x-24=0 \iff (x-6)(x+4)=0$

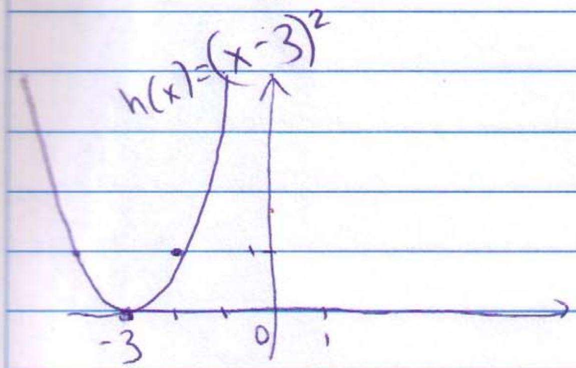
$\iff x=6$ or $x=-4$

The domain is $\{x \mid x \neq 6, x \neq -4\}$

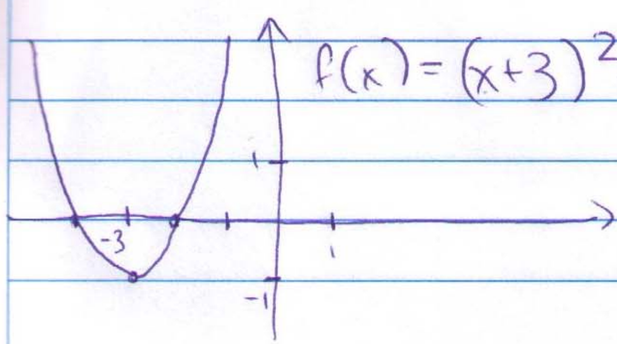
Interval notation: the domain is $(-\infty,-4) \cup (-4,6) \cup (6,\infty)$

10.





Shift 3 units to the left.



Shift 1 unit down.