1. (a) $\left(256^{\frac{1}{4}}\sqrt{5}\right)^2 = \left(256^{\frac{1}{4}}\right)^2\left(\sqrt{5}\right)^2$

$= 256^{\frac{2}{4}} \cdot \left(5^{\frac{1}{2}}\right)^2$

$= 256^{\frac{1}{2}} \cdot 5^{\frac{1}{2}\cdot 2}$

$= \sqrt{256} \cdot 5^1$

$= 16 \cdot 5$

$= 80$

1. (b) $2\sqrt{20} - 3\sqrt{45} = 2\sqrt{4\cdot 5} - 3\sqrt{9\cdot 5}$

$= 2(\sqrt{4})(\sqrt{5}) - 3(\sqrt{9})(\sqrt{5})$

$= 2\cdot 2\sqrt{5} - 3\cdot 3\sqrt{5}$

$= 4\sqrt{5} - 9\sqrt{5}$

$= -5\sqrt{5}$

2. (a) $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{5\sqrt{5}}{3(\sqrt{5})^2}$

$= \frac{\cancel{5}\sqrt{5}}{3\cdot\cancel{5}} = \frac{\sqrt{5}}{3}$

2. (b) $$\frac{2-\sqrt{5}}{2+\sqrt{5}} = \frac{2-\sqrt{5}}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{(2-\sqrt{5})^2}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$= \frac{4-4\sqrt{5}+(\sqrt{5})^2}{4-5}$$

$(a-b)^2 = a^2-2ab+b^2$

$(a-b)(a+b) = a^2-b^2$

$$= \frac{9-4\sqrt{5}}{-1}$$

$$= -9+4\sqrt{5}$$

3. (a) $(x^6+16x^3+64)+3x(x^4-5x^3+4x^2)$

$= x^6+16x^3+\boxed{64}+3x^5-15x^4+12x^3$

$= x^6+3x^5-15x^4+16x^3+12x^3+\boxed{64}$

$= x^6+3x^5-15x^4+28x^3+64$

3. (b) $$\frac{3x}{x-4} + \frac{2x}{x+3} = \frac{3x(x+3)}{(x-4)(x+3)} + \frac{2x(x-4)}{(x+3)(x-4)}$$

$$= \frac{3x^2 + 9x + 2x^2 - 8x}{(x+3)(x-4)}$$

$$= \frac{5x^2 + x}{(x+3)(x-4)} = \frac{x(5x+1)}{(x+3)(x-4)}$$

4. (a) $$3x^2 - 12x - 15 = 3(x^2 - 4x - 5)$$

$$\overset{(*)}{=} 3(x-5)(x+1)$$

Need a, b for which
$a \cdot b = -5$
$a + b = -4$
$a = -5, b = 1$ works
(*)

(b) $$3 - 27x^2 = -(27x^2 - 3)$$

$$= -3(9x^2 - 1)$$

$$\overset{(*)}{=} -3(3x-1)(3x+1)$$

$9x^2 = (3x)^2$
$1 = 1^2$
$9x^2 - 1 = (3x)^2 - 1^2$
difference of squares:
$a^2 - b^2 = (a+b)(a-b)$
(*)

5. $x^4 - x^3 + x^2 - x + 1 = 1x^4 - 1x^3 + 1x^2 - 1x + 1$

$x + 1 = x - (-1)$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -1 & 1 \\ & \downarrow & -1 & 2 & -3 & 4 \\ \hline & 1 & -2 & 3 & -4 & 5 \end{array}$$

↑ Remainder

Since the remainder is not 0, $x+1$ does not divide $x^4 - x^3 + x^2 - x + 1$. It is not a factor.

6. (a) $3x^2 - 10x - 8 = 0$

(*) $\Rightarrow$ $3x^2 - 12x + 2x - 8 = 0$

$\Rightarrow$ $3x(x-4) + 2(x-4) = 0$

$\Rightarrow$ $(3x+2)(x-4) = 0$

$\Rightarrow$ $3x + 2 = 0$ or $x - 4 = 0$

$\Rightarrow$ $x = -\frac{2}{3}$ or $x = 4$

The solution set is $\left\{-\frac{2}{3}, 4\right\}$.

(*)

Factor $3x^2 - 10x - 8$

$3(-8) = -24$

Need a, b for which

$a \cdot b = -24$

$a + b = -10$

$a = -12$, $b = 2$ works.

6. (b) $x(x-8) = -12$

$\Rightarrow x^2 - 8x + 12 = 0$

$\overset{(*)}{\Rightarrow} (x-6)(x-2) = 0$

$\Rightarrow x-6=0$ or $x-2=0$

$\Rightarrow x = 6$ or $x = 2$

The solution set is $\{2, 6\}$.

$(*)$ $\left\{ \begin{array}{l} \text{Need } a, b \text{ for which} \\ a \cdot b = 12 \\ a + b = -8 \\ a = -6, b = -2 \text{ works} \end{array} \right.$

6. (c) $4(x-1) = 3x - 2$

$\Rightarrow 4x - 4 - (3x - 2) = 0$

$\Rightarrow 4x - 4 - 3x + 2 = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

The solution set is $\{2\}$.

7. Solve the inequalities...

(a) $8 - 4(2-x) \le -2x$

$\Rightarrow 8 - 8 + 4x \le -2x$

$\Rightarrow 4x \le -2x$

$\Rightarrow 6x \le 0$

$\Rightarrow x \le 0$

The solution set is $\{x \mid x \le 0\}$, that is, all x in the interval $(-\infty, 0]$.

7. (b) $|-x - 2| \ge 1$

$\Rightarrow -x - 2 \le -1$ or $-x - 2 \ge 1$

$\Rightarrow -x \le 1$ or $-x \ge 3$

$\Rightarrow x \ge -1$ or $x \le -3$

The solution set is all x in ~~the~~ $(-\infty, -3] \cup [-1, \infty)$, or, in set notation, $\{x \mid x \le -3 \text{ or } x \ge -1\}$.

8. (a) $\cancel{y=a}$ $y = \underbrace{m}_{\text{slope} = -2} x + b$

So $y = -2x + b$

Since the line passes through the point $(-1, 1)$,

$$1 = -2(-1) + b$$

$$\Rightarrow 1 = 2 + b$$

$$\Rightarrow -1 = b$$

So $\boxed{y = -2x - 1}$

(b) In general, the standard form for the equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$ where (h,k) is the ~~vertex~~ center and r is the radius of the circle.

So the circle with center $(-10, -20)$ and radius 2 has the equation

$$\underbrace{(x+10)^2}_{x-(-10)} + \underbrace{(y+20)^2}_{y-(-20)} = \underbrace{4}_{2^2}$$

9. (a) $f(x)=\sqrt{\dfrac{2}{x-1}}$

Restrictions:

* $x-1 \neq 0 \Rightarrow x \neq 1$

* $\dfrac{2}{x-1} \geq 0 \Rightarrow \dfrac{x-1}{2} \geq 0$

$\Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$

So: $\left.\begin{array}{l} x \neq 1 \\ x \geq 1 \end{array}\right\} \Rightarrow x > 1$

The domain of the function f is $(1, \infty)$.

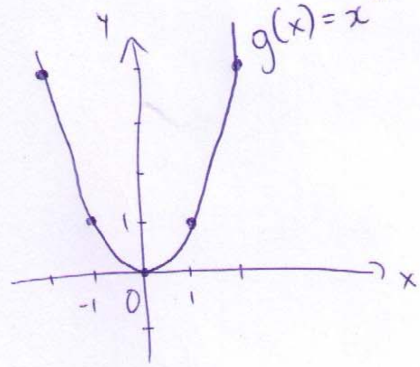
9. (b) $g(x)=\sqrt{3x-12}$

Restriction: $3x-12 \geq 0 \Rightarrow 3x \geq 12$

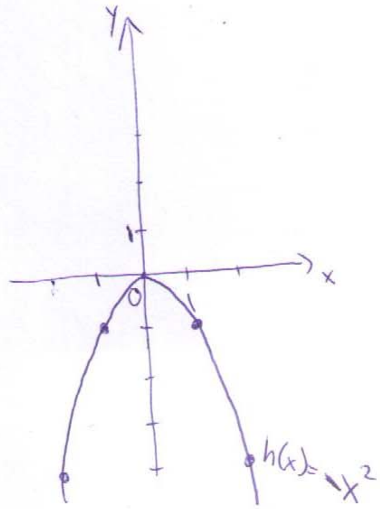
$\Rightarrow x \geq 4$

The domain of the function g is $[4, \infty)$.

10. Sketch $f(x) = 3 - x^2$, Starting from the graph of $g(x) = x^2$.



Multiply by (-1) : flip over the x-axis.



Add 3: shift up by 3 units