Solution of Midterm

(1) (a) $(7-3\sqrt{3})^2 = 49 - 42\sqrt{3} + (3\sqrt{3})^2$
$= 49 - 42\sqrt{3} + 27$
$= 76 - 42\sqrt{3}$

(b) $4\sqrt{24} - 2\sqrt{72} = 4\sqrt{2^2 \cdot 6} - 2\sqrt{2^2 3^2 \cdot 2}$
$= 8\sqrt{6} - 12\sqrt{2}$

(2)

(a) $$\frac{5}{3\sqrt{2}} = \frac{5}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{6}$$

(b) $$\frac{\sqrt{2}}{\sqrt{3}-3\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}-3\sqrt{2}} \cdot \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{3}+3\sqrt{2}}$$

$$= \frac{\sqrt{6}+6}{(\sqrt{3})^2-(3\sqrt{2})^2}$$

$$= \frac{\sqrt{6}+6}{3-18} = -\frac{\sqrt{6}+6}{15}$$

(3)

(a) $4x(3x^4-4x^3+6x^2-1)-2x^2(x^2-4x+3)$
$= 12x^5 - 16x^4 + 24x^3 - 4x - 2x^4 + 8x^3 - 6x^2$
$= 12x^5 - 18x^4 + 32x^3 - 6x^2 - 4x$

(b) $\frac{3x}{x-4} + \frac{2x}{x+3} = \frac{3x(x+3)+2x(x-4)}{(x-4)(x+3)}$

$= \frac{3x^2+9x+2x^2-8x}{(x-4)(x+3)}$

$= \frac{5x^2+x}{(x-4)(x+3)} = \frac{\cancel{x(x+5)}\ x(5x+1)}{(x-4)(x+3)}$

(4)

(a) $3x^2-2x-8 = 3x^2-6x+4x-8$

$= 3x(x-2)+4(x-2)$

$= (x-2)(3x+4)$

(b) $64-27x^3 = 4^3-(3x)^3$

$= (4-3x)(\underbrace{16+12x+9x^2}_{\text{prime}})$

$= (4-3x)(9x^2+12x+16)$

(5)

$$\begin{array}{r|rrrr} -3 & -4 & 5 & 0 & 8 \\ & & 12 & -51 & 153 \\ \hline & -4 & 17 & -51 & 161 \end{array}$$

Remainder is $= 161$

$\neq 0$.

Then $(x+3)$ is not a factor of $-4x^3+5x^2+8$

6

$$2(3+2x) = 3(x-4)$$

a

$$6+4x = 3x-12$$
$$x = -18$$

b

$$2x^2 - x - 3 = 0$$

$$2x^2+2x-3x-3 = 0$$
$$2x(x+1)-3(x+1) = 0$$
$$(x+1)(2x-3) = 0$$
$$\Rightarrow x = -1 \quad \text{or} \quad x = \frac{3}{2}$$

c

$$\sqrt{12-x} = x$$

$$12-x = x^2$$

$$x^2+x-12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = -4 \quad \text{or} \quad x = 3$$

The only solution is $x = 3$

$x = -4$ is impossible solution

7. (a) $|-x-2| \geq 1$

is equivuvalent to:

$-x-2 \geq 1 \quad \text{or} \quad -x-2 \leq -1$

$-x \geq 3 \quad \text{or} \quad -x \leq 1$

$x \leq -3 \quad \text{or} \quad x \geq -1$

Solution $= (-\infty, -3] \cup [-1, \infty)$

(b) $x+8 < \frac{1}{2}(x-4)$

$2x+16 < x-4$

$2x-x < -20$

$x < -20$

Solution $= (-\infty, -20)$

8. (a) $x=2$ is a vertical line passing by $(2,0)$

$y=4$ is the only Horizontal line perpendicular to $x=2$ and passing by $(3,4)$

(8)

$$x^2+y^2+4y-12=0$$

$$x^2+y^2+4y=12$$

$$x^2+y^2+4y+4=12+4$$

$$x^2+(y+2)^2=16$$

This is the equation of a circle with center $(0,-2)$ and radius 4

(9)

$$f(x)=\sqrt{16-x^2}$$

Condition $16-x^2 \geq 0$

$$(4-x)(4+x) \geq 0$$

		-4		4	
$(4-x)$	$+$		$+$	0	$-$
$4+x$	$-$	0	$+$		$+$
$(4-x)(4+x)$	$-$		$+$		$-$

$\Rightarrow$ $16-x^2$ is positive between -4 and 4

Domain $=[-4,4]$

(b) $g(x) = \dfrac{x}{\sqrt{x-3}}$

Condition: $x - 3 > 0$

$x > 3$

Domain $= (3, \infty)$

(10)