PART A (35 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET.

A1. If U and V are the endpoints of vectors $\vec{u} = (5, 0, -2)$ and $\vec{v} = (-3, 1, -1)$ respectively, and $\vec{x} = \overrightarrow{UV}$, find $\vec{x}$.

A: $(2, 1, -3)$	B: $(2, -1, -5)$	C: $(8, -1, -1)$	D: $(-8, 1, 1)$	E: $(-8, 1, -3)$

A2. Find the vector $\vec{w} = 2\vec{u} - 3\vec{v}$ if $\vec{u} = 2\vec{i} + \vec{j} + 3\vec{k}$ and $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$.

A: $(1, 5)$	B: $(1, -1, 0)$	C: $(1, 5, 0)$	D: $(-3, 0, -5)$	E: $(4, 5, 5)$

A3. What is the magnitude of the vector $\vec{u} = (1, -1, 2, -2, -3, 5)$?

A: $2\sqrt{11}$	B: $4\sqrt{11}$	C: $\sqrt{34}$	D: 4	E: $11\sqrt{2}$

A4. If $\vec{u} = (5, 0, -2, 1)$ and $\vec{v} = (-3, 1, -1, 1)$ find $\vec{u} \bullet \vec{v}$.

A: 15	B: -12	C: -11	D: -16	E: $(-15, 0, 2, 1)$

A5. Which of the following statements is/are **true**?

(i) For any vectors $\vec{p}$ and $\vec{q}$ in $\Re^m$, with endpoints P and Q respectively, $d(\vec{p}, \vec{q}) = \left\|\overrightarrow{PQ}\right\|$.
(ii) For any vectors $\vec{u}$ and $\vec{v}$ in $\Re^3$, $\vec{v} \times \vec{u}$ is orthogonal to both $\vec{u}$ and $\vec{v}$.
(iii) For any vector $\vec{w} = (a, b, c)$ in $\Re^3$, the vector $\vec{x} = (b, -a, c)$ is orthogonal to $\vec{w}$.

A: (i) only	B: (i) and (ii) only	C: (ii) only
D: (ii) and (iii) only	E: all of (i), (ii) and (iii)	

A6. Which one of the following is an equation of the line through $P(1, 1)$ parallel to $\vec{v} = (2, 1)$?

A: $(2, 1) \bullet (\vec{x} - (1, 1)) = 0$	B: $\vec{x}(t) = (2, 1) + t(1, 1)$	C: $\vec{x}(t) = (1 - t)(1, 1) + t(2, 1)$
D: $2x + y = 3$	E: $\vec{x}(t) = (1, 1) + t(2, 1)$	

A7. Which one of the following is an equation of the line through $P(1, 1)$ perpendicular to $\vec{v} = (2, 1)$?

A: $x - 2y = -1$	B: $\vec{x}(t) = (2, 1) + t(1, 1)$	C: $\vec{x}(t) = (1 - t)(1, 1) + t(2, 1)$
D: $2x + y = 3$	E: $\vec{x}(t) = (1, 1) + t(2, 1)$	

A8. Which of the following is a normal vector for the line through $P(1,1)$ perpendicular to the line $3x+2y=5$?

A: $(3,2)$	B: $(2,-3)$	C: $(-2,-3)$	D: $(1,-1)$	E: $(1,1)$

A9. Consider the plane Π containing the points $P(0,1,-2)$ and $Q(1,1,-1)$ and $R(2,2,1)$. Which one of the following vectors is a normal to this plane?

A: $(-1,-1,1)$	B: $(-1,2,1)$	C: $(0,1,2)$	D: $(2,-1,1)$	E: $(3,-3,0)$

A10. Which one of the following points lies on the hyperplane $\mathcal{H}$ in $\Re^5$ which has standard form equation $x_1-x_2+2x_3-3x_4+4x_5=5$?

A: $(0,0,0,0,0)$	B: $(1,1,1,1,1)$	C: $(1,0,0,0,1)$	D: $(1,1,3,2,5)$	E: $(1,-1,2,-3,4)$

A11. Find the augmented matrix for the following linear system of equations.

$$\begin{aligned} x-y-3&=0\\ 3x+2y&=1\\ 1+y&=-x \end{aligned}$$

A: $\left[\begin{array}{rr\|r} 1 & -1 & -3\\ 3 & 2 & 1\\ -1 & 1 & 1 \end{array}\right]$	B: $\left[\begin{array}{rrr\|r} 1 & -1 & -3 & 0\\ 0 & 3 & 2 & 1\\ 0 & 1 & 1 & -1 \end{array}\right]$	C: $\left[\begin{array}{rr} 1 & -1\\ 3 & 2\\ 1 & 1 \end{array}\right]$
D: $\left[\begin{array}{rr\|r} 1 & -1 & 3\\ 3 & 2 & 1\\ 1 & 1 & -1 \end{array}\right]$	E: $\left[\begin{array}{rrr\|r} 1 & -1 & -3 & 0\\ 3 & 2 & -1 & 0\\ 1 & 1 & 1 & 0 \end{array}\right]$	

A12. Which one of the following is **not** in row-reduced echelon form?

A: $\begin{bmatrix} 1 & 0 & 2\\ 0 & 1 & 0\\ 0 & 0 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0 \end{bmatrix}$	D: $\begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0 \end{bmatrix}$	E: $\begin{bmatrix} 0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0 \end{bmatrix}$

A13. How many solutions does the system of linear equations shown below have?

$$\begin{array}{rcrcrcr} x & - & y & + & 2z & = & 2\\ & & 4y & - & 2z & = & 1\\ & & 2y & - & z & = & 1 \end{array}$$

A: a three-parameter family of solutions	B: a two-parameter family of solutions
C: a one-parameter family of solutions	D: a unique solution
E: no solution	

A14. How many solutions does the system of linear equations shown below have?

$$\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 2 \\ -2x & + & 4y & - & 2z & = & -4 \end{array}$$

A: a three-parameter family of solutions	B: a two-parameter family of solutions
C: a one-parameter family of solutions	D: a unique solution
E: no solution	

A15. Consider the system of linear equations whose row-reduced echelon form augmented matrix is shown below. Which of the following represents the solution(s), if any, to this SLE?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A: $(-2, 1, 0)$	B: $(-2, 1, 1)$	C: $(-2, 1, t)$	D: $(-2t, 1-t, t)$	E: the system has no solution

Use the following in questions 16 and 17.

The augmented matrix for a linear system of three equations in three unknowns is:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & c-2 & c^2-4 \end{array}\right]$$

A16. For what value(s) of c does this system have a parametric family of solutions?

A: no values of c	B: $c = \pm 2$ only	C: $c = 2$ only	D: all $c \neq \pm 2$	E: all $c \neq 2$

A17. For what value(s) of c is the system inconsistent?

A: no values of c	B: $c = \pm 2$ only	C: $c = 2$ only	D: all $c \neq \pm 2$	E: all $c \neq 2$

A18. If A is a 2×3 matrix, B is a 3×3 matrix and C is a 3×2 matrix, which one of the following operations is **not** defined?

A: ABC	B: CAB	C: BAC	D: $AB + C^T$	E: $CA + B$

A19. If A is a 3×4 matrix while B and C are both 4×2 matrices, find the dimensions of the matrix $D = A(B - C)$, if it is defined.

A: D is not defined	B: 3×2	C: 3×4	D: $3 \times 4 \times 2$	E: 6

A20. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and O is the 2×2 zero matrix, find the value of k for which $A^4 + kA^2 = O$.

A: -1	B: 2	C: -2	D: 4	E: -4

A21. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$. If $A^{-1} = B = [b_{ij}]$, find b_{21}.

A: 1	B: -1	C: 2	D: -2	E: 3

A22. If $A = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, what is the rank of A?

A: undefined	B: 0	C: 1	D: 2	E: 3

A23. Find the rank of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$.

A: 0	B: 1	C: 2	D: 3	E: 4

A24. Which one of the following is **true**?

(i) If A is a 4×4 matrix with rank 4 then the linear system $A\vec{x} = \vec{b}$ must have a unique solution.
(ii) If A is a 5×4 matrix with rank 4 then the linear system $A\vec{x} = \vec{b}$ must have a parametric family of solutions.
(iii) If the rank of A is 4 and the rank of $\left[A \mid \vec{b}\right]$ is 5 then the corresponding linear system $A\vec{x} = \vec{b}$ has a one-parameter family of solutions.
(iv) If the rank of A is 4 and the rank of $\left[A \mid \vec{b}\right]$ is also 4, then the corresponding linear system $A\vec{x} = \vec{b}$ must have exactly 4 unknowns.

A: (i)	B: (ii)	C: (iii)	D: (iv)	E: none of them

A25. If A is an $m \times n$ matrix with rank 3 and the rank of $\left[A \mid \vec{b}\right]$ is also 3, which one of the following statements is **false**.

A: The system $A\vec{x} = \vec{b}$ must be consistent.	B: m must be at least 3.
C: The system $A\vec{x} = \vec{b}$ must have a unique solution.	D: n must be at least 3.

A26. Consider the system of linear equations $A\vec{x} = \vec{b}$ where A is a $6 \times n$ matrix. Which one of the following statements is **true**?

A: If $n = 7$, the system must have infinitely many solutions.
B: If $n = 7$ and $\vec{b} = \vec{0}$, the system must have infinitely many solutions.
C: If $n = 5$, the system must have infinitely many solutions.
D: If $n = 5$ and $\vec{b} = \vec{0}$, the system must have infinitely many solutions.
E: None of A, B, C, D.

A27. Find $\det \begin{bmatrix} 0 & 2 \\ 3 & 2 \end{bmatrix}$.

A: -6	B: -4	C: 0	D: 4	E: 6

A28. Find the 3,2-cofactor of the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 2 & -1 & 5 \\ 3 & 2 & 3 \end{bmatrix}$.

A: -8	B: 1	C: -1	D: 4	E: -4

A29. Which one of the following is **false**?

A: $\det \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 2 & 0 \end{bmatrix} = 0$	B: $\det \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 3 & 2 & 1 \end{bmatrix} = 0$	C: $\det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 4$
D: $\det \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ 0 & 4 & 4 \end{bmatrix} = 4$	E: $\det \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 4 & 4 \end{bmatrix} = 4$	

A30. If $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 5$, find $\det \begin{bmatrix} 2a & b & c \\ 2d & e & f \\ 6g & 3h & 3i \end{bmatrix}$.

A: 30	B: -30	C: $\frac{5}{6}$	D: $-\frac{5}{6}$	E: -5

A31. If $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 5$, find $\det \begin{bmatrix} g+6h & h & i \\ d+6e & e & f \\ a+6b & b & c \end{bmatrix}$.

A: 30	B: -30	C: $\frac{5}{6}$	D: $-\frac{5}{6}$	E: -5

A32. Suppose A and C are two 2×2 matrices with $\det A = 1$ and $\det C = 0$. Let $\vec{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $\vec{b} = \begin{bmatrix} b_1 & b_2 \end{bmatrix}^T$. Which one of the following statements is **false**?

A: The linear system $A\vec{x} = \vec{b}$ always has a unique solution.
B: The linear system $C\vec{x} = \vec{0}$ has infinitely many solutions.
C: The linear system $C\vec{x} = \vec{b}$ is always inconsistent.
D: A is invertible.
E: C is not invertible.

A33. Suppose that the linear system

$$\begin{array}{ccccccc} ax & + & by & + & cz & = & k \\ dx & + & ey & + & fz & = & l \\ gx & + & hy & + & jz & = & m, \end{array}$$

has $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} = 4$, $\det \begin{bmatrix} k & b & c \\ l & e & f \\ m & h & j \end{bmatrix} = 20$, $\det \begin{bmatrix} a & k & c \\ d & l & f \\ g & m & j \end{bmatrix} = 12$,

and $\det \begin{bmatrix} a & b & k \\ d & e & l \\ g & h & m \end{bmatrix} = 8$.

In the solution for this system, what is the value of z?

A: 2	B: 3	C: 4	D: 5	E: 8

A34. Suppose that A is an $n \times n$ matrix with $\det A = \frac{1}{2}$. Which of the following statements is/are **false**?

(i) $A^{-1} = 2 \text{ Adj } A$

(ii) $\frac{1}{2}(\text{Adj } A)A = I_n$ where I_n is the $n \times n$ identity matrix.

(iii) If $\text{Adj } A = [c_{ij}]$ (i.e. c_{ij} is the (i, j)-entry of Adj A), then c_{ij} is the i, j-cofactor of A^T.

A: (i) only	B: (ii) only	C: (iii) only	D: (ii) and (iii) only	E: all of them

A35. If $\text{Adj } A = \begin{bmatrix} 12 & 6 & -16 \\ 4 & 2 & 16 \\ 12 & -10 & 16 \end{bmatrix}$, where $A = \begin{bmatrix} a & 1 & 2 \\ b & 6 & -4 \\ c & 3 & 0 \end{bmatrix}$, find the value of $\det A$.

A: 12	B: 16	C: 32	D: 64	E: -4

PART B (25 marks)

NOTE: You must show sufficient work to justify your answers for all questions in Part B. Within the same question, you may use the result of a calculation done in an earlier question part without doing that calculation again.

5 marks B1. Let $P(2,2,1)$ and $Q(3,1,2)$ be the endpoints of vectors $\vec{p}$ and $\vec{q}$, respectively.

(a) If $\vec{u}$ is a unit vector in the opposite direction to $\vec{p}$, find $\vec{u}$.

(b) Find the area of the parallelogram determined by $\vec{p}$ and $\vec{q}$.

(c) Write a standard form equation for the plane containing vectors $\vec{p}$ and $\vec{q}$.

5 marks B2. In each of the following, bring the augmented matrix $\left[A \mid \vec{b}\right]$ to row-reduced echelon form and find all solutions to the corresponding system of linear equations $A\vec{x} = \vec{b}$.

(a) $\left[A \mid \vec{b}\right] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ with $\vec{x} = (x, y, z)$.

(b) $\left[A \mid \vec{b}\right] = \left[\begin{array}{cccc|c} 1 & 2 & -2 & 3 & 1 \\ 2 & 5 & -4 & 5 & -1 \end{array}\right]$ with $\vec{x} = (x_1, x_2, x_3, x_4)$.

4 marks B3. Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$.

(a) If $C = AB$, find C.

(b) If $C = A^T + B$, find $\det C$.

2 marks B4. It is known that the matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is nonsingular with $A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$.

Find all solutions to the system of linear equations shown below.

$$\begin{array}{ccccccc} ax & + & by & + & cz & = & 3 \\ dx & + & ey & + & fz & = & 2 \\ gx & + & hy & + & iz & = & 1 \end{array}$$

5 marks B5. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix}$.

(a) Find A^{-1} using row-reduction of the appropriate augmented matrix.

(b) Find $\det A$.

(c) **Use your answers to parts (a) and (b)** to find Adj A. (You **must** show *how* you found your answer to get the mark.)

4 marks B6. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) Find $\det A$ by expanding along **column 2**.

(b) **Without finding the inverse of** A, find $\det\left(2A^{-1}\right)$. (You **must** show *how* you calculated your answer to get the marks.)

This page is intentionally left blank. It may be used if you need extra space for any of the written answer questions. Be sure to indicate which question you are answering, and also to direct the marker to this page for that question.