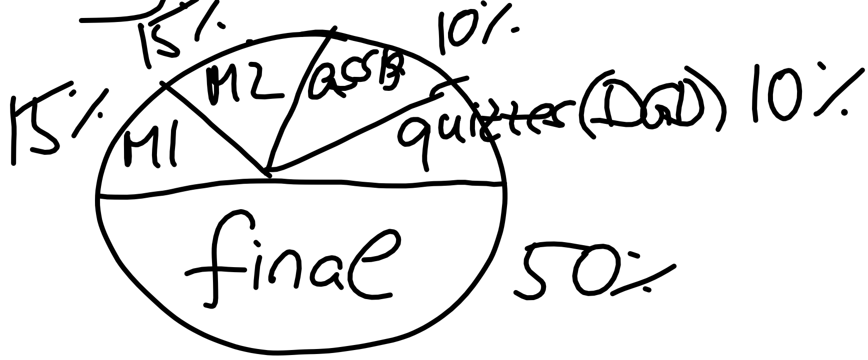
MAT1322A

Early tranc. 7th (6th ed.)



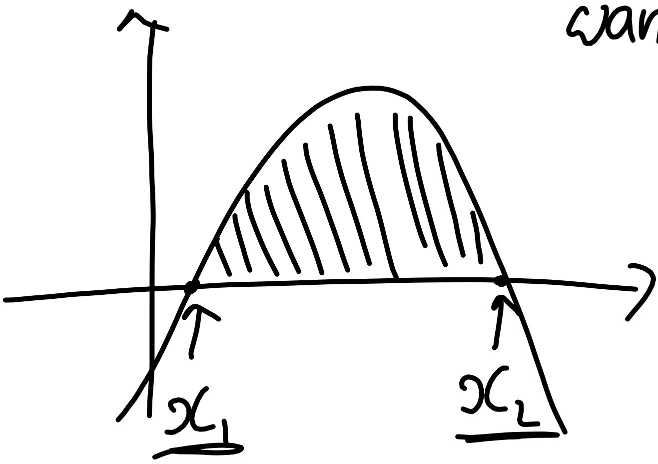
midterm: FEB 10th
MAR 16th

office hours: KED 205G
WED 11 – 12^{30} (from 2nd week on)
or by appointment!

efink@uottawa.ca

Review: definite integrals - area under a curve

want to compute area under curve



$$\boxed{f(x) = -x^2 + 2x - \frac{1}{2}}$$

recall: area is definite integral of $f(x)$ from x_1 to x_2

first step: find x_1, x_2.

they are zeros of $f(x)$.

solve quadratic equation: $-x^2 + 2x - \frac{1}{2} = 0$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-2}}{-2} = 1 \pm \frac{1}{\sqrt{2}}$$

now: integral from $1-\frac{1}{\sqrt{2}}$ to $1+\frac{1}{\sqrt{2}}$ of $f(x)$:

$$A=\int_{1-\frac{1}{\sqrt{2}}}^{1+\frac{1}{\sqrt{2}}} f(x)\,dx = \int_{1-\frac{1}{\sqrt{2}}}^{1+\frac{1}{\sqrt{2}}} -x^2+2x-\frac{1}{2}\,dx$$

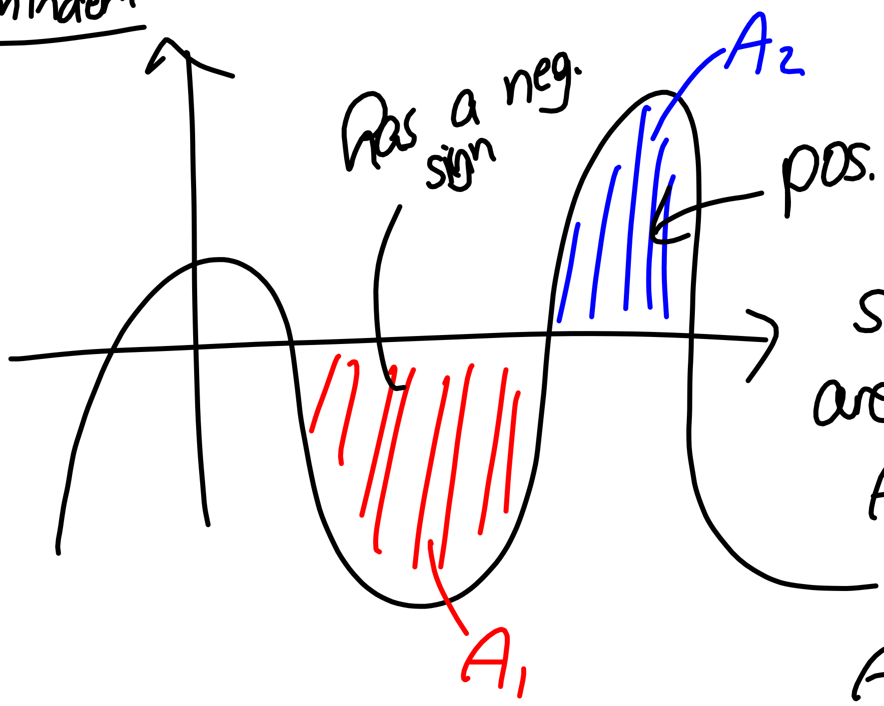
$$=\left[-\frac{x^3}{3}+x^2-\frac{x}{2}\right]_{1-\frac{1}{\sqrt{2}}}^{1+\frac{1}{\sqrt{2}}}$$

= (fill in $1+\frac{1}{\sqrt{2}}$ for x and subtract the same with $1-\frac{1}{\sqrt{2}}$ for x)

$$=\left(-\frac{\left(1+\frac{1}{\sqrt{2}}\right)^3}{3}+\left(1+\frac{1}{\sqrt{2}}\right)^2-\frac{1+\frac{1}{\sqrt{2}}}{2}\right)-\left(-\frac{\left(1-\frac{1}{\sqrt{2}}\right)^3}{3}+\left(1-\frac{1}{\sqrt{2}}\right)^2-\frac{1-\frac{1}{\sqrt{2}}}{2}\right)$$

$$=\underline{\underline{0.471}}$$

reminder:



so for ~~total~~ area:

$A = A_2 - A_1$

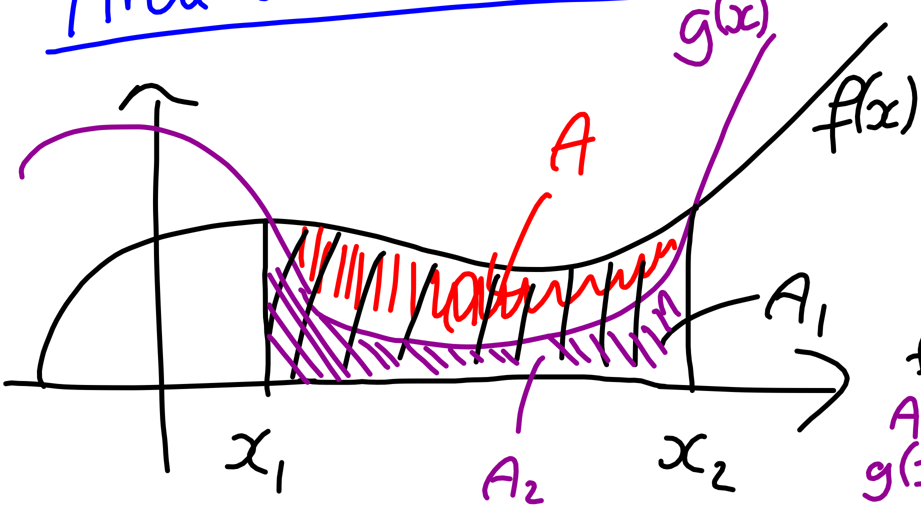
because A_1 is neg!

§6.1 in Stewart:

Area between curves

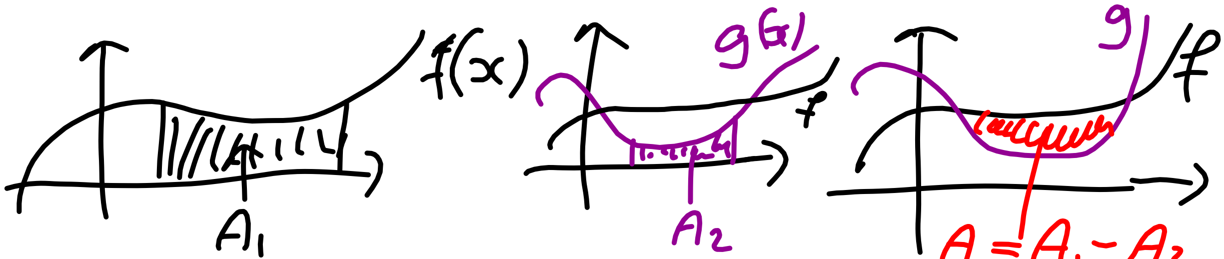
compute A

assume x_1, x_2 given



A_1 is area under $f(x)$ from x_1 to x_2

A_2: area under $g(x)$ from x_1 to x_2



$A = A_1 - A_2$

$$A = A_1 - A_2 = \int_{x_1}^{x_2} f(x)\,dx - \int_{x_1}^{x_2} g(x)\,dx$$

linearity of integral

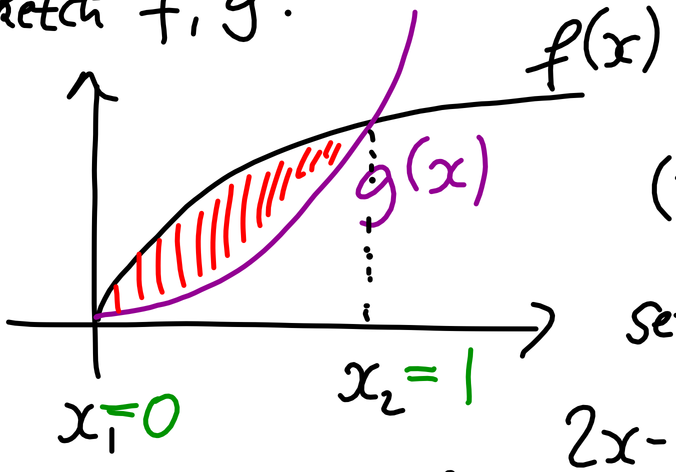
$$\boxed{A = \int_{x_1}^{x_2} f(x) - g(x)\,dx}$$

area between $f(x)$ and $g(x)$ betw. x_1, x_2

Note: we have to find x_1, x_2 by computing their intersection points. (set $f(x) = g(x)$ and solve for x)

Example: $f(x) = 2x - x^2$, $g(x) = x^2$ (Ex 2 p. 416)

(1) sketch f, g:



(2) find x_1, x_2

set $f(x) = g(x)$

$$2x - x^2 = x^2$$

$$2x = 2x^2 \quad /: x \qquad x = 0 \text{ is a sol. } (x_1)$$

$$2 = 2x$$

$$\boxed{x_2 = 1}$$

Here: $f(x)$ is bigger than $g(x)$. So area: use $f(x) - g(x)$

$$A = \int_{x_1=0}^{x_2=1} f(x) - g(x)\,dx = \int_0^1 \underbrace{(2x - x^2)}_{f(x)} - \underbrace{(x^2)}_{g(x)}\,dx$$

linearity of int.

$$= \int_0^1 2x - 2x^2\,dx = 2 \cdot \int_0^1 x - x^2\,dx$$

$$= 2 \cdot \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2 \cdot \left(\underbrace{\left(\frac{1}{2} - \frac{1}{3}\right)}_{= \frac{3}{6} - \frac{2}{6} = \frac{1}{6}} - \underbrace{\left(\frac{0}{2} - \frac{0}{3}\right)}_{=0}\right)$$

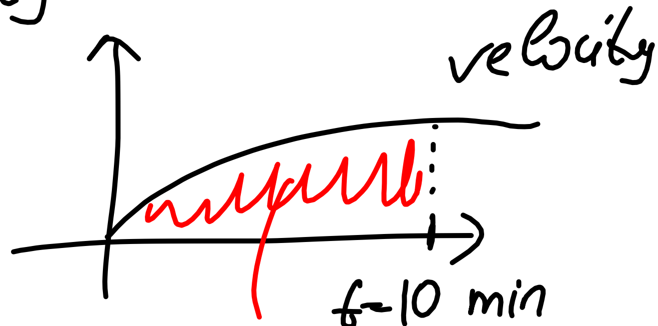
$$= 2\left(\frac{1}{6} - 0\right) = \frac{2}{6} = \underline{\frac{1}{3}}$$

Area between f and g betw. 0 and 1 is $\frac{1}{3}$.

inform: Ex 4 in §6.1: area can be a distance.

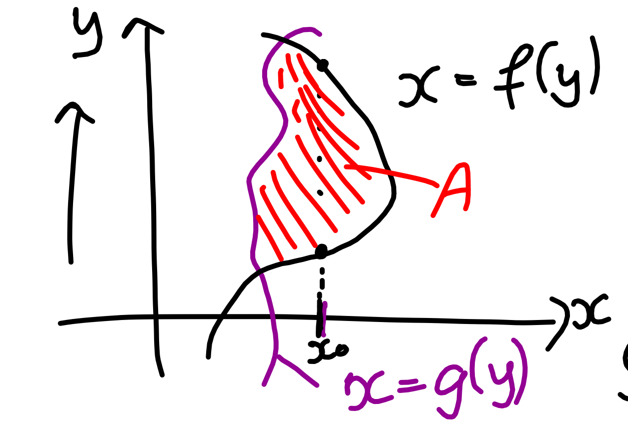
deriv. of dist. $\rightsquigarrow$ velocity

integrate



A is distance travelled in 10 min.

Vertical areas (§ 6.1)



we have two y-values for f(x) at x_0.

→ not a function.

but still curve!

Still want A!!

"rotate picture" to get area.

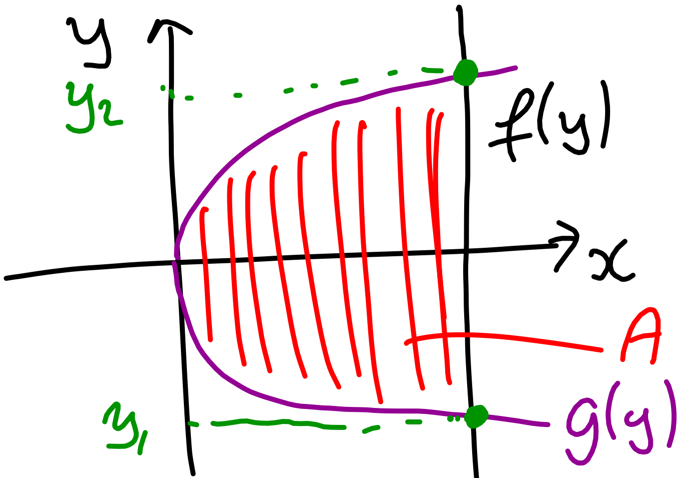
Could compute $f^{-1}(y)$ and $g^{-1}(y)$. But we could also just integrate over y!!

$$\boxed{A = \int_{x_1}^{x_2} f(y) - g(y)\, dy}$$

vertical area.

int. over y

Example: $x = g(y) = y^2$, $f(y) = 9$



(1) find y_1, y_2

set $f(y) = g(y)$

$9 = y^2$

$\pm 3 = y$, $\boxed{\begin{aligned} y_1 &= -3 \\ y_2 &= +3 \end{aligned}}$

need to check: is f or g bigger? → rotate the graph!

here: f is above g!

$$A = \int_{-3}^{3} f(y) - g(y)\,dy = \int_{-3}^{3} 9 - y^2\,dy = \left[-\frac{y^3}{3} + 9y\right]_{-3}^{3}$$

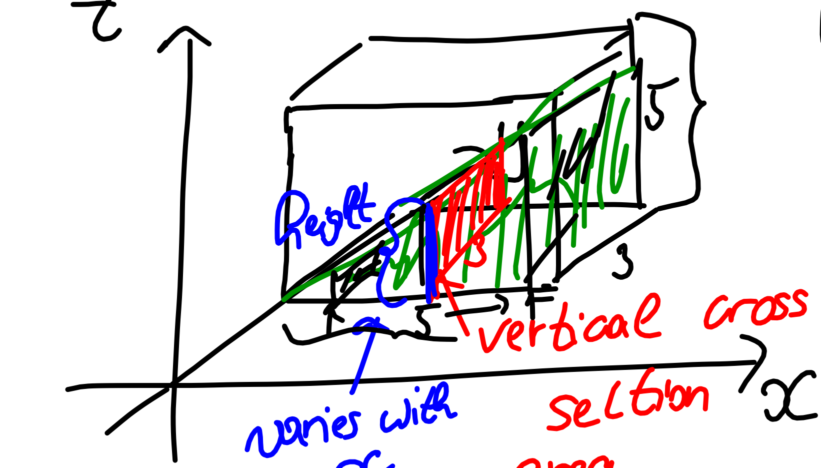
$$= \left(-\frac{27}{3} + 9\cdot 3\right) - \left(-\frac{(-3)^3}{3} + 9\cdot\underbrace{(-3)}\right)$$

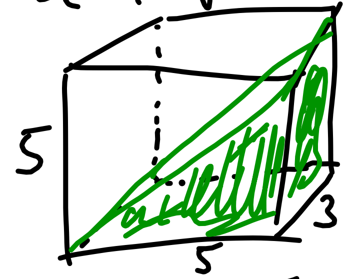
$$= \underline{\underline{36}}$$

more: §6.1 in book!! setting brackets!

Volumes: §6.2 need 3-dimensions

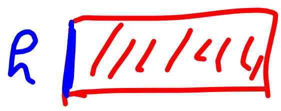
have a half box





x-length 5
y-length 3
z-height 5

find cross section area function.
cross section area depends on x!
area is given by:



cross section!
h changes.
h is proportional to the x-difference.

Now: Volume is the integral over the cross-section function!!

get a cross sect. area function: $h(x) = x$ (↑ height)

(assume cube at origin!) and width $= 3$

area function: $a(x) = h(x) \cdot 3 = 3x$.

$$V = \int_0^5 3x\,dx = \left[3 \cdot \frac{x^2}{2}\right]_0^5 = 3 \cdot \frac{5^2}{2} - 0$$

$$= \underline{\underline{\frac{75}{2}}}$$