Would you like this assignment to be marked? **YES** **NO**

MA129 Lab Report 1 - Pre-Calculus Topics

Name:________________________ Student Number:__________________ Winter 2017

1. [11 *marks*] Simplify each of the following expressions. Where appropriate, express your final answer with positive, rational exponents.

(a) $(t-1)^2(t+2)-(t^2-1)(3+t)$

$$= (t^2-2t+1)(t+2)-(3t^2+t^3-3-t)$$

$$= t^3+2t^2-2t^2-4t+t+2-3t^2-t^3+3+t$$

$$= -3t^2-2t+5$$

(b) $\dfrac{m-1}{m^2-2m-8}+\dfrac{m+1}{m^2+m-2}$

$$\begin{aligned}\frac{m-1}{m^2-2m-8}+\frac{m+1}{m^2+m-2} &= \frac{m-1}{(m-4)(m+2)}+\frac{m+1}{(m-1)(m+2)}\\ &= \frac{(m-1)(m-1)+(m+1)(m-4)}{(m-4)(m+2)(m-1)}\\ &= \frac{m^2-2m+1+m^2-3m-4}{(m-4)(m+2)(m-1)}\\ &= \frac{2m^2-5m-3}{(m-4)(m+2)(m-1)}\end{aligned}$$

(c) $\dfrac{8^{2/3}x^{3/5}(y^{-4})^{1/3}}{x^{-1/5}y^{-1/3}}$

$$\begin{aligned}\frac{8^{2/3}x^{3/5}(y^{-4})^{1/3}}{x^{-1/5}y^{-1/3}} &= \frac{(8^{1/3})^2x^{3/5}y^{-4/3}}{x^{-1/5}y^{-1/3}}\\ &= \frac{4x^{4/5}}{y}\end{aligned}$$

(d) $\left(\dfrac{2}{x}+\dfrac{4}{x-1}\right)\div\left(\dfrac{3}{x-1}-\dfrac{1}{x}\right)$

$$\begin{aligned} &= \frac{2(x-1)+4x}{x(x-1)}\div\frac{3x-(x-1)}{x(x-1)}\\ &= \frac{2x-2+4x}{x(x-1)}\cdot\frac{x(x-1)}{3x-x+1}\\ &= \frac{6x-2}{x(x-1)}\cdot\frac{x(x-1)}{2x+1}=\frac{6x-2}{2x+1}\end{aligned}$$

2. [5 *marks*] If two receivers with resistances R_1 and R_2 are connected in parallel, then the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

relates the total resistance R for the circuit.

(a) Given that R_1 is 3 ohms and R is 2 ohms, find R_2.

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{R_2} \quad \Rightarrow \frac{1}{2} - \frac{1}{3} = \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{R_2} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6} \Rightarrow R_2 = 6$$

(b) Suppose instead that $R = 1.2$ ohms, and R_1 is 1 ohm larger than R_2. Find R_1 and R_2. [Note: R_1 and R_2 cannot be negative.]

$R = 1.2$ and $R_1 = R_2 + 1$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \frac{1}{1.2} = \frac{1}{R_2+1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{1.2} = \frac{R_2 + (R_2+1)}{R_2(R_2+1)}$$

$$\Rightarrow \frac{1}{1.2} = \frac{2R_2+1}{R_2^2 + R_2}$$

$$\Rightarrow R_2^2 + R_2 = 2.4R_2 + 1.2$$

$$\Rightarrow R_2^2 - 1.4R_2 - 1.2 = 0$$

$$\Rightarrow (R_2 - 2)(R_2 + 0.6) = 0$$

Since $R_2 \geq 0$, $R_2 = 2$ ohms and $R_1 = 3$ ohms.

3. [3 *marks*] Solve $-2 < \frac{2+4x}{-3} \leq 4$ for x. State your result using interval notation.

$$-2 < \frac{2+4x}{-3} \leq 4 \quad \Leftrightarrow 6 > 2 + 4x \geq -12$$

$$\Leftrightarrow 4 > 4x \geq -14$$

$$\Leftrightarrow 1 > x \geq -\frac{14}{4} = -\frac{7}{2} \qquad \therefore x \in \left[-\frac{7}{2}, 1\right)$$

4. [4 *marks*] A laptop manufacturer has estimated that the profit in thousands of dollars is given by the expression $-5x^2 + 25x - 15$ where x (in thousands) is the number of units produced. What production range will enable the manufacturer to realize a profit of at least \$15,000?

We need to solve the inequality $-5x^2 + 25x - 15 \geq 15$, or, equivalently, $5x^2 - 25x + 30 \leq 0$.

Solve: $5x^2 - 25x + 30 = 0$

$\Rightarrow 5(x^2 - 5x + 6) = 0$

$\Rightarrow 5(x-3)(x-2) = 0$

$\Rightarrow x = 3, x = 2$

	$(-\infty, 2)$	$(2,3)$	$(3,\infty)$
$5(x-3)(x-2)$	+	−	+

$\therefore$ the solution to $5x^2 - 25x + 30 \leq 0$ is $[2,3]$. A production range of 2000 to 3000 laptops will enable the manufacturer to realize a profit of at least \$15,000.

Grade: $\frac{23}{23}$