

# ELEC 273 Mid-Term Test

October 21, 2017 from 17:00 until 18:30.

Instructors: C.W. Trueman, S.K. Das

Name: \_\_\_\_\_

I.D. #: \_\_\_\_\_

## Instructions

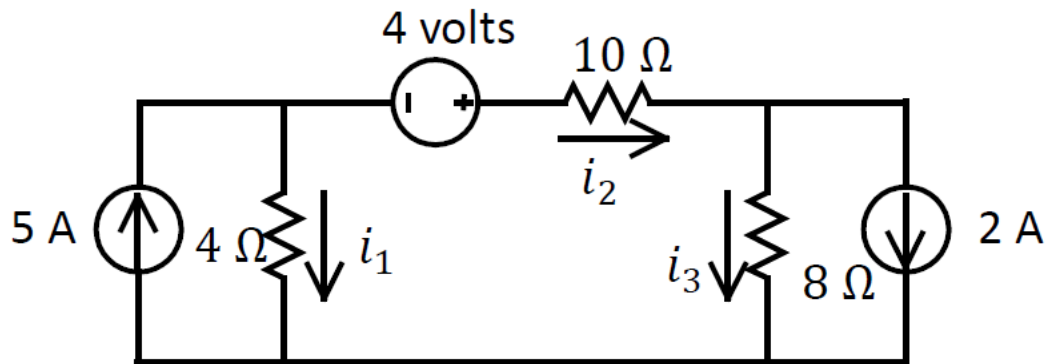
Closed book exam: no notes or books are allowed.

Calculator: You are permitted to have a calculator with an ENCS sticker. No other calculator is allowed.

Return the exam paper with your answer booklet.

You must NOT have a cell phone or other electronic device in your possession.

### 1.(10 marks)

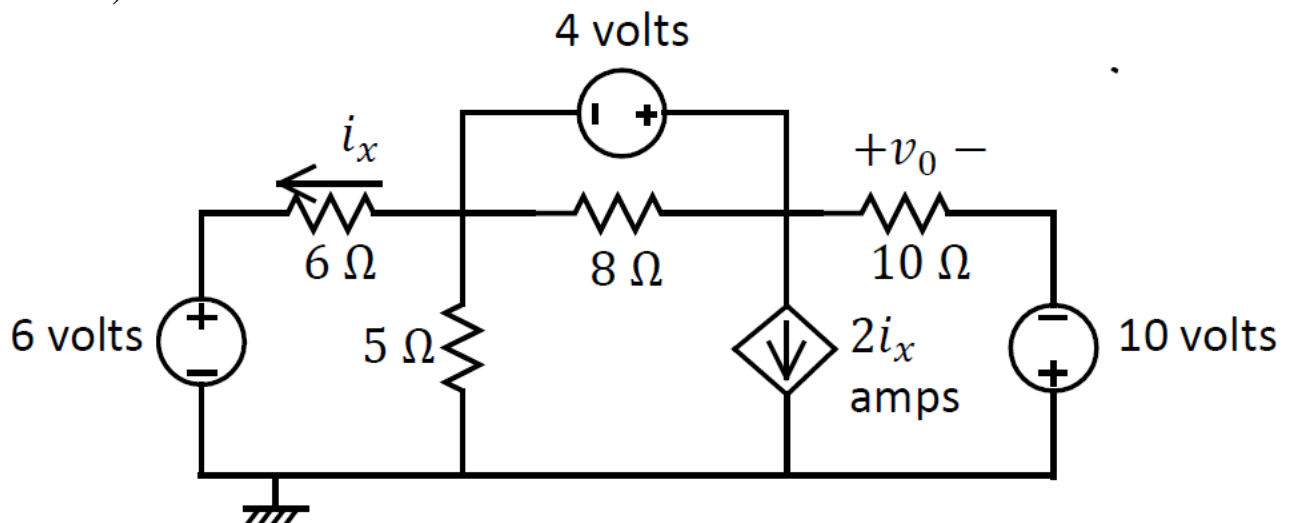


1.1) Use KVL and KCL to write a set of three independent equations using the branch currents  $i_1$ ,  $i_2$  and  $i_3$  as unknowns. (6 marks)

1.2) If  $i_2 = 20/11$  amps, then find  $i_1$ . (1 mark)

1.3) If  $i_2 = 20/11$  amps, then find the power delivered by the 5 A current source. (3 marks)

### 2.(10 marks)



Solve for  $i_x$  and  $v_o$  using nodal analysis, as follows:

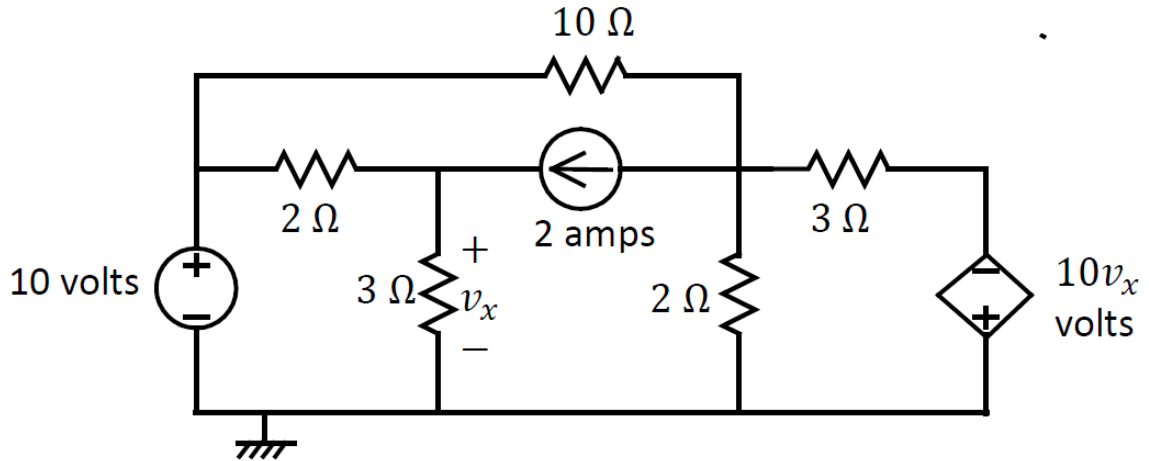
2.1 Choose node voltages and write node equations for this circuit. (5 marks)

2.2 Arrange your node equations into a matrix equation. (1 mark)

2.3 Solve the node equations to find the value(s) of the node voltages. (2 marks)

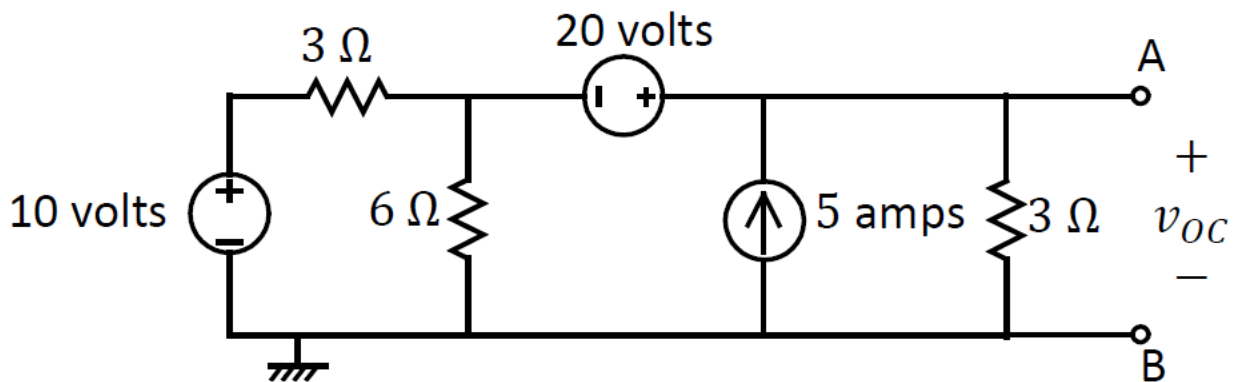
2.4 Find the value of  $i_x$  and of  $v_o$ . (2 marks)

3.(10 marks)



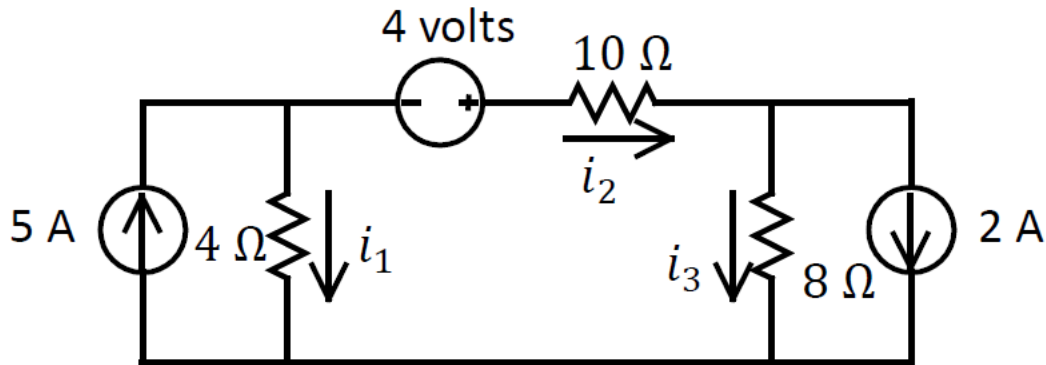
- 3.1) Choose mesh currents and write a set of mesh equations for the circuit shown above. (6 marks)
- 3.2) Eliminate  $v_x$  from your mesh equations. (2 marks)
- 3.3) Arrange your mesh equations into a matrix equation. (2 marks)

4.(10 marks)



Find the Thevenin equivalent circuit at terminals AB using the following steps:

- 4.1) Find the open-circuit voltage  $v_{oc}$  using superposition. (6 marks)
- 4.2) Find the Thevenin equivalent resistance at terminals AB. (2 marks)
- 4.3) A resistor  $R_L$  is connected across terminals AB. What value of  $R_L$  dissipates the maximum amount of power? (1 mark)
- 4.4) Using the value of  $R_L$  from above, how much power does  $R_L$  dissipate? (1 mark)



1.1) Use KVL and KCL to write a set of three independent equations using the branch currents  $i_1$ ,  $i_2$  and  $i_3$  as unknowns. (6 marks)

1.2) If  $i_2 = 20/11$  then find  $i_1$ . (1 mark)

1.3) If  $i_2 = 20/11$  then find the power delivered by the 5 A current source. (3 marks)

**Solution**

See “Solving Circuits Directly with KCL and KCL” in Lecture Notes Set 2.

Also see Tutorial Workshop #3.

1.1)

$$\text{KCL: } 5 - i_1 - i_2 = 0$$

$$\text{KCL: } i_2 - i_3 - 2 = 0$$

$$\text{KVL: } 4i_1 + 4 - 10i_2 - 8i_3 = 0$$

1.2) The problem does not ask the student to solve the equations. Here is the solution.

$$\begin{aligned} i_1 &= 5 - i_2 \\ 4(5 - i_2) + 4 - 10i_2 - 8i_3 &= 0 \\ 20 - 4i_2 + 4 - 10i_2 - 8i_3 &= 0 \\ -14i_2 - 8i_3 &= -24 \\ i_3 &= i_2 - 2 \\ -14i_2 - 8(i_2 - 2) &= -24 \\ -22i_2 + 16 &= -24 \\ -22i_2 &= -40 \\ i_2 &= \frac{40}{22} = \frac{20}{11} \end{aligned}$$

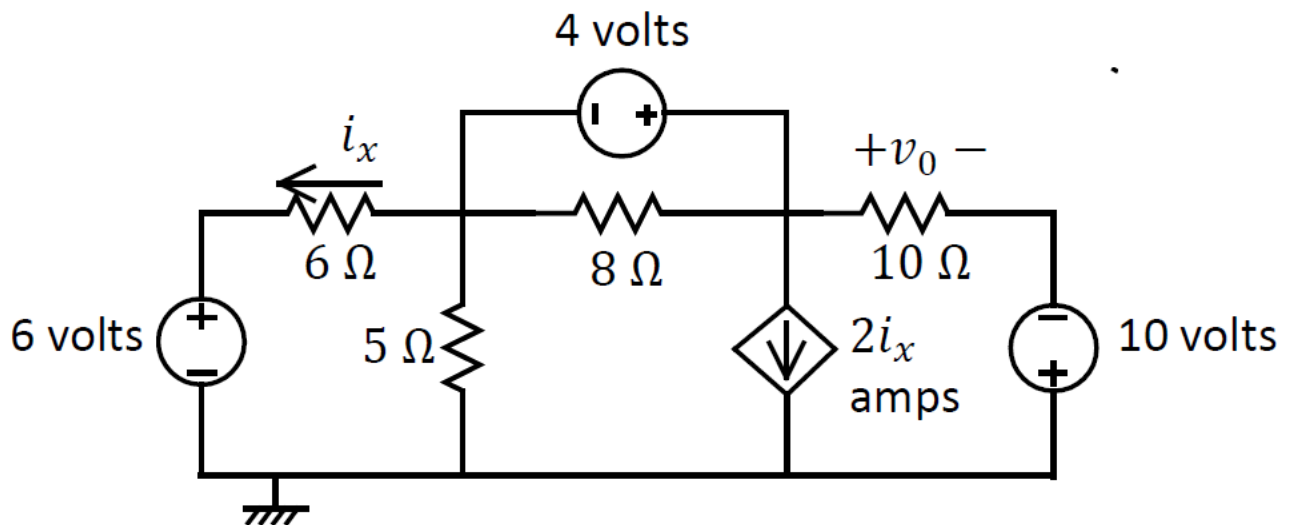
1.2) gives  $i_2 = 20/11$  and asks to find  $i_1$ :

$$i_1 = 5 - i_2 = 5 - \frac{20}{11} = \frac{55-20}{11} = \frac{35}{11} \text{ amps}$$

1.3) The power delivered by the current source is  $P = vi = 5v_1$  where  $v_1$  is the voltage across the current source, which is the voltage across the 4 ohm resistor.

$$\text{Find branch voltage } v_1: \quad v_1 = 4i_1 = \frac{4 \times 35}{11} = \frac{140}{11} \text{ volts}$$

$$\text{Find the power delivered by the 5A source: } p = \frac{140}{11} \times 5 = \frac{700}{11} = 63.6 \text{ watts}$$



2. Solve for  $i_x$  and  $v_0$  using node analysis, as follows:

- 2.1 Choose node voltages and write node equations for this circuit. (5 marks)
- 2.2 Arrange your node equations into a matrix equation. (1 marks)
- 2.3 Solve the node equations to find the value(s) of the node voltages. (2 marks)
- 2.4 Find the value of  $i_x$  and of  $v_0$ . (2 marks)

Solutio

2.1)

$$\frac{6-v_1}{6} - \frac{v_1}{5} + \frac{1}{2} - \frac{1}{2} - 2i_x - \frac{v_2+10}{10} = 0 \quad \text{①}$$

$$i_x = \frac{v_1-6}{6}$$

$$v_2 = v_1 + 4$$

2.2) Matrix equation for  $V_1$  and  $V_2$

$$\begin{aligned} \textcircled{I} \times 30 \quad & 5(6-V_1) - 6V_1 - 60\left(\frac{V_1-6}{6}\right) - 3V_2 - 30 = 0 \\ & 30 - 5V_1 - 6V_1 - 10V_1 + 60 - 3V_2 - 30 = 0 \\ & -21V_1 - 3V_2 = -60 \\ & 21V_1 + 3V_2 = 60 \\ & V_1 - V_2 = 4 \end{aligned}$$

$$\begin{pmatrix} 21 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 60 \\ 4 \end{pmatrix}$$

$$-21V_1 - 3(V_1 + 4) = -60$$

$$-24V_1 - 12 = -60$$

$$24V_1 = 48$$

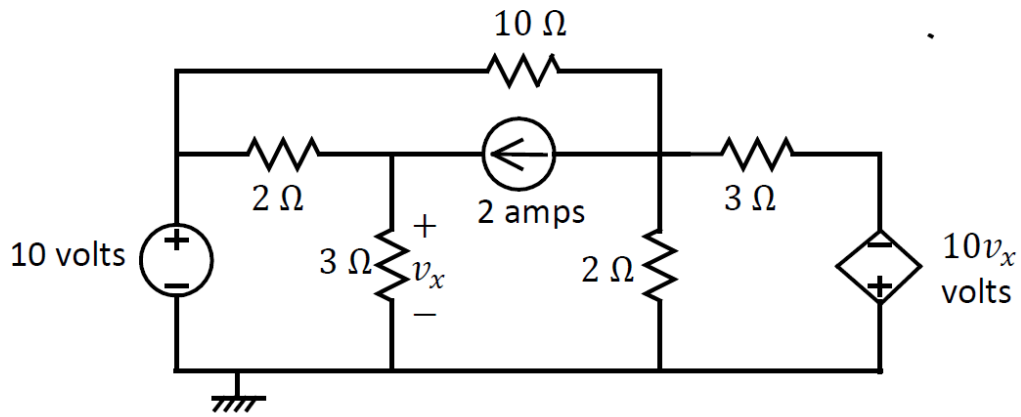
$$V_1 = 2$$

$$V_2 = V_1 + 4 = 6$$

$$2.4) \quad i_x = \frac{V_1 - 6}{6} = \frac{2 - 6}{6} = -\frac{4}{6} = -\frac{2}{3} \text{ A}$$

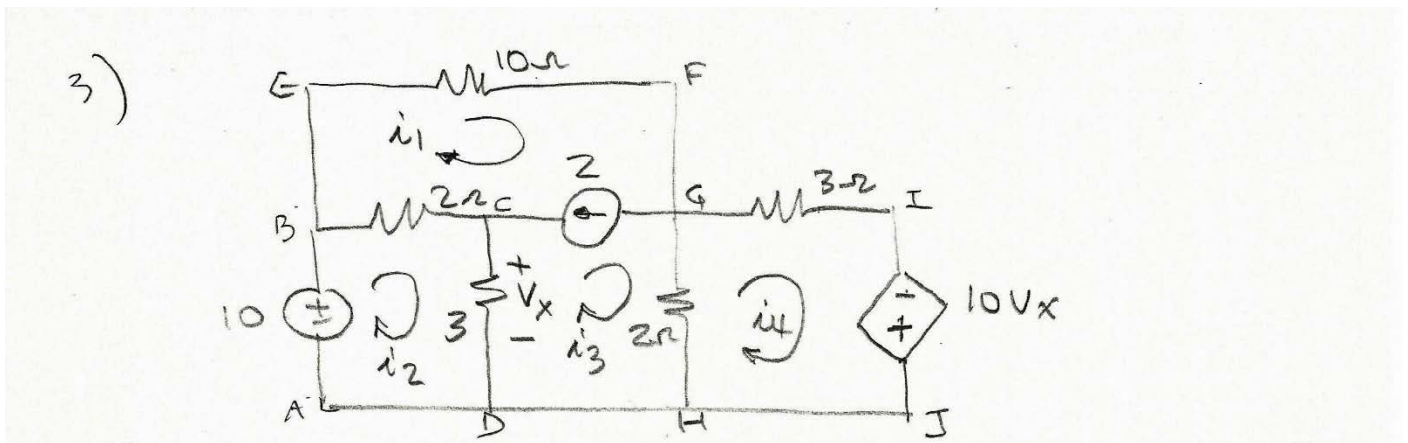
$$V_0 = V_2 + 10 = 6 + 10 = 16$$

Remark: This circuit can be solved "smarter" with only one node voltage  $V_1$  by replacing  $V_2$  with  $V_1 + 4$  in the circuit diagram shown above. Then the matrix in part 2.2) becomes a 1x1 matrix.



- 3.1) Choose mesh currents and write a set of mesh equations for the circuit shown above. (6 marks)  
 3.2) Eliminate  $v_x$  from your mesh equations. (2 marks)  
 3.3) Arrange your mesh equations into a matrix equation. (2 marks)

Solution



3.1) constraint  $i_1 - i_3 = 2$  (I)

KVL ABCDA  $10 - 2(i_2 - i_1) - 3(i_2 - i_3) = 0$  (II)

KVL Supermesh DCBEFGHD

$$3(i_2 - i_3) + 2(i_2 - i_1) - 10i_1 - 2(i_3 - i_4) = 0$$
 (III)

KVL HG I J H

$$2(i_3 - i_4) - 3i_4 + 10 \left[ \overbrace{3(i_2 - i_3)}^{v_x} \right] = 0$$
 (IV)

$$\text{Eqn II} \quad 10 - 2i_2 + 2i_1 - 3i_2 + 3i_3 = 0$$

$$2i_1 - 5i_2 + 3i_3 = -10$$

$$\text{Eqn III} \quad 3i_2 - 3i_3 + 2i_2 - 2i_1 - 10i_1 - 2i_3 + 2i_4 = 0$$

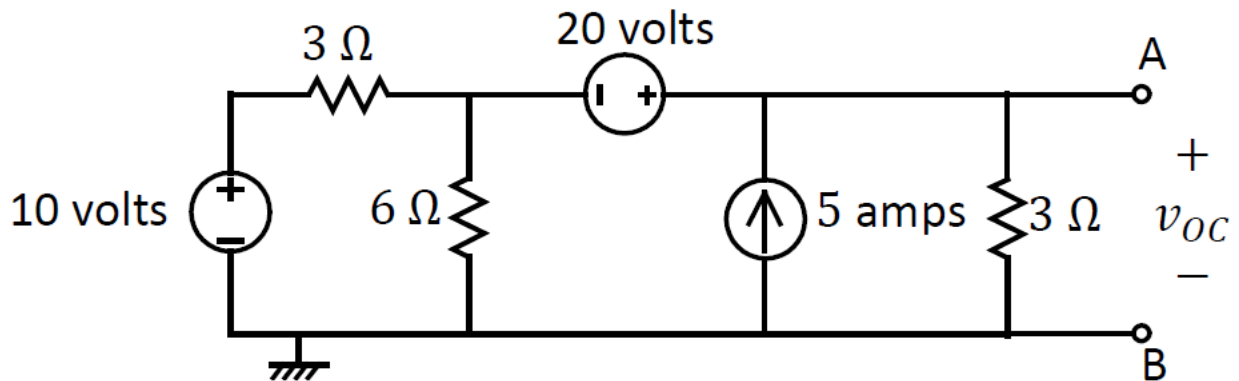
$$-12i_1 + 5i_2 - 5i_3 + 2i_4 = 0$$

$$\text{Eqn IV} \quad 2i_3 - 2i_4 - 3i_4 + 30i_2 - 30i_3 = 0$$

$$30i_2 - 28i_3 - 5i_4 = 0$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & -5 & 3 & 0 \\ -12 & +5 & -5 & +2 \\ 0 & 30 & -28 & -5 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -10 \\ 0 \\ 0 \end{pmatrix}$$

Remark: This circuit can be solved "smarter" with only three unknown mesh currents by replacing  $i_3$  by  $i_1 - 2$  in the circuit diagram. Then the matrix in part 3.3) is a 3x3 matrix.



4. Find the Thevenin equivalent circuit at terminals AB using the following steps;

4.1) Find the open-circuit voltage  $v_{oc}$  using superposition. (6 marks)

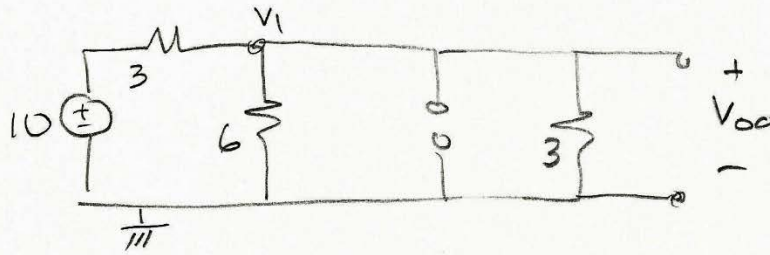
4.2) Find the Thevenin equivalent resistance at terminals AB. (2 marks)

4.3) A resistor  $R_L$  is connected across terminals AB. What value of  $R_L$  dissipates the maximum amount of power? (1 mark)

4.4) Using the value of  $R_L$  from above, how much power does  $R_L$  dissipate? (1 mark)

Solution

4.1)

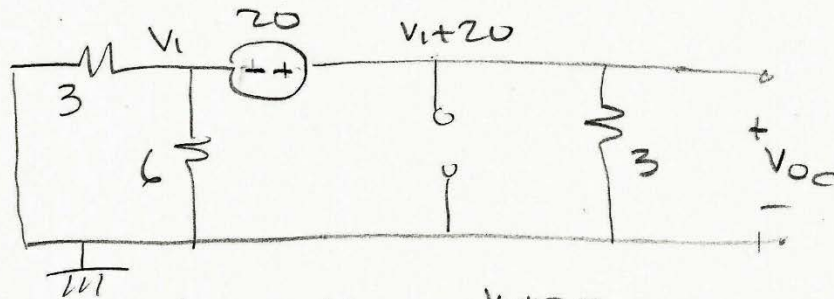


$$10 - V_1 - \frac{V_1}{3} - \frac{V_1}{6} - \frac{V_1}{3} = 0$$

$$60 - 6V_1 - 3V_1 - 6V_1 = 0$$

$$15V_1 = 60$$

$$V_1 = 4 \Rightarrow V_{oc} = V_1 = 4 \text{ V}$$



$$-\frac{V_1}{3} - \frac{V_1}{6} - \frac{V_1+20}{3} = 0$$

$$-6V_1 - 3V_1 - 6V_1 - 120 = 0$$

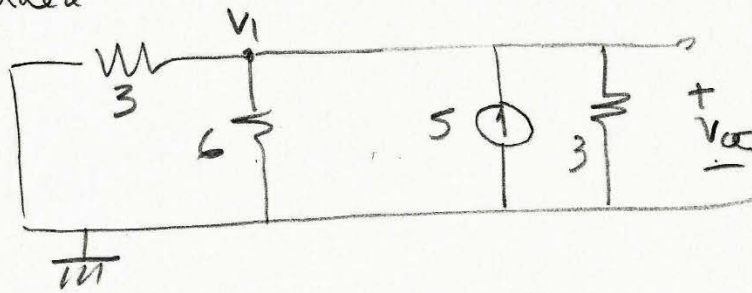
$$-15V_1 = 120$$

$$V_1 = -\frac{120}{15}$$

$$V_{oc} = V_1 + 20 = -\frac{120}{15} + 20 = \frac{-120 + 300}{15}$$

$$= \frac{180}{15} = 12 \text{ V}$$

4.1) Continued



$$-\frac{V_1}{3} - \frac{V_1}{6} + 5 - \frac{V_1}{3} = 0$$

$$-6V_1 - 3V_1 + 90 - 6V_1 = 0$$

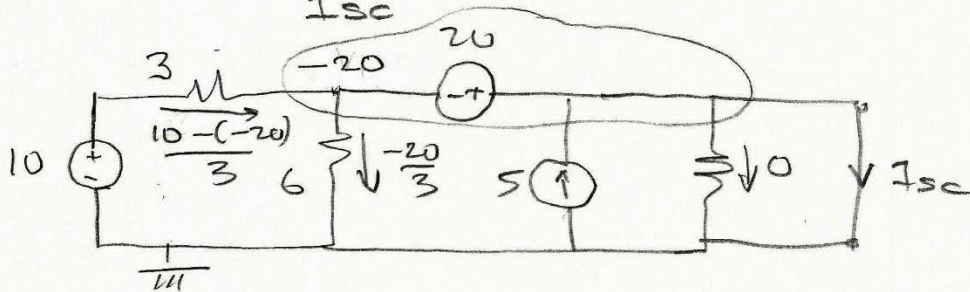
$$-15V_1 = -90$$

$$V_1 = 6 \rightarrow V_{oc} = 6 \text{ V}$$

By Superposition with all three sources active

$$V_{oc} = 4 + 12 + 6 = 22 \text{ volts}$$

4.2) FWD  $R_T = \frac{V_{oc}}{I_{sc}}$

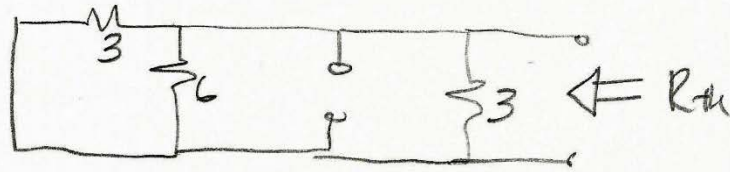


$$\text{KCL} \quad \frac{10 - (-20)}{3} - \frac{-20}{3} + 5 - I_{sc} = 0$$

$$I_{sc} = \frac{30}{3} + \frac{20}{3} + 5 = \frac{60 + 20 + 30}{3} = \frac{110}{3}$$

$$R_T = \frac{V_{oc}}{I_{sc}} = \frac{22}{\frac{110}{3}} = \frac{3 \times 22}{110} = \frac{6}{5} \Omega$$

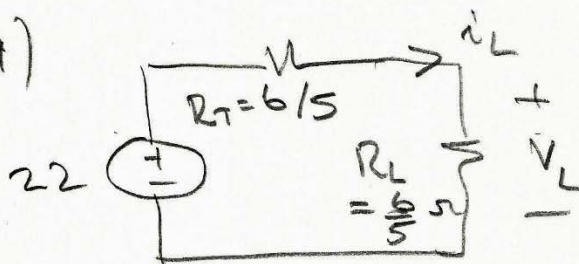
CHECK - Dead Circuit Method



$$R_{th} = (6 \parallel 3) \parallel 3 = \left( \frac{3 \times 6}{9} \right) \parallel 3 = 2 \parallel 3 = \frac{6}{5} \Omega$$

4.3) For max power choose  $R_L = R_T = \frac{6}{5} \Omega$

4.4)



choose  $R_L = R_T$

$$i_L = \frac{V_T}{2R_T}$$

$$V_L = R_T i_L = \frac{V_T}{2}$$

$$P = V_L i_L = \frac{V_T}{2} \times \frac{V_T}{2R_T} = \frac{V_T^2}{4R_T}$$

$$= \frac{22^2}{4 \times 6/5} = 100.8 \text{ watts}$$