ECON 302

Concordia University

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Fall 2018, Assignment 1 - Solutions

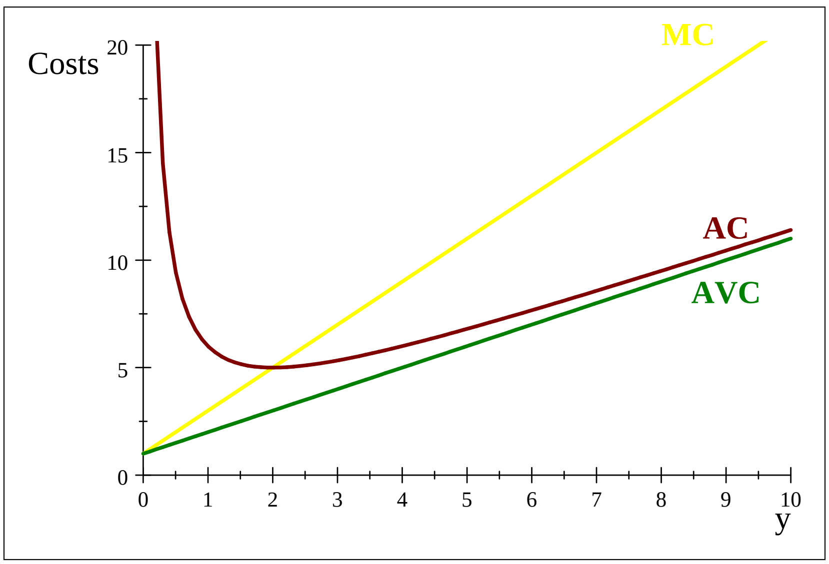
1. (25 pts) A competitive firm has a cost function given by: $C(y) = y^2 + y + 4$.

(a) Derive the firm's marginal cost function $MC(y)$, average variable cost function $AVC(y)$, and average cost function $AC(y)$ and show them on a graph. (5 pts)

$$MC(y) = 2y + 1$$

$$AVC(y) = \frac{VC(y)}{y} = y + 1$$

$$AC(y) = \frac{C(y)}{y} = y + 1 + \frac{4}{y}$$



(b) At what output is the average cost $AC(y)$ minimized? (5 pts)

The average cost $AC(y)$ is minimized when $MC = AC$. Thus, it should be:

$2y + 1 = y + 1 + \frac{4}{y} \Rightarrow y = \frac{4}{y} \Rightarrow y^2 = 4 \Rightarrow y = 2$.

The average cost (AC) is minimized at $y = 2$.

(c) Determine the short-run supply curve for this firm and show this curve on your graph. (5 pts)

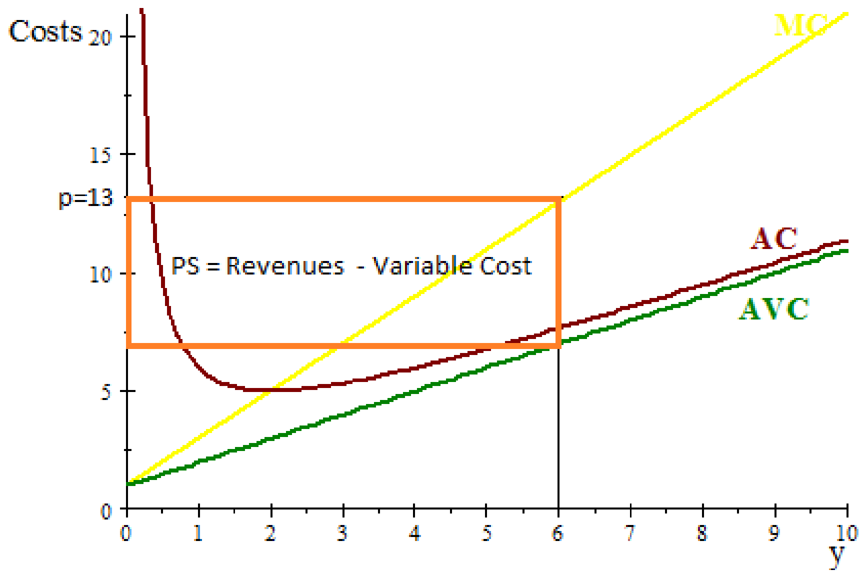
The short-run supply curve is part of the MC curve which is upward-sloping (increasing) and above the AVC curve. Since MC curve is always increasing and above the AVC curve we can conclude that, $p = MC \Rightarrow p = 1 + 2y \Rightarrow y = \frac{p-1}{2}$.

Therefore, the short-run supply curve for the firm is,

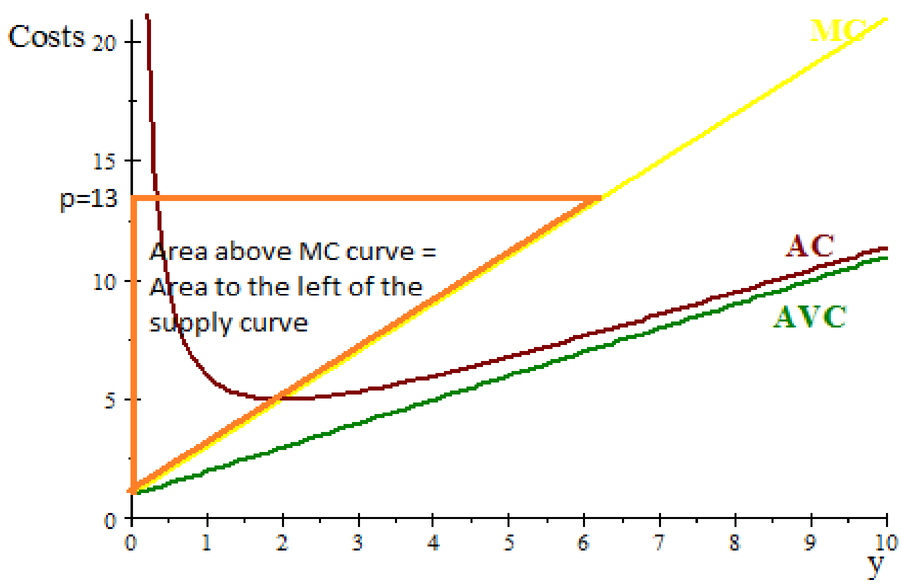
$$S(p) = \frac{p-1}{2}$$

(b) Assume that the market price is $p = 13$. Show on a graph the producer's surplus using the three alternative methods covered in class. Use one of these methods to calculate the producer's surplus. (5 pts + 5 pts)

The first way to calculate the producer's surplus is to take the difference between revenues and variable cost.



The second way to calculate the producer's surplus is to take the area above the MC curve, while the third way is to take the area to the left of the supply curve. In our case, these two areas are the same.



When $p = 13$, from the short-run supply curve we have, $y = \frac{p-1}{2} = \frac{13-1}{2} \Rightarrow y = 6$.

Producer's surplus is the area to the left of the supply curve. Thus $PS = \frac{1}{2} * (13 - 1) * 6 = 36$.

2. (30 pts) A competitive firm has a cost function that satisfies the following: (i) $PS(y) - \Pi(y) = 4$ for all y, (ii) $MC(0) = AVC(0) = 2$ and (iii) $AVC(y) = y + 2$.

(a) Derive the firm's total cost function $C(y)$, marginal cost function $MC(y)$, average variable cost function $AVC(y)$, and average cost function $AC(y)$. (5 pts)

Since $PS(y) - \Pi(y) = 4 \Rightarrow FC = 4$.

Also, since $AVC(y) = y + 2 \Rightarrow VC(y) = y * AVC(y) \Rightarrow VC(y) = y^2 + 2y$.

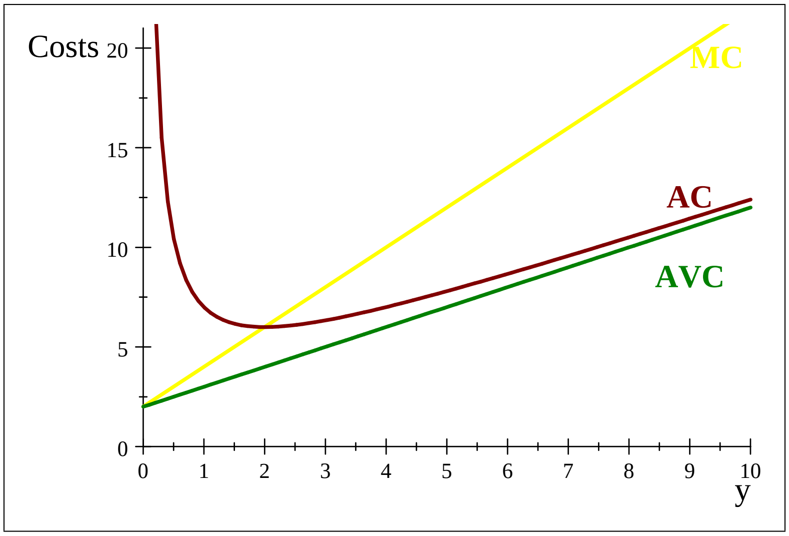
Thus, $C(y) = VC(y) + FC = y^2 + 2y + 4$.

$$C(y) = y^2 + 2y + 4.$$

Also,

$$MC(y) = 2y + 2$$
$$AVC(y) = \frac{VC(y)}{y} = y + 2$$
$$AC(y) = \frac{C(y)}{y} = y + 2 + \frac{4}{y}$$

(b) Show MC, AVC and AC functions on a graph. (5 pts)



(c) Derive the short-run supply curve for this firm. (5 pts)

The short-run supply curve is part of the MC curve which is upward-sloping (increasing) and above the AVC curve. Since MC curve is always increasing and above the AVC curve we can conclude that, $p = MC \Rightarrow p = 2y + 2 \Rightarrow y = \frac{p-2}{2}$.

Therefore, the short-run supply curve for the firm is,

$$S(p) = \frac{p-2}{2}$$

(d) If the market price is $p = 12$. How many units would the firm produce? (5 pts)

If the market price is $p = 12$, then from the short-run supply curve $y = \frac{p-2}{2} = \frac{12-2}{2} \Rightarrow y = 5$.

(e) Determine the associated profit and producer surplus for the firm. (5 pts)

The profit is, $\Pi = py - C(y) = py - (y^2 + 2y + 4)$.

At $p = 12$ and $y = 5$, $\Pi = 12 * 5 - (25 + 10 + 4) = 21 \Rightarrow \Pi = 21$.

The producer surplus is, $PS = py - VC(y) = py - (y^2 + 2y)$.

At $p = 12$ and $y = 5$, $PS = 12 * 5 - (25 + 10) = 25 \Rightarrow PS = 25$.

(f) How do the maximum profit related to the producer's surplus? (5 pts)

PS = Profit + Fixed Cost.

In the short-run, producer's surplus is always greater than profit. Only if fixed cost is zero (in the long-run) are PS and profit the same.

3. (20 pts) Consider three firms out of a competitive industry. They have the following technologies: $C_1(y) = \frac{1}{2}y^2 + 2y$, $C_2(y) = \frac{1}{2}y^2 + 4y$, and $C_3(y) = \frac{1}{2}y^2 + 6y$ respectively.

(a) Derive the firms' individual supply curves. (5 pts)

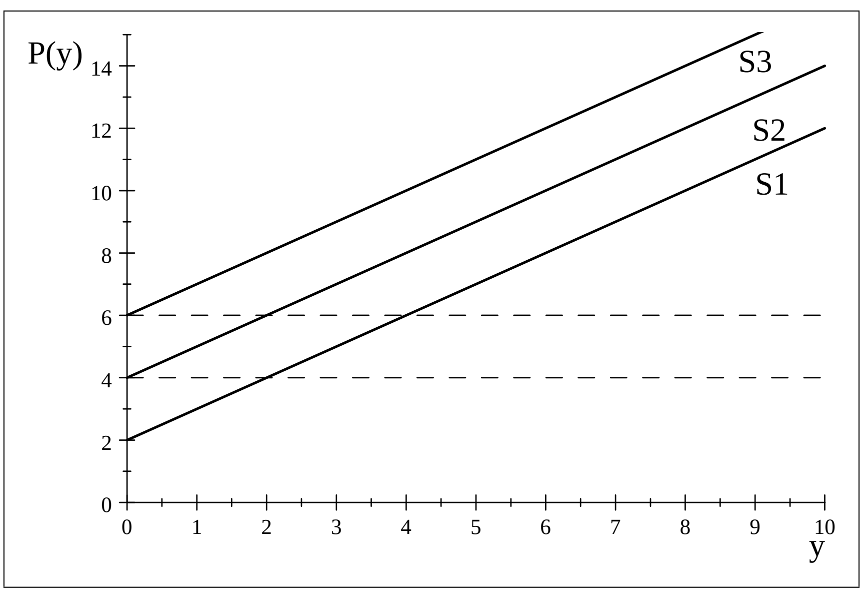
The marginal costs for the three firms are, $MC_1(y) = y+2$, $MC_2(y) = y+4$, and $MC_3(y) = y+6$ respectively.

The variable costs for the three firms are, $AC_1(y) = \frac{1}{2}y+2$, $AC_2(y) = \frac{1}{2}y+4$, and $AC_3(y) = \frac{1}{2}y+6$ respectively.

The firm's individual supply curve is the part of the MC curve which is upward-sloping (increasing) and above the AC curve. Since the MC curve is always increasing and above the AC curve we can conclude that the firms' individual supply curves are the following,

$$S_1(p) = p - 2$$
$$S_2(p) = p - 4$$
$$S_3(p) = p - 6$$

(b) Show these curves on a graph. (5 pts)



(c) Construct the industry supply curve for these three firms and show it on the same graph. (5 pts)

For any price $p \leq 2$, $S(p) = 0$.

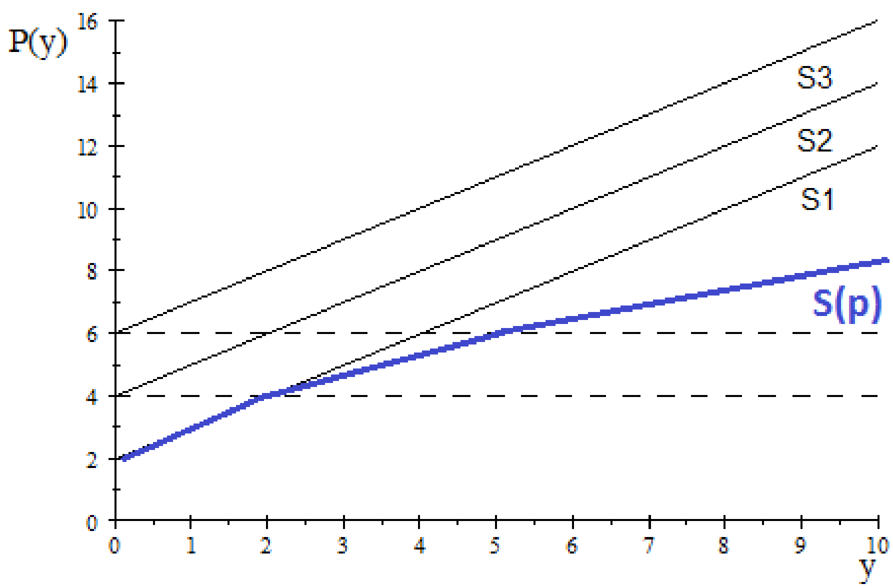
For any price $2 < p \leq 4$, $S(p) = S_1(p) = p - 2$.

For any price $4 < p \leq 6$, $S(p) = S_1(p) + S_2(p) = 2p - 6$.

For any price $p > 6$, $S(p) = S_1(p) + S_2(p) + S_3(p) = 3p - 12$.

Thus, the industry supply curve is,

$$S(p) = \begin{cases} 0 & \text{if } p \leq 2 \\ p - 2 & \text{if } 2 < p \leq 4 \\ 2p - 6 & \text{if } 4 < p \leq 6 \\ 3p - 12 & \text{if } p > 6 \end{cases}$$



(d) At what prices does the industry supply curve have a kink in it? Explain. (5 pts)

The industry supply curve has a kink in it at $p = 4$ and $p = 6$. For any price $p < 6$, Firm 3 reduces its quantity supplied to zero and for any price $p < 4$, Firm 2 reduces its quantity supplied to zero. Thus, the industry supply curve has a kink in it where the market price becomes low enough that some firm reduces its quantity supplied to zero.

4. (25 pts) Consider three firms out of a competitive industry. They have the following technologies: $C_1(y) = y^2 + 4$; $C_2 = y^2 + y + 4$; $C_3 = y^2 + 2y + 4$ respectively.

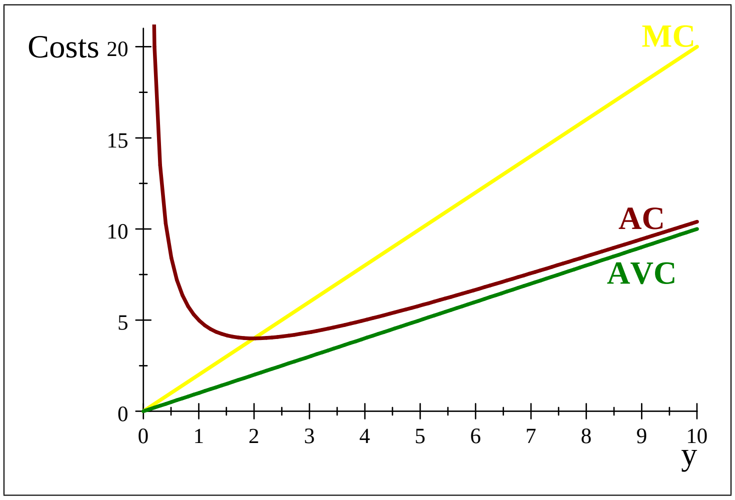
(a) For each firm derive the marginal cost function $MC(y)$, average variable cost function $AVC(y)$, and average cost function $AC(y)$. Show these curves on three graphs, one for each firm. (6 pts)

For Firm 1:

$$MC_1(y) = 2y$$

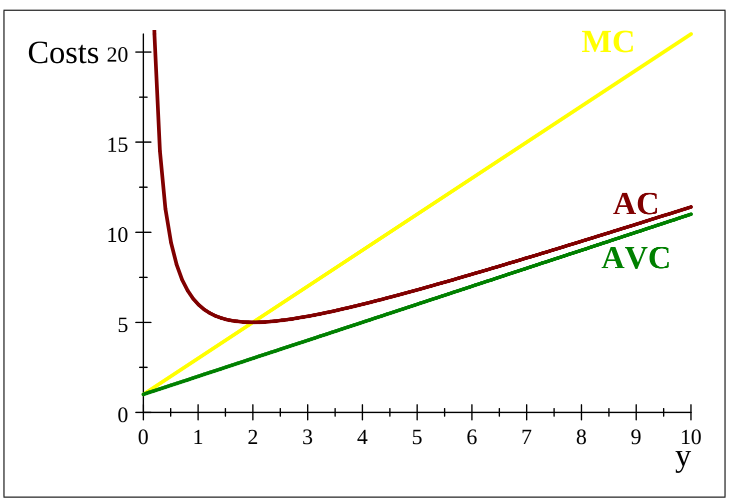
$$AVC_1(y) = y$$

$$AC_1(y) = y + \frac{4}{y}$$



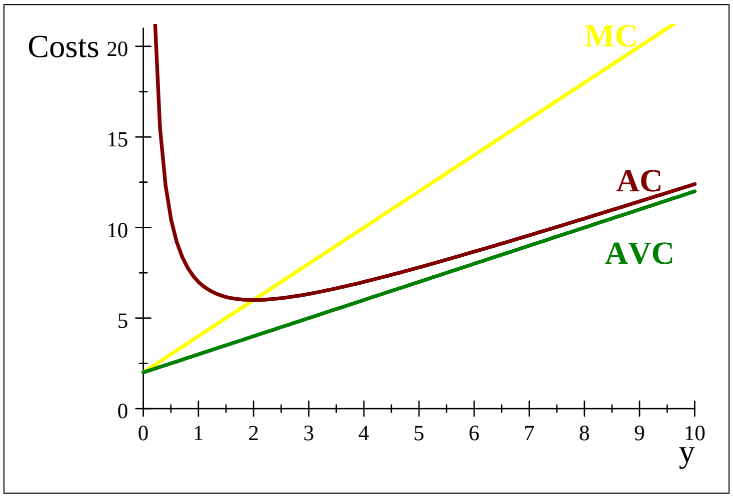
For Firm 2:

$$MC_2(y) = 2y + 1$$
$$AVC_2(y) = y + 1$$
$$AC_2(y) = y + 1 + \frac{4}{y}$$



For Firm 3:

$$MC_3(y) = 2y + 2$$
$$AVC_3(y) = y + 2$$
$$AC_3(y) = y + 2 + \frac{4}{y}$$



(b) Suppose that in the short-run the market price is $p = 5$. Calculate each firm's profit. (6 pts)

For Firm 1:

$p = MC_1 \Rightarrow 5 = 2y \Rightarrow y = 2.5$

$\Pi_1 = py - C_1(y) = 5 * 2.5 - (2.5^2 + 4) = 12.5 - 10.25 = 2.25$, Profit.

For Firm 2:

$p = MC_2 \Rightarrow 5 = 2y + 1 \Rightarrow y = 2$

$\Pi_2 = py - C_2(y) = 5 * 2 - (2^2 + 2 + 4) = 10 - 10 = 0$, Break even.

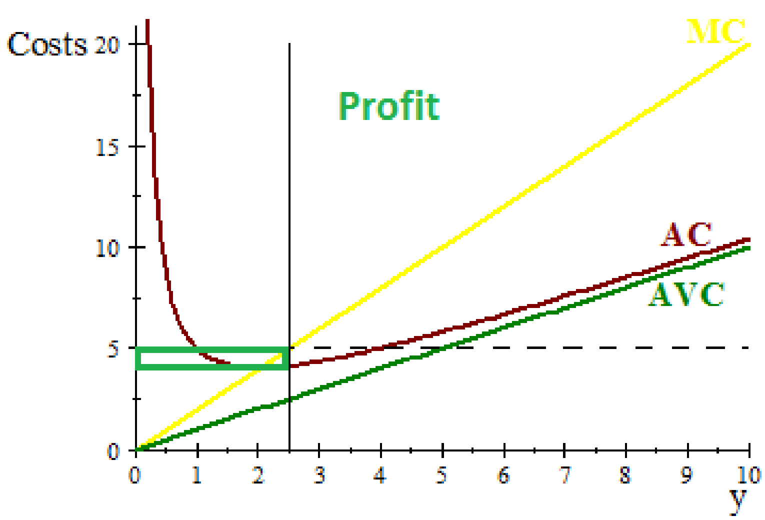
For Firm 3:

$p = MC_3 \Rightarrow 5 = 2y + 2 \Rightarrow y = 1.5$

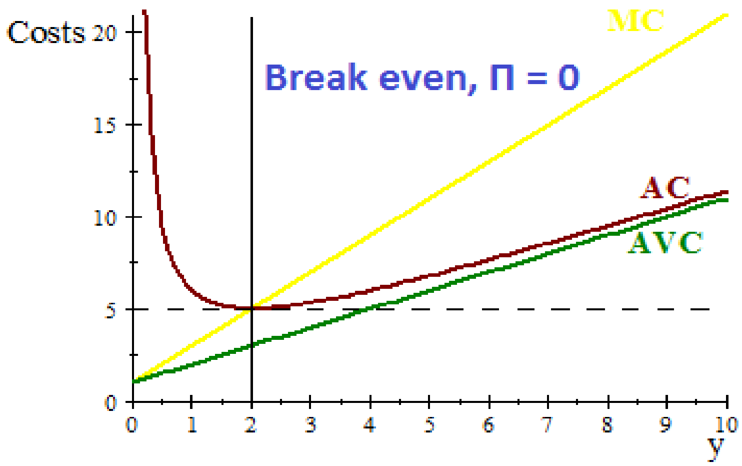
$\Pi_3 = py - C_3(y) = 5 * 1.5 - (1.5^2 + 2 * 1.5 + 4) = 7.5 - 9.25 = -1.75$, Loss.

(c) Show each firm's profit on your graphs. (6 pts)

For Firm 1:



For Firm 2:



For Firm 3:

(d) Write the shutdown condition and explain (in one sentence) why it will be reasonable for a firm to produce zero output. (3 pts)

The shutdown condition is, $p < AVC$ meaning that the revenue from selling y don't even cover the variable cost of production, thus, it will be better for a firm to produce zero output and pay the fixed cost in the short-run.

(e) Should all of these firms produce in the short-run? Explain. (4 pts)

Yes, all firms should produce in the short-run at $p = 5$ because $p > AVC$ for all of them, i.e. the revenue covers the variable cost in all three cases. Notice that, the MC curve is always above the AVC curve, hence $p = MC > AVC$. Even though Firm 3 is making a loss in the short-run ($\Pi_3 = -1.75$), it is better for it to produce since it can cover some part of the fixed cost.