

CONCORDIA UNIVERSITY
Department of Economics

ECON 222 SECTIONS A, B
STATISTICAL METHODS II
Fall 2018 – ASSIGNMENT 1

Due: Tuesday, October 9th, by 4:00 pm (in the instructor's mailbox)

1. **(4 marks)** Calculate the first derivative for the following functions.

a. **(2 marks)** $f(x) = \ln(x^2 + 3x)$

b. **(2 marks)** $f(x) = \frac{1}{\sqrt{x}}$

2. **(4 marks)** The population mean and variance of the random variable X are m and S^2 , respectively.

Prove that, for a sufficiently large sample size n , the sample mean $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ satisfies...

a. **(2 marks)** $E(\bar{x}) = m$

b. **(2 marks)** $\text{var}(\bar{x}) = \frac{1}{n} S^2$

3. (12 marks) Let X and Y be two random variables with a joint probability density function (PDF) of...

$$f(x, y) = 0.5, \quad 0 \leq x \leq y \leq 2$$

- a. (2 marks) Prove that $f(x, y)$ is a proper probability density function.
- b. (2 marks) Calculate the marginal PDF of X as well as its mean and variance.
- c. (2 marks) Calculate the marginal PDF of Y .
- d. (2 marks) Calculate the conditional PDF of X given $Y = 0.5$.

e. (2 marks) Calculate the conditional mean and variance of X given $Y = 0.5$.

f. (2 marks) Briefly explain whether X and Y are independent.

4. (2 marks) Let X have a normal distribution with mean m and variance S^2 . If $Y = aX + b$, where (a and b are constants), calculate the probability density function of Y .

5. (4 marks) A store manager designs a new accounting system that will be cost effective if the mean monthly charge account balance is more than \$170. A sample of 400 accounts is randomly selected. The sample mean balance is \$175 and the sample standard deviation is \$65. Can the manager conclude that the new system will be cost effective?

a. (2 marks) Test whether the new system is cost effective at the $\alpha = 0.05$ level of significance. Be sure to state the null and alternative hypotheses, calculate the appropriate test statistic, state the critical value and how you derive your conclusion.

b. (2 marks) Calculate the corresponding p -value of the test and briefly interpret it.

6. (8 marks) Suppose that Y_1, Y_2, \dots, Y_N is a random sample from a population with mean m and variance S^2 . Rather than using all N observations, consider an estimator of the population mean that uses only the first two observations, $Y^* = \frac{Y_1 + Y_2}{2}$.

a. (2 marks) Prove that Y^* is a linear estimator.

b. (2 marks) Prove that Y^* is an unbiased estimator.

c. (2 marks) Calculate the variance of Y^* .

d. (2 marks) Briefly explain why the sample mean of all N observations is a better estimator than Y^* .

7. (6 marks) Computer Exercise. Complete question C.20 on page 858.

8. (8 marks) Computer Exercise. Complete question C.22 on page 858.