

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	Date	Pages
Final	April 2012	3
Instructors	Course Examiner	
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Special Instructions		
<ul style="list-style-type: none"> ▷ Ruled booklets to be used. ▷ Only approved calculators are allowed. 		

MARKS

[9] 1. (a) Let $f(x) = \frac{x^2+x-6}{2x^2-3x-2}$. Find

$$\frac{2x^2 - 4x + x - 2}{2x(x-2) + 1(x-2)} = \frac{2x^2 - 3x - 2}{(2x+1)(x-2)}$$

(A) $\lim_{x \rightarrow 2} f(x)$

(B) $\lim_{x \rightarrow -1} f(x)$

(b) Given that $\lim_{x \rightarrow 2} g(x) = -3$ and $\lim_{x \rightarrow 2} h(x) = 3$, find the

$$\lim_{x \rightarrow 2} \sqrt{h(x) - g(x)}$$

(c) If $\lim_{x \rightarrow 4} k(x) = 4$, then $\lim_{x \rightarrow 5} k^2(x) = 16$. True or False. Explain your answer.

[10] 2. (a) If $g(x) = 2x^3 - 3x^4 - x\sqrt{x}$, find $g'(x)$.

(b) If $f(x) = \frac{x+1}{x^3}$, find $f'(x)$.

(c) If $y = e^\pi$, find y' .

(d) Find y' if $xy^2 = e^y$.

(e) If $y = \sqrt[3]{x^4 + 2}$, then $y' = ?$

$$\begin{aligned} x(2yy') &= e^y y' \\ 2xyy' &= e^y y' \\ \cancel{2xyy'} - e^y y' &= -y' \\ y'(2xy) &= e^y y' \end{aligned}$$

$$\begin{aligned} y'(\dots) &= 0 \\ y' &= \frac{0}{(\dots)} \end{aligned}$$

- [9] 3. Given the price-demand equation

$$x + 500p = 10,000$$

- (a) Express the demand x as a function of the price p .
- (b) Express the revenue R as a function of the price p .
- (c) Find the elasticity of demand, $E(p)$.
- [9] 4. Let $y = x^2 - 3x + 2$. Suppose $x = -3$, $\Delta x = dx = -0.1$.
- (a) Find Δy and dy . ??
- (b) Display Δy and dy on a graph of $f(x)$.

- [9] 5. The total profit (in dollars) from the sale of x lawn mowers is

$$P(x) = 30x - 0.03x^2 - 750 \quad 0 \leq x \leq 1,000$$

- (a) Find the average profit per mower if 50 mowers are produced.
- (b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.
- (c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.
- [9] 6. A point is moving on the graph of $x^2 + 2y^2 = 41$. When the point is at $(3, -4)$, its y co-ordinate is decreasing by 3 units per second. How fast is the x co-ordinate changing at that moment?

- [6] 7. Evaluate the following integrals [accurate to 2 decimals].

(a) $\int_1^3 (3x + 2e^x - \frac{7}{x}) dx$

(b) $\int_0^5 \frac{5x}{x^2 + 7} dx$

[12] 8. Compute the following:

(a) $\int e^{5x} dx$

(b) $\int (5x^6 - 2x^3) dx$

(c) $\int (x - 8)^{-8} dx$

(d) $\int (x^3 - ex) dx$

(e) $\int x(x^2 + 3)^{-5} dx$

(f) $\int \frac{5x^3}{7 - x^4} dx$

[9] 9. Find the area bounded by $f(x) = 2x^2$ and $g(x) = 4 - 2x$ for $-2 \leq x \leq 2$.

[9] 10. Use the graphing strategy to analyze the function

$$h(x) = \frac{4x}{x-2}$$

State all pertinent information and sketch the graph of h .

[4] 11. Suppose f is continuous at $x = 3$. Decide if f must be differentiable at $x = -3$.

[5] 12. Two countries have the same Lorenz Curve. What conclusion can you draw from this information?

Final Exam Solutions - April 2012.

1. A. i.

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 3x - 2} = \frac{4 + 2 - 6}{8 - 6 - 2} = \frac{0}{0} \text{ I.F.}$$

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 3x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{(x-2)(2x+1)} =$$

$$\lim_{x \rightarrow 2} \frac{x+3}{2x+1} = \frac{5}{5} = 1.$$

ii.

$$\lim_{x \rightarrow -1} \frac{x^2 + x - 6}{2x^2 - 3x - 2} = \frac{1 - 1 - 6}{2 + 3 - 2} = \frac{-6}{3} = -2.$$

$$B. \lim_{x \rightarrow 2} g(x) = -3; \lim_{x \rightarrow 2} h(x) = 3.$$

$$\lim_{x \rightarrow 2} (h(x) - g(x))^{1/2} = \left(\lim_{x \rightarrow 2} h(x) - \lim_{x \rightarrow 2} g(x) \right)^{1/2}$$

$$= (3 - (-3))^{1/2} = \sqrt{6}.$$

C. False. It would depend on the function $h(x)$.

If $h(x) = 4$ then true; if $h(x) = -4x + 20$ then false.

2. A.

$$g(x) = 2x^3 - 3x^4 - x\sqrt{x}$$

$$g(x) = 2x^3 - 3x^4 - x^{3/2}$$

$$g'(x) = 6x^2 - 12x^3 - \frac{3}{2}x^{1/2}$$

B. $f(x) = \frac{x+1}{x^3}$

$$f'(x) = \frac{x^3(1) - (x+1)(3x^2)}{x^6} = \frac{-2x^3 - 3x^2}{x^6}$$

$$f'(x) = \frac{-2x - 3}{x^4}$$

C. $y = e^\pi$

$$y' = 0$$

D. $x y^2 = e^y$

Implicit Differentiation.

$$x(2yy') + y^2(1) = e^y y'$$

$$2xyy' + y^2 = e^y y'$$

$$2xyy' - e^y y' = -y^2$$

$$y'(2xy - e^y) = -y^2$$

$$y' = \frac{-y^2}{2xy - e^y}$$

E. $y = \sqrt[3]{x^4 + 2} = (x^4 + 2)^{1/3}$

$$y' = \frac{1}{3} (x^4 + 2)^{-2/3} (4x^3)$$

$$y' = \frac{4}{3} x^3 (x^4 + 2)^{-2/3}$$

3. $x + 500p = 10,000$

A. $x = -500p + 10,000$

B. $R = xp = (10,000 - 500p)p = 10,000p - 500p^2$

C.

$$E(p) = \frac{-p f'(p)}{f(p)}$$



$$= \frac{-p(-500)}{10,000 - 500p}$$

$$E(p) = \frac{500p}{10,000 - 500p}$$

(4/13)

4. $f(x) = x^2 - 3x + 2$. $x = -3$; $\Delta x = dx = -.1$.

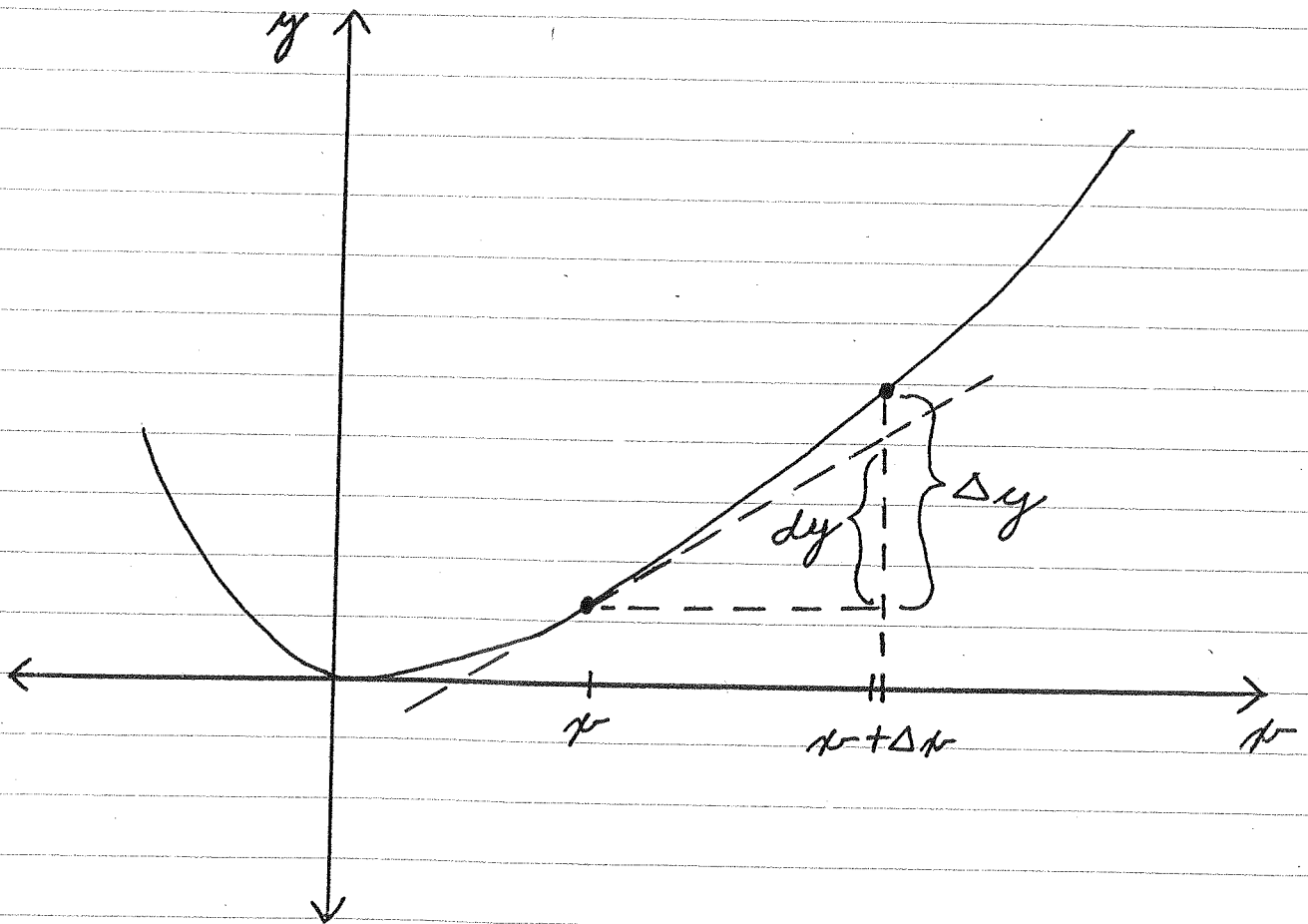
A. $\Delta y = f(-3.1) - f(-3) = \cancel{(2.31)} - (20.91) - (20)$

$\Delta y = .91$.

$dy = f'(x) dx$

$dy = (2x - 3) dx = (2(-3) - 3)(-.1) = .9$.

B.



(5/13)

$$5. P(x) = 30x - .03x^2 - 750 \quad 0 \leq x \leq 1,000$$

$$A. \bar{P} = \frac{P}{x} = \frac{30x - .03x^2 - 750}{x} = 30 - .03x - \frac{750}{x}$$

$$\bar{P}(50) = 30 - .03(50) - \frac{750}{50} = \$13.5$$

$$B. \bar{P}'(x) = -.03 + \frac{750}{x^2}$$

$$\bar{P}'(50) = -.03 + \frac{750}{(50)^2} = .27$$

average profit is increasing at a rate of 27
\$ after 50 mowers produced.

$$C. \bar{P}(51) \approx \bar{P}(50) + \bar{P}'(50)$$

$$\downarrow \approx 13.5 + .27$$

$$\bar{P}(51) \approx \$13.77$$

6/13

$$6. \quad x^2 + 2y^2 = 41. \quad (3, -4); \quad \frac{dy}{dt} = -3; \quad \frac{dx}{dt} ?$$

Related Rates Problem.

$$2x \frac{dx}{dt} + 4y \frac{dy}{dt} = 0$$

$$4y \frac{dy}{dt} = -2x \frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(\frac{-2y}{x} \right) \frac{dy}{dt} = \left(\frac{-2(-4)}{3} \right) (-3) = -8 \text{ units/sec.}$$

$$7. A. \int_1^3 \left(3x + 2e^x - \frac{7}{x} \right) dx$$

$$= \left(\frac{3x^2}{2} + 2e^x - 7 \ln|x| \right) \Big|_1^3$$

$$= \left(\frac{27}{2} + 2e^3 - 7 \ln(3) \right) - \left(\frac{3}{2} + 2e - 0 \right)$$

$$= 12 + 2e^3 - 7 \ln(3) - 2e$$

$$= 39.04422$$

$$\approx 39.04.$$

$$B. \int_0^5 \frac{5x}{x^2+7} dx.$$

$$\text{Let } u = x^2 + 7$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$dx = \frac{du}{2x}$$

$$\therefore \int_0^5 \frac{5x}{u} \left(\frac{du}{2x} \right) = \frac{5}{2} \int_0^5 \frac{du}{u}$$

$$= \frac{5}{2} \ln|u| \Big|_0^5$$

$$= \frac{5}{2} \ln|x^2+7| \Big|_0^5$$

$$= \frac{5}{2} [\ln(32) - \ln(7)]$$

$$= 3.799564$$

$$\approx 3.80.$$

$$8. A. \int e^{5x} dx = \frac{1}{5} e^{5x} + C.$$

$$B. \int (5x^6 - 2x^3) dx = \frac{5}{7} x^7 - \frac{x^4}{2} + C.$$

$$C. \int (x-8)^{-8} dx \quad \text{Let } u = x-8 \\ du = dx$$

$$\int u^{-8} du = \frac{u^{-7}}{-7} = \frac{(x-8)^{-7}}{-7} + C.$$

$$D. \int (x^3 - ex) dx = \frac{x^4}{4} - \frac{e}{2} x^2 + C.$$

$$E. \int x(x^2+3)^{-5} dx \quad \text{Let } u = x^2+3 \\ du = 2x dx \\ dx = \frac{du}{2x}$$

$$\int x u^{-5} \left(\frac{du}{2x} \right) = \int \frac{1}{2} u^{-5} du = \frac{1}{2} \int u^{-5} du$$

$$= \left(\frac{1}{2} \right) \frac{u^{-4}}{-4} = -\frac{1}{8} (x^2+3)^{-4} + C.$$

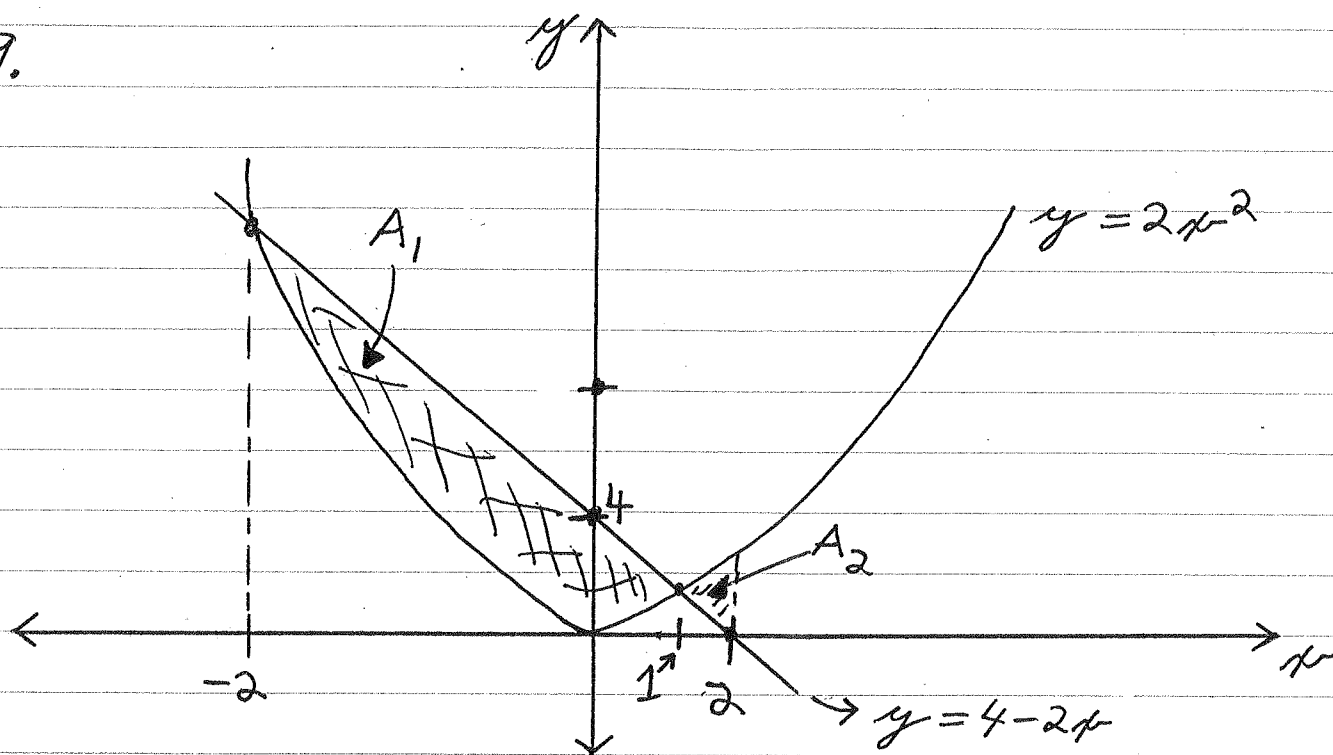
$$F. \int \frac{5x^3}{7-x^4} dx \quad \text{Let } u = 7-x^4 \\ du = -4x^3 dx \\ dx = \frac{du}{-4x^3}$$

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$$\int \frac{5x^3}{u} \left(\frac{du}{-4x^3} \right) = -\frac{5}{4} \int \frac{du}{u} = -\frac{5}{4} \ln|u|$$

$$= -\frac{5}{4} \ln|7-x^4| + C.$$

9.

Total area: A .

$$A = A_1 + A_2 \quad ?$$

$$A_1 = \int_{-2}^2 (\text{T.C.} - \text{B.C.}) dx = \int_{-2}^2 [(4-2x) - (2x^2)] dx$$

$$= \int_{-2}^2 (4-2x-2x^2) dx = \left(4x - x^2 - \frac{2x^3}{3} \right) \Big|_{-2}^2$$

$$A_1 = \left(4 - 1 - \frac{2}{3} \right) - \left(-8 - 4 + \frac{16}{3} \right) = 15 - 6 = 9.$$

$$A_2 = \int_1^2 [\text{T.C.} - \text{B.C.}] dx = \int_1^2 [(2x^2) - (4 - 2x)] dx$$

$$= \int_1^2 (2x^2 + 2x - 4) dx = \left(\frac{2x^3}{3} + x^2 - 4x \right) \Big|_1^2$$

$$= \left(\frac{16}{3} + 4 - 8 \right) - \left(\frac{2}{3} + 1 - 4 \right) = \frac{14}{3} - 1 = \frac{11}{3}$$

\therefore

$$A = A_1 + A_2 = 9 + \frac{11}{3} = \frac{38}{3}$$

10.

$$f(x) = \frac{4x}{x-2}$$

Domain: $\mathbb{R}/2$

x-intercept: 0

y-intercept: 0

V.A.: $x=2$

H.A.: $y=4$

C.P.: \emptyset

P.I.: \emptyset

Increasing: \emptyset

Decreasing: $(-\infty, 2); (2, \infty)$

No absolute max. or min. points

No local max. or min. points.

C.V. $(2, \infty)$

C.D. $(-\infty, 2)$

11/13

$$f(x) = \frac{4x}{x-2}$$

C.P.P

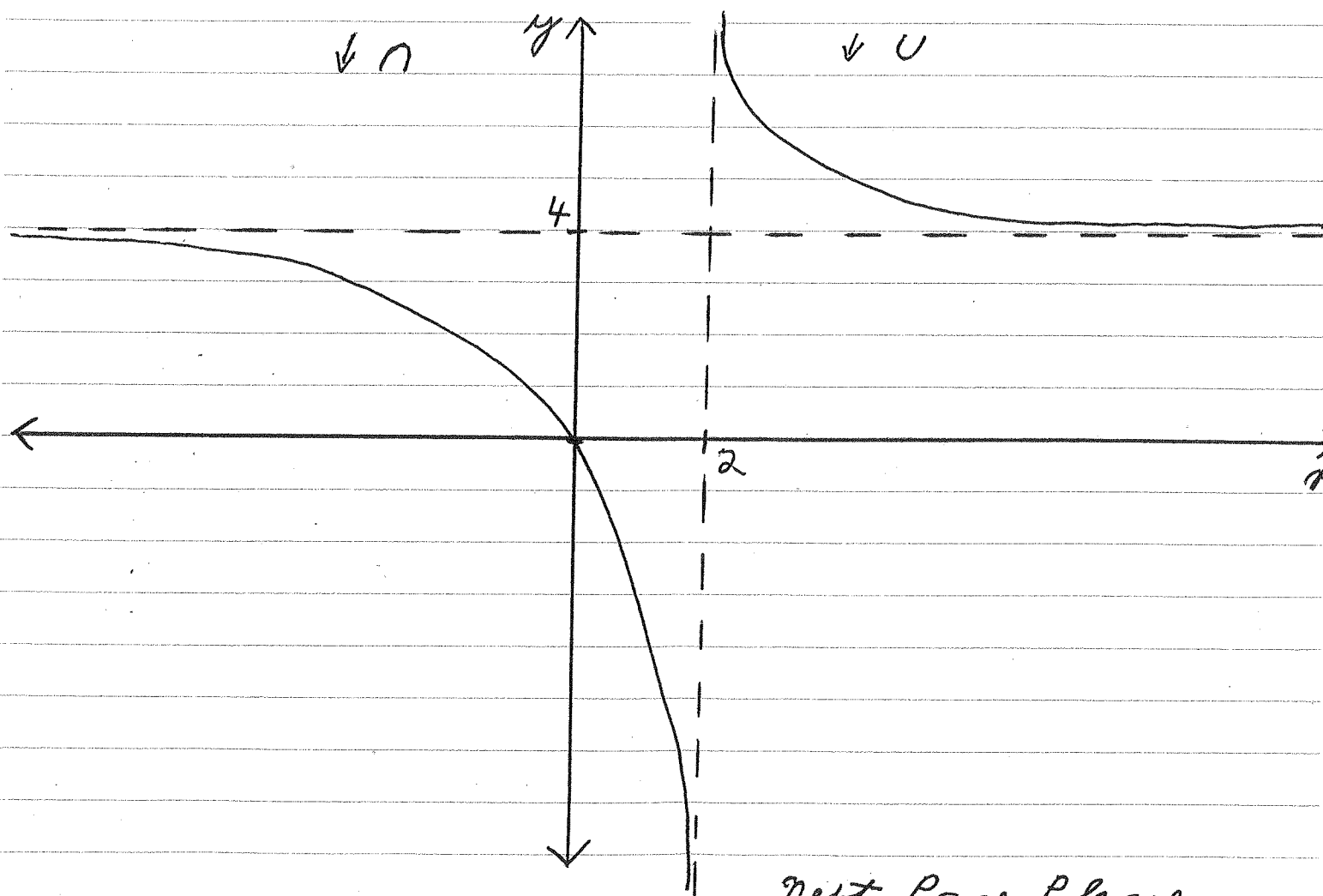
$$f'(x) = \frac{(x-2)(4) - (4x)(1)}{(x-2)^2} = \frac{-8}{(x-2)^2} = 0 \Rightarrow \text{no C.P.}$$

P.I.?

$$f'(x) = -8(x-2)^{-2}$$

$$f''(x) = 16(x-2)^{-3} (1) = 0$$

$$\frac{16}{(x-2)^3} = 0 \Rightarrow \text{no P.I.}$$



Next Page, Please.

No C.P., can not use S.D.T. Must use Concavity Test and F.D.T.

F.D.T.

$$f'(0) < 0 \downarrow$$

$$f'(3) < \downarrow$$

Concavity Test

$$f''(0) < 0 \text{ C.D. } \cap$$

$$f''(3) > 0 \text{ C.V. } \cup$$

11. Undecidable! Whether the function $f(x)$ is differentiable at $x = -3$, is independent of $f(x)$ being continuous at $x = 3$.
12. The conclusion being that the two countries have the same Income Distribution. The Gini Index's are the same.