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Problem 1

$$G(s) = \frac{1}{Js^2+Bs}$$

(1) $F(s) = K\,E(s)$

Closed - loop transfer function

$$\frac{Y}{R} = \frac{KG}{1+KG} = \frac{K}{Js^2+Bs+K}$$

$$\xi = \frac{B}{2\sqrt{JK}} \quad ; \qquad K = \frac{B^2}{4J\xi^2}$$

The system does not oscillate (overdamped) for $\xi > 1$ that is

For $\xi = 1 \Rightarrow K = \frac{B^2}{4J}$ critical value

For $\xi > 1 \Rightarrow K < \frac{B^2}{4J} \Rightarrow \boxed{K < \frac{B^2}{4J} = 0.25}$

(2) With PD controller the characteristic polynomial becomes

$$Js^2 + (B+K_d)s + K = 0$$

$$\xi = \frac{B+K_d}{2\sqrt{JK}} = 0.45$$

$$\omega_n = \sqrt{\frac{K}{J}} = 3$$

$$\Rightarrow \boxed{K = \omega_n^2 J = 4 \times 9 = 36}$$

$$\Rightarrow \boxed{K_d = 2\xi\sqrt{KJ} - B = 8.8}$$

Problem 2 : Note this solution refers to J=2; the text of the exam contained the typo J=4

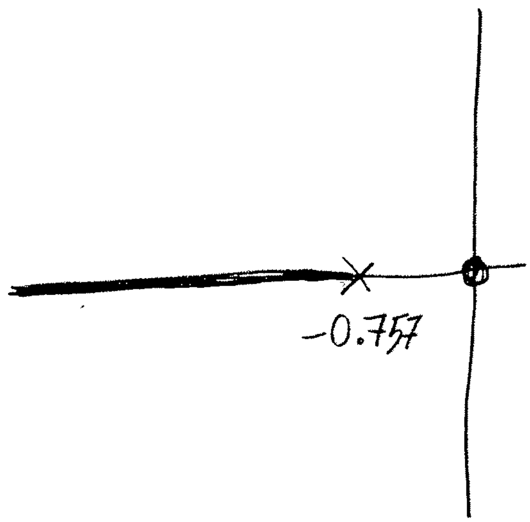
Characteristic polynomial

$$Js^3 + (B+K_d)s^2 + Ks + K_i = 0$$

$$a(s) = Js^3 + Bs^2 + Ks + K_i$$
$$b(s) = s^2$$

$$1 + \frac{K_d s^2}{Js^3 + Bs^2 + Ks + K_i} = 0$$

(2) Root loci exist on the left of an odd number of open-loop poles and zeros: 3



- two zeros at $s=0$
- one pole at $s=-757$

$\Rightarrow$ Root loci exist on the real interval

$$\boxed{s \in (-\infty, -0.757]}$$

Real roots $s=0$, $s=2.63$

$\Rightarrow$ $\boxed{s=0}$ is a break in/break away since it is real and it belongs to the root locus plot

NOT INCLUDED IN THE EXAM

(3) $n=3$, $m=2$

Angle: $\gamma_0 = \frac{180}{1} = 180°$

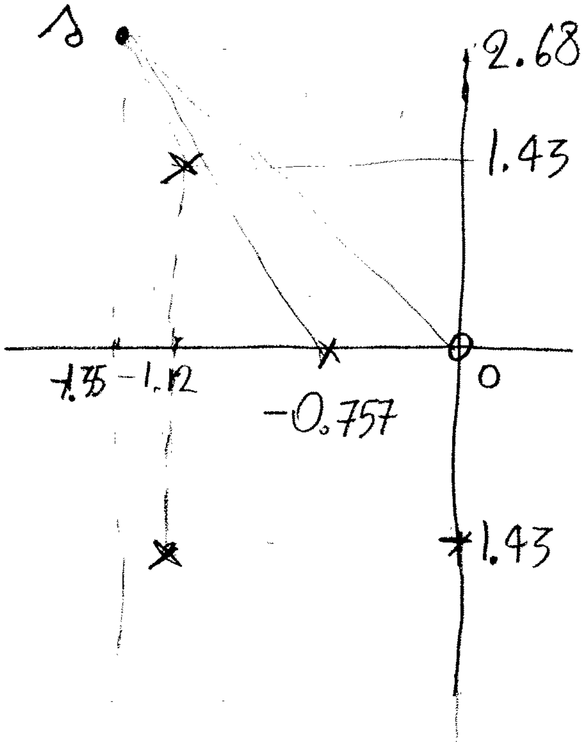
(4)

$\xi = 0.45$

$\omega_n = 3$

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -1.35 \pm j2.68$$

$$\angle \frac{b(s)}{a(s)}\Big|_{s} = 2\angle s - \angle s+0.757 - \angle s+1.12 \pm 1.43j$$



$$\angle s = 90° + \tan^{-1}\left(\frac{1.35}{2.68}\right) = 90° + 26.7° = 116.7°$$

$$\angle s+0.753 = 90° + \tan^{-1}\left(\frac{1.35-0.757}{2.68}\right) = 90° + 12.5° = 102.5$$

$$\angle \bar{s}+1.12+1.43j = 90^\circ + \tan^{-1}\left[\frac{1.35-1.12}{2.68+1.43}\right] =$$

$$= 90^\circ + 3.2^\circ = 93.2^\circ$$

$$\angle \bar{s}+1.12-1.43j = 90^\circ + \tan^{-1}\left[\frac{1.35-1.12}{2.68-1.43}\right] =$$

$$= 90^\circ + 10.4^\circ = 100.4^\circ$$

Therefore

$$\angle \frac{b(\bar{s})}{a(\bar{s})} = 2(116.7^\circ) - (102.5^\circ + 93.2^\circ + 100.4^\circ)$$

$$= -62.6^\circ \neq \pm 180^\circ(2K+1) \quad \forall K$$

$\Longrightarrow$ $\bar{s}$ does not belong to the root locus

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(5) The phase contribution equals the opposite angle def.

$$\psi = 180^\circ - \angle \frac{b(\bar{s})}{a(\bar{s})} = +180^\circ - (297.4^\circ) = -117.4^\circ$$

$$\boxed{\left| \angle G_c(\bar{s}) \right| = 117.4^\circ}$$

(4) – Alternative solution of point (4)

$$a(\bar{s}) = 2\bar{s}^3 + 6\bar{s}^2 + 10\bar{s} + 5 = 12.6 - 25.8j$$

$$b(\bar{s}) = -5.35 - 7.23j$$

$$\frac{b(\bar{s})}{a(\bar{s})} = 0.144 - 0.278j$$

$$\angle \frac{a(\bar{s})}{b(\bar{s})} = \tan^{-1}\left[\frac{Im\left(b(\bar{s})/a(\bar{s})\right)}{Re\left(b(\bar{s})/a(\bar{s})\right)}\right] = -62.6^\circ$$

Note: with the typo $J = 4$ we would obtain

$$a(\bar{s}) = 65.8 - 34.9j$$

$$\frac{b(\bar{s})}{a(\bar{s})} = -0.0179 - 0.119j$$

$$\angle \frac{b(\bar{s})}{a(\bar{s})} = -98.5^\circ$$

The typo would not affect the answer obtained with the other procedure