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MCG 3307: Control Systems II - Mid-term exam

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Policy

The present exam is closed book and closed notes. Illegible work and loose sheets will not be graded. All electronic devices, with the exception of non programmable calculators, must be turned off during the exam.

Problem 1

Consider the translational mechanical system in Fig. 1. This simple model is ofter used in engineering and science to describe the dynamics systems: for example it can be used to describe the motion of a car in appropriate conditions. The governing equation of the system is

$$M\ddot{y}(t) = f_B(t) + f(t) \tag{1}$$

where the external force associated with the damper is $f_B(t) = -B\dot{y}$, and $f(t)$ is the external force exerted by a automatic controller to the system. Define the Laplace transforms of the displacement y and of the control force f to be, respectively, $Y(s)$ and $F(s)$. Assume homogeneous initial conditions. The Laplace transformed governing equation is then $Ms^2Y(s) + BsY(s) = F(s)$, and therefore the transfer function $G(s)$ between the displacement $Y(s)$ and the control force $F(s)$ is

$$G(s) = \frac{1}{Ms^2 + Bs} \tag{2}$$

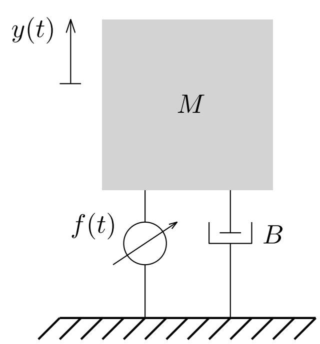

Fig. 1: Sketch of a translational mechanical system (oscillating mass) with external force $f(t)$ exerted by a automatic controller.

1. (**4pt**) Consider the control force to be the output of a proportional controller, with the error in the Laplace domain given by $E(s) = R(s) - Y(s)$. For $M = 4$ and $B = 2$ determine the range of the proportional controller gain K such that the system does not oscillate. (*Hint: consider the closed loop transfer function and base your analysis on it*)
2. (**4pt**) Consider a proportional plus derivative controller (PD controller) in the feed forward, and find the proportional and the derivative gains, K and K_{d}, such that the damping ratio ζ and the undamped natural frequency ω_n equal, respectively, 0.45 and 3 rad/s.

Problem 2

Consider the system in Problem 1 with $F(s)$ given by the output of a proportional-integral-derivative (PID) controller in the feed-forward with gains K, K_{i}, and K_{d} respectively. The reference input $R(s)$ is a desired displacement, and therefore the feedback gain is unity. The characteristic polynomial is $Ms^3 + (B + K_{\mathrm{d}})s^2 + Ks + K_{\mathrm{i}} = 0$. Assume $M = 2$, $B = 6$, $K = 10$, $K_{\mathrm{i}} = 5$.

1. (**2pt**) By isolating K_{d} identify the polynomials $a(s)$ and $b(s)$ to write the characteristic polynomial in the form

$$1 + K_{\mathrm{d}}\frac{b(s)}{a(s)} = 0 \tag{3}$$

2. (**4pt**) The open loop poles are located at

$$s = -0.757, \quad s = -1.12 \pm 1.43j \tag{4}$$

   Determine the intervals of existence of root loci on the real axis.
3. (**2pt**) Find the angles of the asymptote(s) of the root locus plot.
4. (**4pt**) Consider a pair of complex conjugates points $\bar{s}_{1/2}$ (in the stable region of the s plane) determined by the values of ζ and ω_n given in Point 2 in Problem 1, and find if they belong to the root loci of $G(s)$. (*Hint: use the phase condition*)
5. (**2pt**) Determine the phase contribution ψ of a lead compensator with transfer function $G_{\mathrm{c}}(s)$ such that $\bar{s}_{1/2}$ belong to the root locus of $G_{\mathrm{c}}G(\bar{s}_{1/2})$.