Linear Assignment #4

Question 1.

a) 3(-1-2i)-(-4)(1+3i)

=-3-6i+4+12i

=6i+1

b) (-1-2i) (1+3i)

=-1 -3i -2i -6i^2

= -6i^2-5i-1

= 6 -5i -1

= -5i -5

c) z× $\frac{1}{w}$ = z ×$\frac{\vec{w}}{|w^2|}$

= (-1 -2i) × $\frac{1+3i}{1+3i^2}$

= $\frac{-1-3i-2i-6i^2}{10}$

=$\frac{5-5i}{10}$

=$\frac{1}{2}-\frac{1}{2}i$

d)

Question 2

a)$(3-\lambda)\begin{bmatrix}3-\lambda & -1\\ -1 & 3-\lambda\end{bmatrix}+\begin{bmatrix}-1 & -1\\ -1 & 3-\lambda\end{bmatrix}-\begin{bmatrix}-1 & 3-\lambda\\ -1 & -1\end{bmatrix}$

=(3-λ) $[(3-\lambda)(3-\lambda)-(-1)(-1)]+[(-1)(3-\lambda)-(-1)(-1)]-[(-1)(-1)-(-1)(3-\lambda)]$

=$(3-\lambda)(\lambda^2-6\lambda+8)-2(4-\lambda)$

=$3\lambda^2-18\lambda+24-\lambda^3+6\lambda^2-8\lambda-8+2\lambda$

$=-\lambda^3 + 9\lambda^2 - 24\lambda + 16$

$= (1-\lambda)\dfrac{-\lambda^2 + 8\lambda - 16}{-\lambda^3 + 9\lambda^2 - 24\lambda + 16}$

$=(1-\lambda)(-\lambda^2 + 8 - 16)$

$=(-\lambda^2 + 4\lambda + 4\lambda - 16)$

$=(\lambda - 4)^2\ (\lambda - 1)$

Eigen values are: 1, 4 with a multiplicity
y of 2.

b) $\left[A - 1I\middle|\vec{0}\middle|\right] \begin{bmatrix} 3 & -1 & 3 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \Rrightarrow \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} Reduce \Rrightarrow R_1 \leftrightarrow R_3 \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{bmatrix}$

$2R_1 + R_3, R_1 - R_2 \begin{bmatrix} -1 & -1 & 2 \\ 0 & -3 & 3 \\ 0 & -3 & 3 \end{bmatrix}$

$R_2 - R_3 \begin{bmatrix} -1 & -1 & 2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}, \qquad R_2 x - \frac{1}{3} \begin{bmatrix} -1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$R_2 + R_1 \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad R_1(-1) \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = x_3$

$x_2 = -x_3 \qquad\qquad x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} x_3 any\ scalar.$ This is the basis.

$x_3 = free$

$\left[A - 4I\middle|\vec{0}\middle|\right] \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \Rrightarrow \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

$R_1 - R_2, R_1 - R_3 \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = -x_2 - x_3$

$x_2 = free$

$x_3 = free$

Basis= $x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $x_2, x_3 any\ scalar$

$c) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$