Study Guide Exam 1

Information Midterm 1:

- Wednesday, October 3
- 5:30-6:50 PM (normal lecture time and place)
- Topics on the Exam: lectures 1–6, textbook sections 1.1–2.4.

The following list of topics is not exhaustive, it is just meant to highlight the most important aspects of each topic.

Topics to Keep in Mind:

- Many problems will require use of algebra and trigonometry reviewed in the "Supplemental material" posted to Brightspace; e.g., factorization of polynomials, properties of exponents, and important values of trig functions..
- Solving inequalities, for example $2x - 5 \leq 8$.
- Understanding the definition of absolute value as a piecewise functions, e.g. $|x - 7|$;
- Properties of lines (parallel, perpendicular, increasing, decreasing); finding equations of a line (using the point-slope as well as slope intercept formulas).
- You should also know the general quadratic formula. This formula states that if $b^2 - 4ac \geq 0$ then the solutions of the equation $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Basic facts about functions: finding domains of functions, performing algebraic operations on functions (like adding, subtracting, multiplying, dividing and composing functions)
- Limit Techniques: simplifying an expression before evaluating the limit, calculating one-sided limits (especially useful to study functions that have absolute values and piece-wise defined functions), limits at infinity.

- You should know what it means for a function to be continuous at $x = a$, that is, that $\lim_{x\to a} f(x) = f(a)$.

- You should know the list of continuous functions given in class, and that continuity is preserved under the elementary algebraic operations, for example, that the sum of two continuous functions is a (new) continuous function, the product of continuous functions is a (new) continuous function, etc.

- You should know how to find a general formula for the average rate of change of a function on an interval, particular on intervals of the form $[a, a+h]$, and that the average rate of change of a function on an interval is the slope of the secant line to the function on that interval.

- **You should know the limit definition of the derivative of a function $f(x)$.** That is, if asked "write the definition of the derivative of a function $f(x)$" you should write "the derivative of f at x is given by $f'(x) = \lim_{h\longrightarrow 0} \frac{f(x+h)-f(x)}{h}$". If asked to find a derivative using the limit definition, you can't use any of the differentiation rules to find it; you must find $\lim_{h\longrightarrow 0} \frac{f(x+h)-f(x)}{h}$ using the limit laws and techniques.

- Geometric interpretation of derivatives in terms of slopes of tangent lines, namely, if $y = f(x)$ is the curve representing the graph of a function $f(x)$ then $f'(x)$ is the slope of the tangent line to the curve going through the point $(x, f(x))$ (which is the instantaneous rate of change of $f(x)$ at the point x).

- Interpretation of Derivatives as rates of change, namely, if $y(x)$ describes a quantity y as a function of a quantity x then $\frac{dy}{dx}$ is the (instantaneous) rate of change of y with respect to x. For example, if $x(t)$ is the position of a particle then $\frac{dx}{dt}$ is the (instantaneous) rate of change of the position x with respect to the time t, that is, $\frac{dx}{dt}$ is the velocity of the particle. (If, e.g., x is expressed in meters and t in seconds, then the units of dx/dt would be meters per second.)

- Differentiation Rules:
  - (i) $\frac{d}{dx}[c] = 0$ (c, a constant)
  - (ii) $\frac{d}{dx}[x^n] = nx^{n-1}$ for every real number n
  - (iii) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$
  - (iv) $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$
  - (v) $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

- Product and Quotient Rules
  - (vi) Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$
  - (vii) Quotient Rule: $\frac{d}{dx}\left[\frac{hi(x)}{lo(x)}\right] = \frac{lo(x)\frac{d}{dx}[hi(x)] - hi(x)\frac{d}{dx}[lo(x)]}{(lo(x))^2}$

- Chain Rule

(viii) General Chain Rule: $\frac{d}{dx}[g(f(x))] = g'(f(x))f'(x)$ or

$$\frac{d}{dx}[g(f(x))] = g'(f(x))\frac{d}{dx}[f(x)].$$

(ix) Consequence "General Power Rule" (or "The Power Rule for Functions"): $\frac{d}{dx}\left[\left(f(x)\right)^n\right] = n\left(f(x)\right)^{n-1}\frac{d}{dx}[f(x)]$.

Most Common Calculus Exam Mistakes

1. **MISTAKE**: $(x+y)^2 = x^2+y^2$. Powers don't behave that way. The correct way to expand this expression gives

$$(x+y)^2 = x^2+2xy+y^2 \tag{1}$$

2. **MISTAKE**: $\frac{1}{x+y} = \frac{1}{x}+\frac{1}{y}$. The rule for adding fractions gives

$$\frac{1}{x}+\frac{1}{y} = \frac{x+y}{xy} \tag{2}$$

3. **MISTAKE**: $\frac{1}{x+y} = \frac{1}{x}+y$. This error comes from carelessness about what's in the denominator

4. **MISTAKE**: $\sqrt{x+y} = \sqrt{x}+\sqrt{y}$. There is no simplified way to write $\sqrt{x+y}$

5. **MISTAKE**: If $x < y$ then $kx < ky$ where k is *any* constant. This is is true only when k is a positive constant. If k is negative you need to reverse the inequality.

6. **MISTAKE**: $\left|\frac{x-2}{x+1}\right| = 3$ implies $\frac{x-2}{x+1} = 3$: again this is partially correct because you must also work with the equation $\frac{x-2}{x+1} = -3$. The first equation $\frac{x-2}{x+1} = 3$ gives $x = -5/2$ and the second equation $\frac{x-2}{x+1} = -3$ gives $x = -1/4$ so the complete solution is $x = -5/2$ or $x = -1/4$

7. **Forgetting to simplify fractions in limits**: It is not correct to say that $\lim_{x\longrightarrow 1}\frac{x^2-1}{x-1} = \frac{0}{0}$ and therefore the limit is undefined. Precisely the point here is that before evaluating a limit you have to do some algebraic manipulation. For example

$$\lim_{x\longrightarrow 1}\frac{x^2-1}{x-1} = \lim_{x\longrightarrow 1}\frac{(x-1)(x+1)}{x-1} = \lim_{x\longrightarrow 1}(x+1) = 2 \tag{3}$$

8. **MISTAKE**: $ax = bx$ implies $a = b$. This is fine only if x is not 0. For example $2x = 3x$ implies that $x = 0$ not that $2 = 3$.

9. **MISTAKE**: $\frac{d}{dx}\left(2^{\pi}\right) = \pi 2^{\pi-1}$. 2^{π} is just a number so its derivative must be 0

$$\frac{d}{dx}\left(2^{\pi}\right) = 0 \tag{4}$$

10. **NOT USING ONE-SIDED LIMITS FOR PROBLEMS WITH CONTINUITY**: For example, if you want to find the value c that makes $J(x) = \begin{cases} 2x^2 + cx - 1 & \text{if } x < 1 \\ \sqrt{x+3} & \text{if } x \geq 1 \end{cases}$ continuous you can't just evaluate both formulas at 1 and say that $2\left(1\right)^2 + c\left(1\right) - 1 = \sqrt{1+3}$. What you need to set equal are the respective lateral limits and the value of $J(1)$, that is, you must solve

$$\lim_{x \longrightarrow 1^-} J(x) = \lim_{x \longrightarrow 1^+} J(x) = J(1) \tag{5}$$

which ends up giving in the end the original equation you were trying to solve.

11. **NOT USING THE DEFINITION OF THE DERIVATIVE WHEN ASKED TO USE IT**: if a problem says find $f'(x)$ using the definition of the derivative, they mean compute the limit

$$\lim_{h \longrightarrow 0} \frac{f(x+h) - f(x)}{h} \tag{6}$$

You shouldn't find it with the rules of the derivatives.

12. **MISTAKE**: $(f(x)g(x))' = f'(x)g'(x)$. Remember that the correct product rule for derivatives is

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x) \tag{7}$$

13. **MISTAKE**: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)}{g'(x)}$. Remember that the correct quotient rule for derivatives is

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2} \tag{8}$$

14. **MISREADING THE PROBLEM**: it happens a lot. After you finish a problem on the exam, go back and read the question again.

15. **NOT USING COMMON SENSE:** for example, if the problems asks you to find the area of a region it can't be the case that your final answer is a negative number. Always check that your answer is plausible. For example, it would be strange that if you are asked the volume of a sphere of radius one feet you end up saying that it is 10 trillion cubic feet.

16. **DIFFERENT NOTATIONS FOR THE DERIVATIVES**: you should know the following ways to calculate the derivative of a function $y = f(x)$ (these are all

the same, the only difference is their notation)

$$\begin{cases} f'(x) = \lim_{h \longrightarrow 0} \frac{f(x+h)-f(x)}{h} \\ \\ f'(a) = \lim_{h \longrightarrow 0} \frac{f(a+h)-f(a)}{h} \\ \\ f'(x) = \frac{dy}{dx} = \frac{df}{dx} = \lim_{\triangle x \longrightarrow 0} \frac{\triangle y}{\triangle x} = \lim_{\triangle x \longrightarrow 0} \frac{y(x+\triangle x)-y(x)}{\triangle x} = \lim_{\triangle x \longrightarrow 0} \frac{f(x+\triangle x)-f(x)}{\triangle x} \end{cases} \tag{9}$$