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Assignment1 Comp 335

(1). a. $L^2 \subseteq L$ if and only if $L = L^+$

1. If $L = L^+$ then $L^2 \subseteq L$

L is any Language over an alphabet

$L^2 = L.L$

$L^+ = L^1 \cup L^2 \cup L^3 \dots$

$LL = L^2 \subseteq L^+$

but $L^+ = L$

$\Rightarrow L^2 \subseteq L$

2 If $L^2 \subseteq L$ then $L = L^+$

Base case $L \subseteq L$

Assume $L^2 \subseteq L$

then $L^n \subseteq L$

$L^{n+1} = L.L^n$

$L^{n+1} \subseteq L.L$

$L^{n+1} \subseteq L^2$

$L^{n+1} \subseteq L$

for $n \geq 1$ $L^n \subseteq L$ therefore $L^+ = L^1 \cup L^2 \cup L^3 \dots L^n \subseteq L$

it is also true that $L \subseteq L^+$ $n \geq 1$

$\Rightarrow L^+ = L$

b If $L \subseteq R$ then $\min\{|x| : x \in L\} \geq \min|y| : y \in R\}$

Proof by contradiction If $L \subseteq R$ then $\min\{|x| : x \in L < \min|y| : y \in R\}$

1 if $L \subseteq R$ the $w \in L$ and $w \in R$

$\min|w| = \min|w|$ not $<$ since $w \in L$ and $w \in R$

2 If $L \subseteq R$, assume $R = L \cup \{w_i, w_i \notin L\}$. $L = w_1 w_2 \dots w_n$

$\min|x| \in L > \min|w_i| . w_i \in R$.

2. a $L_1L_2 = L_2$

- $L_1 = \{\lambda\}$
  $L_2 = \omega_1\omega_2 \dots \omega_n$
  $L_1L_2 = \lambda\omega_1\omega_2 \dots \omega_n = \omega_1\omega_2 \dots \omega_n = L$

- $L_1 = \emptyset$
- $L_2 = \omega_1\omega_2 \dots \omega_n$
  $L_1L_2 = \omega_1\omega_2 \dots \omega_n$

- $L_1 = \{a^n \quad n \geq 0\}$
  $L_2 = \{a^m b^p \quad m, p \geq 0\}$
  $L_1L_2 = \{a^n . a^m b^p \quad m, n, p \geq 0\}$
  $= \{a^{n+m} b^p \quad m, n, p \geq 0\}$
  $= \{a^Q b^p \quad Q, p \geq 0\}$. (integer close under addition)

- $L_1 = \{a^m b^n \quad m, n \geq 0\}$
  $L_2 = \{b^p \quad p \geq 0\}$
  $L_1L_2 = \{a^m . b^n . b^p \quad m, n, p \geq 0\}$
  $= \{a^m b^{n+p} \quad m, n, p \geq 0\}$
  $= \{a^m b^Q \quad m, Q \geq 0\}$ (integer close under addition

b. $L_1\Sigma^* = L_1$

- $L_1 \{a^n \quad n \geq 0\}$
  $\Sigma^* \{\lambda, a, aa . aaa \dots\}$ $(\Sigma = \{a\})$
  $L_1\Sigma^* = \{a^n \lambda, a^n a, a^n aa \dots \quad n \geq 0\}$
  $= \{a^{n+0}, a^{n+1}, a^{n+2} \dots \quad n \geq 0\}$.
  $= \{a^m, a^m, a^m \dots \quad m \geq 0\} = L_1$

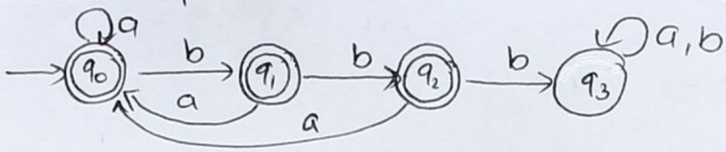
- $L_1 = \omega_1\omega_2 \dots \omega_n$
  $\Sigma^* = \emptyset^* = \{\lambda\}$
  $L_1\Sigma^* = \omega_1\omega_2 \dots \omega_n \lambda = \omega_1\omega_2 \dots \omega_n = L_1$

- $L_1 = w_1 w_2 \ldots w_n$

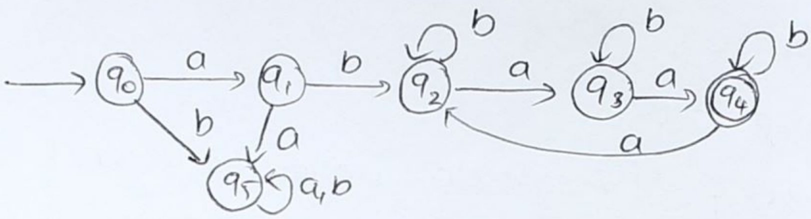
$\Sigma^* = \{\lambda\}^* = \{\lambda\}$

$L_1 = w_1 w_2 \ldots w_n \lambda = w_1 w_2 \ldots w_n = L_1$

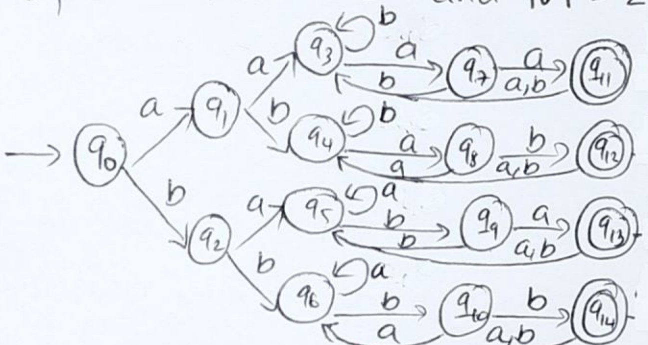
3. a. $L_1 = \{w : w \in \Sigma^*$ where w does not have bbb as a substrings



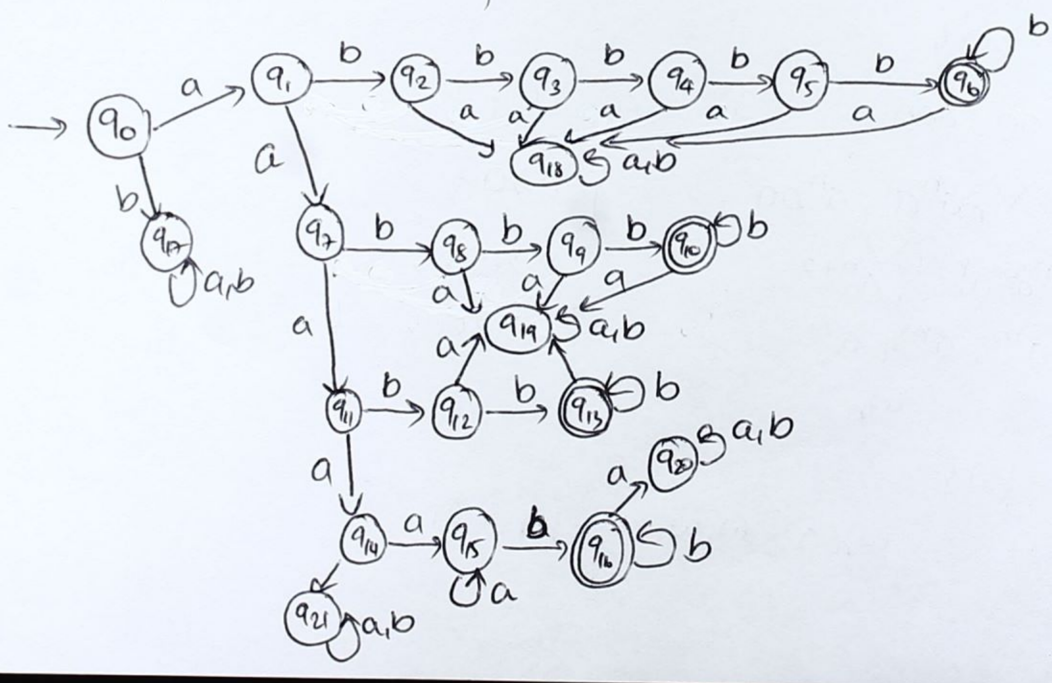
b. $L_2 = \{w : w \in \Sigma^+$ where $n_a(w) \bmod 3 = 0$ and w begins with ab$\}$



c. $L_3 \{vwv, v, w \in \Sigma^*$ and $|v| = 2\}$



d. $L_4 = \{a^m b^n \quad m.n > 4\}$



5. $L = \{a^n b^{n+1} : n \geq 0\}$

$\bar{L} = \Sigma^* - L$

$\bar{L}$ the set of all string over Σ except the string belong to $a^n b^{n+1}$ $n \geq 0$

$\bar{L} = \{\lambda, a^n, b^n, a^{n+1}b^n, b^n a^{n+1}, b^{n+1}a^n \dots\}$

4. Regular expression for $(L_1 \cup L_2)L_3$

L_1: $(a^* + ba + bba)^*(\lambda + b + bb)$

L_2: $ab\,b^*ab^*a(b^* + b^*ab^*ab^*a)^*$

L_3: $aa(a+b)^*aa + ab(a+b)^*ab + ba(a+b)^*ba + bb(a+b)^*bb$

$(L_1 \cup L_2)L_3 = ((a^* + ba + bba)^*(\lambda + b + bb) + abb^*ab^*a(b^* + b^*ab^*ab^*a)^*)$
$(aa(a+b)^*aa + ab(a+b)^*ab + ba(a+b)^*ba + bb(a+b)^*bb)$

7. NFA $L_1 \cup \bar{L}_3$